CHIRAL DYNAMICS, FIELD-CURRENT IDENTITY, AND THE K_{l3} FORM FACTORS

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Based on the notions of chiral dynamics and the field-current identity as applied to the broken SU(3), theoretical expressions for the form factors $f_\pm(t)$ are obtained. A small ξ ($|\xi| \ll 1$) and a large $|\lambda_-|$ (an order of magnitude larger than λ_+) are predicted. [However, the product $\xi\lambda_-$ is expected to be small.]

We report here the predictions of chiral dynamics[1] and the field-current identity[2] on the K_{l3} form factors, conventionally denoted by $f_\pm(t)$. The model we use to predict these form factors is a straightforward generalization of the phenomenological Lagrangian of, e.g., Lee and Nieh[1] to the chiral SU(3)⊗SU(3), broken in such a way that the SU(3)-symmetry-breaking part transforms like the $I=0$ member of an octet and the chiral SU(2)⊗SU(2) is broken only by the finite pion mass. We shall first present the results and discuss experimental implications thereof. We shall then discuss the model and outline the derivation of the results briefly [paragraph (III) below; the experimentally oriented reader may well skip this paragraph].

(I) Denoting the K_{l3} hadronic matrix element by

$$\langle\pi^-(q)|S_\mu(0)|K^0(p)\rangle = (p_+)_\mu f_+(p^2,q^2,t) + (p_-)_\mu f_-(p^2,q^2,t) \quad (1)$$

as usual, where $p_\pm = p \pm q$, and $t = p_-^2$, we have

$$f_+(p^2,q^2,t) = \frac{m_{K^*}^2}{m_{K^*}^2 - t}\left[\frac{1}{2}\left(\frac{f_K}{f_\pi}+\frac{f_\pi}{f_K}\right) - \frac{1}{2}\beta^2\frac{f_K}{f_\pi}(1+\delta)\frac{t}{m_{K^*}^2}\right], \quad (2)$$

$$f_-(p^2,q^2,t) = \frac{1}{2}\left(\frac{f_K}{f_\pi}-\frac{f_\pi}{f_K}\right) + \frac{p^2-q^2}{2m_{K^*}^2}\beta^2\frac{f_K}{f_\pi}(1+\delta) - \frac{p^2-q^2}{m_{K^*}} f_+(p^2,q^2,t), \quad (3)$$

where $\beta \equiv f_\pi g/m_\rho$, which is numerically close to $1/\sqrt{2}$,[3] and f_K and f_π are the kaon and pion decay constants, and δ is an arbitrary parameter,[4] which, from the decay widths of the ρ and A_1 mesons, is expected to be $\simeq -\frac{1}{3}$.[5] On the mass shell ($p^2 = m_K^2$, $q^2 = m_\pi^2$), we have

$$f_+(t) \cong \frac{m_{K^*}^2}{m_{K^*}^2 - t}\left(1 - \frac{1+\delta}{4}\frac{f_K}{f_\pi}\frac{t}{m_{K^*}^2}\right),$$

$$f_-(t) \cong \left(\frac{f_K}{f_\pi} - 1\right) + \frac{m_K^2 - m_\pi^2}{m_{K^*}^2} \times \left[\frac{1+\delta}{4}\frac{f_K}{f_\pi} - f_+(t)\right], \quad (4)$$

where we have neglected terms of second order or higher in the SU(3)-symmetry breaking. Expressions (2) and (3), in addition to exhibiting the K^* poles implied by the field-current identity,[2] satisfy (i) the Gatto-Ademollo[6] condition and (ii) the Callan-Treiman[7] condition:

(i) In the limit $p = q$, Eqs. (1)-(3) give

$$\lim_{p\to q}[p_+^\mu f_+(p^2,q^2,t) + p_-^\mu f_-(p^2,q^2,t)] = p_+^\mu \frac{1}{2}\left(\frac{f_K}{f_\pi}+\frac{f_\pi}{f_K}\right). \quad \text{(i)}$$

Note that $f_K/f_\pi = 1 +$ first order in the SU(3) breaking, so that $\frac{1}{2}(f_K/f_\pi + f_\pi/f_K) - 1$ is second order in the SU(3)-symmetry breaking.

(ii) In the soft-pion limit $q^\mu \to 0$, we have

$$\lim_{q_\mu\to 0}[p_+^\mu f_+(p^2,q^2,t) + p_-^\mu f_-(p^2,q^2,t)] = p^\mu (f_K/f_\pi) \quad \text{(ii)}$$

which is the result of Callan and Treiman.[7]

(II) The linear approximation to $f_+(t)$ of Eq. (4) gives

$$f_+(t) \simeq 1 + \lambda_+ t/m_\pi^2$$

with

$$\lambda_+ = \frac{m_\pi^2}{m_{K^*}^2}\left(1 - \frac{1+\delta}{4}\frac{f_K}{f_\pi}\right) \simeq 0.018, \quad (5)$$

where we used $f_K/f_\pi \simeq 1.28$, $\delta = -\frac{1}{3}$. (For $\delta = 0$, we obtain 0.016.) This is to be compared with[8] $\lambda_+(K^0) = 0.013 \pm 0.009$, $\lambda_+(K^+) = 0.023 \pm 0.008$.

The linear approximation to $f_-(t)$ of Eq. (4) gives

$$f_-(t) = \left[\left(\frac{f_K}{f_\pi} - 1\right) - \frac{m_K^2 - m_\pi^2}{m_{K^*}^2}\left(1 - \frac{1+\delta}{4}\frac{f_K}{f_\pi}\right)\right] - \frac{m_K^2 - m_\pi^2}{m_{K^*}^2}\frac{m_\pi^2}{m_{K^*}^2}\left(1 - \frac{1+\delta}{4}\frac{f_K}{f_\pi}\right)\frac{t}{m_\pi^2}. \quad (6)$$

Here we face the curious situation that the constant term and the pole term in Eq. (3) nearly cancel each other at $t = 0$ [or the two terms in the square bracket of Eq. (6)]. In the usual parametrization of $f_-(t)$,

$$f_-(t) = \xi(1 + \lambda_- t/m_\pi^2), \quad (7)$$

we find

$$\xi = 0.026 \ (0.054) \quad (8)$$

and

$$\lambda_- = -0.20 \ (-0.083) \quad (9)$$

with $\delta = -\frac{1}{3}$ (0).

What is happening physically is this: The first two terms on the right-hand side of Eq. (3) may be regarded as the model-dependent representation of the contributions of higher mass intermediate states to $f_-(t)$. The pole-dominance hypothesis,[9] as expressed formally through the field-current identity, does not require $f_-(t)$ to be dominated by the K^*-pole term. [It does call for $f_+(t)$ to be dominated by the K^* pole.] It happens that these two terms and the third term (K^*-pole term) in Eq. (3) nearly cancel each other (numerically) near $t = 0$. This gives a very small value of ξ, and a large value of λ_-, an order of magnitude larger than λ_+. It is therefore misleading to say that λ_- is characteristic of the inverse square mass of the intermediate states which contribute importantly to f_-.[10]

The numerical estimates (5), (8), and (9) are subject to an uncertainty of about 20%, arising from the inexactness of the partially conserved axial-vector current (PCAC) condition (as exemplified by the 10% discrepancy in the Goldberger-Treiman relation) and the ambiguity in handling the SU(3)-breaking effects. In particular, the values of ξ and λ_- are sensitive to the variation in δ (but the product $\xi\lambda_-$ is not). In any case, if we take seriously the values of Eqs. (5), (8), and (9) (the latter two for $\delta = -\frac{1}{3}$), we obtain for the branching ratio $R \equiv \Gamma(K_{\mu 3})/\Gamma(K_{e3}) \simeq 0.68$ to be compared with the experimental values[11] $R^+ = 0.73 \pm 0.03$ and $R^0 = 0.75 \pm 0.08$. The goodness of the fit of our parameters to all the available experimental data (the branching ratio R, longitudinal and average transverse polarizations of the muon) may be estimated from the work of Auerbach et al.,[11] assuming that the constant-ξ curves vary continuously in the plot of χ^2 vs λ_- in the range $-1 \leq \xi \leq 1$. According to Fig. 2(a) of Ref. 11, our parameters give $\chi^2 \simeq 15$ (in the present instance,[12] $\chi^2 = 14.1$ corresponds to a probability of 5%). On the other hand, if the errors on the transverse polarization and the branching ratio are doubled arbitrarily, we obtain $\chi^2 \simeq 5$ [see Fig. 2(b) of Ref. 11]. Thus we conclude that our parameters are not unsatisfactory from the standpoint of fitting all the available data, without prejudice to any particular experiment.[13]

(III) We shall outline briefly the derivation of Eqs. (2) and (3), deferring the details to another communication. The octet (or nonet; it does not matter for the present consideration) of the 0^- mesons is assumed to undergo a nonlinear transformation[14] under the chiral SU(3) transformation. To lowest order in the pseudoscalar meson fields, these fields undergo a c-number constant translation under the chiral transformation. The pertinent part of the Lagrangian which is chiral-SU(3) ⊗ SU(3) invariant and satisfies the field-current identity is[15]

$$\mathcal{L}_0 = -\tfrac{1}{4}\,\mathrm{Tr}(\partial_\mu U_\nu - \partial_\nu U_\mu + i\frac{g}{\sqrt{2}}[P_\mu, P_\nu] + i\frac{g}{\sqrt{2}}[U_\mu, U_\nu]) - \tfrac{1}{4}\,\mathrm{Tr}(D_\mu P_\nu - D_\nu P_\mu)^2 + \tfrac{1}{2}m^2(U_\mu^2 + P_\mu^2) + \tfrac{1}{2}\,\mathrm{Tr}(D_\mu \varphi^{(0)} - g f^{(0)} P_\mu)^2; \quad D_\mu = \partial_\mu + \frac{ig}{\sqrt{2}}[U_\mu,\ ], \quad (10)$$

where U_μ, P_μ, and $\varphi^{(0)}$ are the usual 3×3 matrix representations of the 1^+, 1^+, and 0^- fields, g is the gauge parameter, and $f^{(0)}$ is a constant. There is also the possibility of adding a "nonminimal"

$SU(3)\otimes SU(3)$-invariant coupling,[4] the relevant part of which is of the form

$$\mathrm{Tr}(\partial_\mu U_\nu - \partial_\nu U_\mu)[\partial_\mu \varphi^{(0)}, \partial_\nu \varphi^{(0)}]. \quad (11)$$

To these we must add terms which will ensure the PCAC condition for pions (i.e., the isotopic chiral symmetry is broken only by the finite pion mass; the PCAC conditions are not met for strangeness-changing and hypercharge currents), and break the mass degeneracies within members of octets (or nonets). For the latter, we can consider both the scalar-mixing and vector-mixing models,[16] but insofar as the ρ, K^*, π, and K mesons are concerned, the outcome is formally identical. As in the case of the chiral $SU(2)\otimes SU(2)$,[1] there is a bilinear coupling of the $D_\mu\varphi^{(0)}$ and P_μ fields, which induces scale transformations on the 0^- fields. Denoting by φ_π and φ_K the pseudoscalar fields associated with the physical pions and kaons, we write

$$C_\pi\varphi_\pi = \varphi_\pi^{(0)}, \quad C_K\varphi_K = \varphi_K^{(0)}, \quad (12)$$

where C_π and C_K, as well as $m_{KA}^2/m_{K^*}^2$, depend on the details of SU(3) breaking. The pion and kaon decay constants, f_π and f_K, are related to $f^{(0)}$ by

$$C_\pi f_\pi = C_K f_K = f^{(0)}. \quad (13)$$

The $K^*K\pi$ vertex implied in Eqs. (10) and (11) can be written without reference to C_π and C_K by virtue of Eq. (13). It is contained in

$$\mathcal{L}_{K^*K\pi} = \frac{i}{\sqrt{2}} g (K^{*-})^\mu \left[\frac{1}{\sqrt{2}}\left(\frac{f_K}{f_\pi}\pi^0 \partial_\mu K^+ - \frac{f_\pi}{f_K}\partial_\mu \pi^0 K^+\right) + \left(\frac{f_K}{f_\pi}\pi^+ \partial_\mu K^0 - \frac{f_\pi}{f_K}\partial_\mu \pi^+ K^0\right)\right]$$
$$- \frac{i}{\sqrt{2}} g\beta^2 \frac{f_K}{f_\pi}(1+\delta)\left(\frac{1}{\sqrt{2}}\partial_\mu \pi^0 \partial_\nu K^+ + \partial_\mu \pi^+ \partial_\nu K^0\right)[\partial^\mu (K^{*-})^\nu - \partial^\mu (K^{*-})^\mu] + \cdots, \quad (14)$$

where the omitted terms refer to the other species of the K^* mesons and we have used the particle symbols for the corrsponding particle fields (i.e., $\pi = \varphi_\pi$, etc.). The parameter δ is the measure of the "nonminimal" coupling[4] of Eq. (11). The strangeness-changing current S_μ is given by

$$S_\mu = V_\mu^{\ 4} - iV_\mu^{\ 5} = (m_{K^*}^2/g)(U_\mu^{\ 4} - iU_\mu^{\ 5}). \quad (15)$$

From Eqs. (14) and (15) follow our results, Eqs. (2) and (3).

(IV) The recent work of Mann and Primakoff[17] is at variance with our results, Eqs. (8) and (9). In their work (see also d'Espagnat and Gaillard[18]), they make use of the Fubini-Furlan sum rule[19] and the Gatto-Ademollo theorem (i) and the Callan-Treiman relation (ii). They approximate the so-called leakage terms by functions linear in t in such a way that (i) and (ii) are satisfied. As a consequence, their expression for $f(t)$, defined as

$$f(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t), \quad (16)$$

becomes the square root of a polynomial function in t. Indeed, because of the connection between $f(t)$ and the leakage terms dictated by the Fubini-Furlan sum rule, their $f(t)$ would in general be the square root of a polynomial function in t unless the leakage terms were specifically chosen to prevent this, e.g., as $-[1 - t^2/(m_K^2 - m_\pi^2)] + [F(t)]^2$, where $F(t)$ is not the square root of a polynomial in t. In our case $f(t)$ may be evaluated directly from Eqs. (2) and (3):

$$f(t) \cong 1 + \frac{t}{m_K^2 - m_\pi^2}\left(\frac{f_K}{f_\pi} - 1\right). \quad (17)$$

It can be shown[20] that if they had parametrized the leakage terms in such a way that $f(t)$ becomes a linear function of t, they would have obtained essentially our results (8) and (9). Thus we conclude that the MP-type analysis is sensitive to the parametrization of the so-called leakage terms.

I have enjoyed discussions on this subject with A. K. Mann, H. T. Nieh, H. Primakoff, and J. Schwinger.

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3668B, and the Alfred P. Sloan Foundation.

[1]See the references cited in B. W. Lee and H. T. Nieh, Phys. Rev. 166, 1507 (1968). See, in particular, S. Weinberg, Phys. Rev. Letters 18, 188 (1967); J. Schwinger, Phys. Letters 24B, 473 (1967).

[2]T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters 18, 1029 (1967); T. D. Lee and B. Zumino, Phys. Rev. 163, 1670 (1967).

[3]This is the substance of the Fayazuddin-Riazuddin-Kawarabayashi-Suzuki relation. For a recent discussion on this subject and reference to earlier discussions, see L. S. Brown and R. L. Goble, Phys. Rev. Letters 20, 346 (1968).

[4]The parameter δ is the same as that of H. J. Schnitzer and S. Weinberg, Phys. Rev. 164, 1828 (1967), and is related to the parameter χ of J. Wess and B. Zumino, Phys. Rev. 163, 1727 (1967).

[5]Because of the uncertainty in the ρ and A_1 widths, this value may be uncertain by as much as 100%. We use this value as a guide to the role δ plays in the analysis.

[6]M. Ademollo and R. Gatto, Phys. Rev. Letters 13, 264 (1964).

[7]C. G. Callan and S. T. Treiman, Phys. Rev. Letters 16, 153 (1966).

[8]These experimental values are taken from W. Willis, rapporteur's talk at the International Conference on Elementary Particles and High Energy Physics, Heidelberg, Germany, 1967 (to be published).

[9]For an exhaustive bibliography on this subject, see Lee and Zumino, Ref. 2, footnote 3.

[10]This is of course true, for example, in a model in which the entire contribution to $p_+^\mu f_+ + p_-^\mu f_-$ comes from the K^* intermediate state, or in a model in which the divergence of this quantity is dominated by a nearby pole corresponding to a low-mass scalar excitation. In this connection, see the discussions of P. Dennery and H. Primakoff, Phys. Rev. 131, 1334 (1963); H. T. Nieh, Phys. Rev. 164, 1780 (1967).

[11]L. B. Auerbach, A. K. Mann, W. K. McFarlane, and F. J. Sciulli, Phys. Rev. Letters 19, 464 (1967).

[12]In the discussion in Ref. 11, the number of degrees of freedom N is 7.

[13]I am indebted to Professor P. Grannis and Professor W. Lee for discussions bearing on the statistical analysis of the type discussed here.

[14]For discussions on this subject, see the references cited in footnote 1, and J. Schwinger, 1967 Brandeis Summer Institute Lectures (unpublished); S. Weinberg (to be published); W. A. Bardeen (to be published).

[15]The construction we adopt here is very similar to that of Schwinger, Ref. 1.

[16]S. R. Coleman and H. J. Schnitzer, Phys. Rev. 134, B863 (1964); N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. 157, 1376 (1967).

[17]A. K. Mann and H. Primakoff, Phys. Rev. Letters 20, 32 (1967), hereafter referred to as MP.

[18]B. d'Espagnat and M. K. Gaillard, Phys. Letters 25B, 346 (1967).

[19]S. Fubini and G. Furlan, Physics 1, 229 (1965).

[20]It may be remarked that Eq. (17), evaluated at the end of the t interval where $t=(m_K-m_\pi)^2$, is very similar to MP Eq. (11) which is $f(t=(m_K-m_\pi)^2)=1+(f_K/f_\pi-1)(m_K-m_\pi)/m_K$. Using Eq. (17) and the linear approximation $f_+(t)=1+\lambda_+t/m_\pi^2$, we recover our results (8) and (9).

S-WAVE π-N SCATTERING LENGTHS, CURRENT ALGEBRA, AND $\rho+\rho'$ DOMINANCE

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Various predictions of the $\rho\pi\pi$ coupling constant $f_\rho^2/4\pi$ are discussed. A $\rho+\rho'$ meson dominance model is considered with $M_{\rho'}\simeq 1600$ MeV and $\Gamma_{\rho'}\simeq 200$ MeV. This model predicts a value of the S-wave π-N scattering length formula a_1-a_3 in better agreement with experiment than the ρ-dominance model when $\Gamma_\rho=128$ MeV, and the modified Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin relation is well satisfied.

A number of people[1] have used the SU(2)⊗SU(2) commutation relations (CCR)[2] and the partially conserved axial-vector current (PCAC) hypothesis[3] to derive an interesting expression for the scattering length a_T of pion scattering on any isospin-carrying target:

$$a_T=(\mu_{\pi t}/4\pi F_\pi^2)(\vec{T}_\pi\cdot\vec{T}_t), \tag{1}$$

where $\vec{T}_\pi$ and $\vec{T}_t$ are the isospins of the pion and the target particle, and F_π is defined by $\langle 0|A_\mu^1-iA_\mu^2|\pi^+\rangle=-i\sqrt{2}q_\mu F_\pi$. The reduced mass is denoted by $\mu_{\pi t}=m_\pi m_t/(m_\pi+m_t)$. If we adopt the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF)[4] relation $F_\pi^2=F_\rho^2/2m_\rho^2$, and the ρ-dominance and universality relation $F_\rho^2=m_\rho^4/f_\rho^2$, then in place of Eq. (1) we obtain

$$a_T=-2(f_\rho^2/4\pi)(\mu_{\pi t}/m_\rho^2)(\vec{T}_\pi\cdot\vec{T}_t). \tag{2}$$

This relation was derived directly from ρ dominance and universality by Sakurai.[5] It can be derived simply by considering the "effective" Yukawa potential for single ρ exchange between