SU(3) singlet is taken into account, the "unbroken"-SU(3) decay-rate predictions (well satisfied by established supermultiplets) are consistent with present crude experimental values for most resonances, provided only that the $\Sigma\pi/\Lambda\pi$ ratio from "1660" decay is large. The significant inconsistencies which do exist are due entirely to the $\Xi^*(1815)$ decay rates. See M. Goldberg et al., Nuovo Cimento 45A, 169 (1966); N. Masuda and S. Nukomo, to be published.

[20] See P. Schlein, in Lectures in Theoretical Physics, edited by Wesley E. Brittin et al. (University of Colorado Press, Boulder, Colorado, 1965), Vol. VIIB, p. 111; J. Leitner, ibid., p. 43.

[21] For example, the observation of a $Y_1^*(1385) \rightarrow \Sigma^0\pi^+$ signal of a size compatible with the accepted $\Sigma^0\pi^+/\Lambda\pi^+$ branching ratio, etc.

[22] Using the accepted M and Γ parameters of the $Y^*(1385)$ and $Y^*(1910)$, the best "three-resonance" fit (χ^2 probability of 84%), which is shown as the solid curve of Fig. 2(c), yields the values $M = 1702 \pm 11$ MeV, $\Gamma = 108 \pm 24$ MeV for the $Y_1^*(1695)$. This is consistent with the world average given in our first paragraph, obtained by averaging out best values of M and Γ with those quoted by the other experiments.

MEASUREMENT OF THE LOW-ENERGY END OF THE μ^+ DECAY SPECTRUM*

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Precise measurements[1-4] above 25 MeV have shown that with appropriate radiative corrections,[5,6] the Michel formula[7]

$$N(y;\rho,\eta) = 4(1+2\eta y_0)^{-1}(y^2-y_0^2)^{1/2}[3y(1-y) + (\tfrac{2}{3})\rho(4y^2-3y-y_0^2) + 3y_0\eta(1-y)] \quad (1)$$

(where $y = E/E_{\rm max}$, E = total electron energy, $E_{\rm max} = 52.83$ MeV, and $y_0 = m_e/E_{\rm max}$)[8] gives an excellent fit to the upper half of the μ-decay spectrum when the parameters ρ and η are given the values $\rho = \frac{3}{4}$, $\eta = 0$ corresponding to a $V-A$ theory. For experiments in the 25- to 53-MeV range, however, the correlation between the parameters is such[2] that η can be derived only by assuming a precise value (e.g., $\frac{3}{4}$) for ρ, and ρ can be derived only by assuming a precise value (e.g., zero) or a range of possible values (e.g., $-\frac{1}{2}$ to $+\frac{1}{2}$) for η. By contrast, in our energy range (1-7 MeV) ρ and η are almost decoupled, so that we can make a significant two-parameter fit.

The significance of the low-energy yield becomes evident if we analyze the decay process in the charge-retention ordering. The ν-$\bar{\nu}$ correlation then plays the same role as the e-$\bar{\nu}$ correlation in β decay and the parameter η of Eq. (1) is a measure of that correlation through the electron "recoil" spectrum.

Our spectrometer was a 10-liter hydrogen bubble chamber in a 21-kG field exposed to a beam of stopping μ^+ and π^+ from the Chicago synchrocyclotron. The film was scanned so as to produce two distinct samples. Sample 1 covered the whole spectrum as seen in every hundredth frame and sample 2, taken from all other frames, consisted of events with projected radius $r < 2.5$ cm. This radius corresponds, for zero dip-angle events scanned at 2× magnification, to ~7 MeV/c. The spectrum from sample 2 is shown in Fig. 1.

The calibration of the chamber as a low-momentum electron spectrometer is described elsewhere.[9] Briefly, the standards used were the following: (a) two internal-conversion electron lines of energies ~$\frac{1}{2}$ and 1 MeV (momenta of 0.875 and 1.414 MeV/c) from a $\rm Bi^{207}$ source deposited in a thin layer on 1-mil polyester strips stretched through the chamber, (b) the high-energy cutoff of the μ-decay spectrum

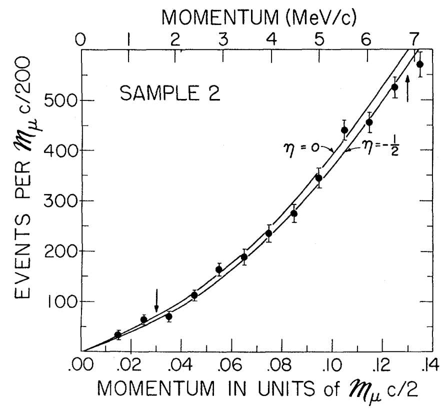

FIG. 1. Low end of spectrum. Points show distribution of events below 7.4 MeV/c found among 530 000 decays of all momenta. Curves show spectra calculated for $\eta = 0$ and $\eta = -0.5$ assuming $\rho \equiv \frac{3}{4}$, radiative corrections as in Ref. 5, and spectrometer resolution as given by Eq. (2). Arrows show limits of momentum range used in analysis (2444 events in this range).

(52.83 MeV), and (c) Dalitz electron pairs from radiative π^- capture in hydrogen[10] (pair energy = 129.6 MeV).

The momentum distribution $f(p)$ corresponding to an electron line of momentum p_i (MeV/c) was found to be

$$f(p)=\frac{(p/p_i)^{0.7}}{|p-p_i|^{2.7}+(\frac{1}{2}p_i\Gamma)^{2.7}}, \tag{2}$$

where, to good approximation, $\Gamma=(8.2+24/p_i)$ is the full width at half-maximum in percent. The shape of the lower half of this function is not directly verified for $p_i>1.4$ MeV/c (except insofar as the 130-MeV Dalitz pairs provide a valid test) but since (a) the measured values of Γ agree with the calculated values at $\frac{1}{2}$, 1, 52, and 65 MeV, (b) the upper half is well determined and gives a good fit at 52 MeV, (c) the lower half varies as expected with magnetic field (7, 14, and 21 kG) at $\frac{1}{2}$ and 1 MeV, and (d) the large radiative losses which contribute to the extreme low end of $f(p)$ usually cause visible δ rays or kinks which are taken into account by detailed measurement, we assume that the shape is adequately represented by (2) throughout our momentum range.

As Eq. (1) is valid only for unpolarized μ's, we should expect an error if muons emitted along the field $\vec{B}$ and those emitted opposite to $\vec{B}$ were detected with different efficiency. The following checks were made: (a) The spectrum from beam muons which moved nearly perpendicularly to $\vec{B}$ was analyzed separately. The result is not significantly different from that yielded by muons from π^+ decay in the chamber. (b) When the cutoff chosen for the dip angle λ of the electron track with respect to the plane perpendicular to $\vec{B}$ is reduced from $|\sin\lambda|<0.7$ to $|\sin\lambda|<0.4$, the result is again not significantly affected (Table I).

Momentum-dependent corrections are made for annihilation in flight (~$\frac{1}{2}$%), systematic momentum errors (~1%), measuring inefficiency (~6-16%),[11] and scanning inefficiency (~4-15%) (figures refer to sample 2). The determination of scanning inefficiency is based on two scans of $\frac{4}{5}$ of the film and four scans of the remaining $\frac{1}{5}$. The quadruple-scan data provide corrections for the tendency of different scanners to miss the same events.

We estimate that for sample 2 the errors associated with the various corrections contribute as follows to the error in the determin-

Table I. Measured values for η and ρ.

Sample[a] (or Experimenter)	Momentum Range (units: $m_\mu c/2$)	η	ρ	Remarks
1	0.05-0.90	≡0	0.760 ± 0.037	fit: χ^2 = 14.4; 16 deg. freedom
	0.05-0.25	-0.4 ± 1.0	≡3/4	Low mom. end of sample 1
2	0.03-0.13	-0.32 ± 0.29	≡3/4	fit: χ^2 = 8.5; 9 deg. freedom
	0.03-0.13	-0.24 ± 0.61	0.76 ± 0.07	χ^2 = 8.5; 8 deg. freedom
	0.03-0.13	-0.24 ± 0.56	≡3/4	On basis of shape alone
	0.03-0.13	-0.13 ± 0.36	≡3/4	μ^+ beam only (μ dip 0°)
	0.03-0.13	-0.49 ± 0.36	≡3/4	π^+ beam only (μ dip 0°-90°)
	0.03-0.13	-0.24 ± 0.36	≡3/4	dip cut off reduced from 44° to 24°
PLANO (ref 12)	whole spectrum	(-2.0 ± 0.9)	0.745 ± 0.025	Author discounts value for η
PEOPLES (ref 1)	> 0.4	+0.05 ± 0.5	≡3/4	η ≡ 0 yields ρ = 0.7503 ± 0.0026
SHERWOOD (ref 2)	> 0.5	-0.7 ± 0.6	≡3/4	η ≡ 0 yields ρ = 0.760 ± 0.009
FRYBERGER (refs 3,4)	> 0.45	-0.7 ± 0.5	≡3/4	η ≡ 0 yields ρ = 0.762 ± 0.008
Refs 1-4 combined	> 0.4	$\lvert\eta\rvert<0.5$	0.752 ± 0.005	Error on ρ csp. to assumed limits on η
Samp. 2 and refs 1-4	0.03-0.13, >0.4	-0.31 ± 0.30	0.751 ± 0.003	See fig 2

[a] Sample 1: events from every hundredth frame. Sample 2: events of projection radius <2.5 cm from all other frames.

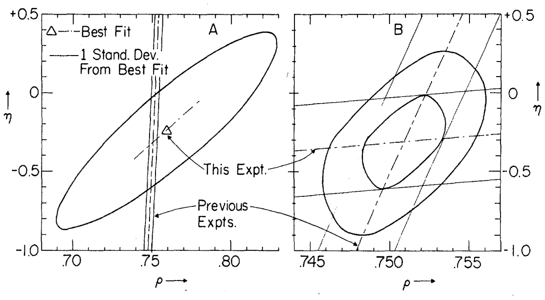

FIG. 2. ρ-η correlation. (a) The two-parameter fit is indicated by the triangle and ellipse (at one standard deviation). The nearly vertical lines show the correlation from the higher momentum measurements of Refs. 1-4 where the slopes and intercepts are weighted averages (slope for Ref. 1 inferred from uncertainties given for ρ and η). (b) The one- and two-standard-deviation contours which result when the data of this and the previous experiments are combined. (ρ scale greatly expanded.)

ation of η: $\Delta f(p) \to \Delta\eta = 0.12$, $\Delta\Gamma \to \Delta\eta = 0.11$, $\Delta p \to \Delta\eta = 0.12$, measuring inefficiency $\to \Delta\eta = 0.08$, scanning inefficiency $\to \Delta\eta = 0.11$, and statistical error $\to \Delta\eta = 0.16$. Combining these errors we obtain $\eta = -0.32 \pm 0.29$ for $\rho \equiv \frac{3}{4}$. The two-parameter fit gives $\eta = -0.24 \pm 0.61$, $\rho = 0.76 \pm 0.07$. Consistent results are obtained by analyzing sample 2 on the basis of the shape alone, and by analyzing the portion of sample 1 below 13 MeV (Table I). Our figure $\rho = 0.760 \pm 0.037$ ($\eta \equiv 0$) for sample 1 agrees well with the more precise values (Table I).

When we combine our data with those from previous measurements as indicated in Fig. 2, we obtain the result

$$\eta = -0.31 \pm 0.30, \quad \rho = 0.751 \pm 0.003. \tag{3}$$

Although no significant limit has heretofore been placed on η by direct measurement[12] (except by assuming ρ to be precisely $\frac{3}{4}$) it has been possible to predict limits as follows: For a V–A theory, $\eta \equiv 0$. For a two-component neutrino hypothesis, η is restricted by the relationship $\eta^2 \le \frac{1}{4}(1-\xi^2)$ to the range $|\eta| \le 0.11^{+0.03}_{-0.11}$, where ξ (measured value,[13] 0.975 ± 0.015) is the asymmetry parameter.[14] If we make no assumption about S, T, and P then η is limited by the requirement that (1) be everywhere positive to $\eta > [\frac{1}{3}\rho - 1]$ or, for $\rho = \frac{3}{4}$, to $\eta > -\frac{1}{2}$; and it is further limited by the relationship[4] $\eta^2 \le (1-h^2)$ to the range $|\eta| \le 0.49$, where h is the helicity (average of measured values $= 1.00 \pm 0.13$).[4] Our result (3) is seen to fall within this limit.

We are continuing the experiment under improved conditions, and V. L. Telegdi and his associates are preparing an independent measurement in this laboratory at somewhat higher momenta using a wire spark-chamber spectrometer.

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[8] When terms of order y_0 are included, as is necessary at low energies, Eq. (1) is most compactly written in terms of the total energy variable $y = E/E_{\max}$. In presenting our data, however, we display the momentum $x = p/p_{\max} = (y^2 - y_0^2)^{1/2}$ as in Refs. 1-4. The theoretical figures are transformed accordingly.

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[11] Additional measurements have raised the efficiency values above those indicated in Fig. 6 of Ref. 9. Note that there is no uncertainty in the over-all measuring inefficiency.

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