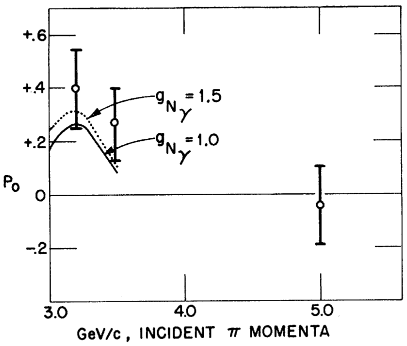

FIG. 1. Polarization in $\pi^-p\to\eta n$ at $|t|\approx 0.2$; solid curve calculated for $g_{N_\gamma}=1.0$ and dotted curve for $g_{N_\gamma}=1.5$.

state, and thus we have $\mathrm{Im}f_{\mathrm{res}}^{\eta n}<0$ and $\mathrm{Im}g_{\mathrm{res}}^{\eta n}>0$ in the region of $0<|t|<0.2$. Thus, Eq. (8) provides a definite positive polarization at 3.20 GeV/c. This gives a good qualitative explanation of the experimental data.[1]

It is also possible to obtain the factor g_{N_γ} in Eq. (2) by fitting Eq. (8) to the polarization data in $\pi^-p\to\eta n$ at 3.20 and 3.47 GeV/c.[1] The polarization calculated[9] by two possible values of g_{N_γ} is shown in Fig. 1, where the experimental data are also plotted. The differential cross sections calculated by Eqs. (1) and (2) with values of g_{N_γ} as 1.0 and 1.5 are in good agreement with the experimental data.[8] However, this is no longer true with values of g_{N_γ} higher than about 1.5. Since the ratio g_{N_γ} is related to the ratio of F and D coupling of N_γ resonance as[10]

$$g_{N_\gamma}=-3^{-1/2}(1-4f),$$

where $f+d=1$, the ratio F/D should be bigger than 2.3.[11] It is thus demonstrated that the ratio F/D can be obtained through the polarization data in $\pi^-p\to\eta n$ around the resonance. However, for more precise determination of the ratio we obviously need better data. It is also desirable to have polarization measurements around $N(2190)$. We could thus compare the ratio F/D of N_γ at different resonances.

We wish to acknowledge the assistance of R. George and R. Miller in the numerical calculations.

[1]D. D. Drobnis, J. Lales, R. C. Lamb, R. A. Lundy, A. Moretti, R. C. Niemann, T. B. Novey, J. Simanton, A. Yokosawa, and D. D. Yovanovitch, Phys. Rev. Letters 20, 274 (1968).

[2]R. J. N. Phillips and W. Rarita, Phys. Rev. 140, B200 (1965), and Phys. Rev. Letters 15, 807 (1965), and Phys. Letters 19, 598 (1965); R. K. Logan and L. Sertorio, Nuovo Cimento 52A, 1022 (1967).

[3]D. D. Reeder and K. V. L. Sarma, Nuovo Cimento 51A, 169 (1967).

[4]I. Mannelli and P. Sonderegger, private communication.

[5]Details of a similar model have been discussed by R. C. Lamb, R. A. Lundy, T. B. Novey, A. Yokosawa, and D. D. Yovanovitch, Phys. Rev. Letters 20, 353 (1968).

[6]R. C. Arnold, private communication.

[7]P. Astbury et al., Phys. Letters 23, 396 (1966).

[8]O. Guisan et al., Phys. Letters 18, 200 (1965).

[9]Resonance parameters of $N(2650)$ used in this calculation are similar to those used in Ref. 5.

[10]A. W. Martin and K. C. Wali, Phys. Rev. 130, 2455 (1963).

[11]It should be noted here that we reach the same conclusion about the ratio F/D even if we adopt the alternative solution of $b_{2A}^{\eta n}>0$ since the first term in Eq. (8) is much smaller than the second term due to the fact that the average of $|\mathrm{Im}f_{\mathrm{res}}^{\eta n}|\approx 0$ in the region of $0.14<|t|<0.25$.

DETERMINATION OF THE *KNY* COUPLING CONSTANTS USING MODIFIED DISPERSION RELATIONS*

C. H. Chan and F. T. Meiere
Physics Department, Purdue University, Lafayette, Indiana
(Received 19 January 1968)

The *KNY* coupling constants are determined using a new procedure which includes a criterion for checking the self-consistency of the necessary extrapolation into the unphysical region.

Several determinations of the *KNY* coupling constants have been made using forward *KN* dispersion relations in which the imaginary part of the scattering amplitude in the unphysical region is found by extrapolation from the experimentally accessible physical region. Most recently, Kim[1] has performed this extrapolation using multichannel effective-range theory (EFR) and his results fit quite nicely into the SU(3) description of these coupling constants.[2] However, they differ significantly from similar analyses[3] which perform the necessary extrapolation using a constant scattering-length approximation (CSL). Undoubtedly, the EFR parametrization is favored on theoretical grounds, but in view of the theoretical significance of these coupling constants, we consider it interesting to have some independent method for their determination. We assert that the family of modified dispersion relations given below provides one such method and, furthermore, that it contains a criterion for the internal self-consistency of the calculation, which is lacking in previous determinations.

Our starting point is the observation that all parametrizations for extrapolation of the amplitude into the unphysical region provide both the real and imaginary parts of the amplitude, and that once their parameters are determined, one knows the real part as accurately as the imaginary part.[4] Since conventional dispersion relations use only the imaginary part, dispersion relations involving the real part in the unphysical region should provide an independent determination of the coupling constants. Gilbert[5] and Liu and Okubo[6] have written dispersion relations for πN scattering involving the real and imaginary parts in the physical region. However, we found a different class of dispersion relations more suited for the KN case.

If $T(\nu)$ is the invariant forward scattering amplitude for $\bar{K}N$ scattering,[7] where ν is the laboratory energy of the kaon, we determine the coupling constants (referred to simply as g_Y^2) from the dispersion relation $F_\beta(\nu)=T(\nu)/(\nu-m_K)^{1-\beta}(\nu-\nu_c)^\beta$, where m_K is the kaon mass, ν_c is the lowest πY threshold, and β is an arbitrary parameter, $0<\beta<1$. The factor $(\nu-m_K)^{1-\beta}\times(\nu-\nu_c)^\beta$ is chosen so that it introduces a short cut joining the branch points ν_c and m_K, and is taken to be positive for real $\nu>m_K$ and negative for real $\nu<\nu_c$. In the physical region, $\operatorname{Im}F_\beta(\nu)$ is proportional to $\operatorname{Im}T(\nu)$, but in the unphysical region it is proportional to $\cos\pi\beta\times\operatorname{Im}T(\nu)+\sin\pi\beta\operatorname{Re}T(\nu)$. Thus, as β varies, contributions from the unphysical region are determined by combinations that range from $\operatorname{Re}T$ to essentially $\pm\operatorname{Im}T$, while contributions from the physical region are determined always by the total cross sections.

This modification has several advantages in the KN case. In the physical region, unlike the modified dispersion relations of Liu and Okubo,[6] we need only to know the $\operatorname{Im}T$. Experimentally, $\operatorname{Im}T$ is well determined from threshold to 20 BeV/c through the known total cross sections, while very little direct information on $\operatorname{Re}T$ is available over most of the physical region. This is in contrast to the unphysical region where, as mentioned, the same procedure for determining $\operatorname{Im}T$ determines $\operatorname{Re}T$. Thus the input to the dispersion relations for $F_\beta(\nu)$ is as well known as for the conventional dispersion relations. Also, the high-energy behavior is $F_\beta(\nu)\sim T(\nu)/\nu$,[8] independent of β, so that the high-energy behavior of the modified amplitude does not restrict the allowed values of β as it does for Liu and Okubo in the πN case.

Furthermore, we assert that the stability of the results for the coupling constants against variations in β provides a sensitive check on whether the form of the extrapolated amplitude used in the unphysical region is consistent with the known total cross sections.

Starting from the $\bar{K}N$ amplitude corresponding to isospin I, equal to 0 or 1, which isolates either the Λ or Σ pole, our basic equations are obtained from the dispersion relation for $F_\beta(\nu)$ evaluated at KN threshold $\nu=-m_K$ (evaluated at $\bar{K}N$ threshold, they reduce to an identity). For example, for $I=0$,

$$\frac{2g_{KN\Lambda}{}^2X(\Lambda)}{(\nu_\Lambda+m_K)(m_K-\nu_\Lambda)^{1-\beta}(\nu_c-\nu_\Lambda)^\beta}=+\frac{1}{4\pi^2}\int_0^\infty\frac{dk}{\nu}\left(\frac{\nu+m_K}{\nu+\nu_c}\right)^\beta[2\sigma_{K^+p}(\nu)-\sigma_{K^+n}(\nu)]$$
$$-\frac{1}{4\pi^2}\int_0^\infty\frac{dk}{\nu}\left(\frac{\nu-m_K}{\nu-\nu_c}\right)^\beta[2\sigma_{K^-p}(\nu)-\sigma_{K^-n}(\nu)]+\frac{1}{\pi}\int_{\nu_c}^{m_K}\frac{d\nu}{\nu+m_K}$$
$$\times\frac{\cos\pi\beta\operatorname{Im}T(\nu)+\sin\pi\beta\operatorname{Re}T(\nu)}{(m_K-\nu)^{1-\beta}(\nu-\nu_c)^\beta}-\frac{T(-m_K)}{(2m_K)^{1-\beta}(\nu_c+m_K)^\beta},\quad(1)$$

Table I. *KNY* coupling constants using the EFR parametrization. Also given are the values for the integrals for the $KN\Lambda$ case.

β	$g_{KN\Lambda}^2$	$g_{KN\Sigma}^2$	Integral over unphysical region (mb)	Integral over physical region (mb)
0.1	14.6 ± 1.5	0.5 ± 0.5	11.32	−10.08
0.2	14.7 ± 1.6	0.3 ± 0.5	7.16	−5.86
0.3	14.4 ± 1.7	0.2 ± 0.6	5.64	−4.34
0.4	14.0 ± 1.7	0.1 ± 1.0	4.80	−3.52
0.5	13.5 ± 1.8	0.0 ± 1.3	4.25	−2.99
0.6	13.1 ± 1.8	−0.2 ± 2.0	3.84	−2.62
0.7	12.6 ± 1.9	−0.3 ± 2.7	3.51	−2.33
0.8	12.0 ± 1.9	−0.5 ± 3.5	3.23	−2.10
0.9	11.1 ± 2.0	−0.9 ± 4.9	2.92	−1.92

where $g_{KN\Lambda}^2$ is the renormalized $KN\Lambda$ coupling constant[9]

$$X(\Lambda) = [m_K^2 - (m_\Lambda - m_N)^2]/4m_N^2,$$

and ν_Λ and ν_c are the positions of the Λ pole and $\Sigma\pi$ threshold, respectively. The analogous equation for $I=1$ gives $g_{KN\Sigma}^2$. We point out here that the residue functions $X(Y)$ used by Kim[10] are incorrect. Because of this, his results for the KN coupling constants should be reduced by a factor m_N/m_Λ for $g_{KN\Lambda}^2$ and m_N/m_Λ for $g_{KN\Lambda}^2$. This correction is made when we compare results.

The integrals in Eq. (1) were evaluated numerically for several values of β. It is interesting to note that no principal-value integrals occur, contrary to the conventional case.[2] In the unphysical region and for low-energy $\bar{K}N$ scattering (up to $k = 500$ MeV/c), the EFR parametrization[1] was used. For low-energy KN scattering (up to $k = 300$ MeV/c), the effective range parameters of Goldhaber et al.[11] were used. In the regions from 500 MeV/c for $\bar{K}N$ and 300 MeV/c for KN up to 18 BeV/c, the most recent cross-section measurements available were used.[12] We assumed that above 18 BeV/c, $\sigma_{\bar{K}N} - \sigma_{KN} \propto k^{-1/2}$. Contributions from the asymptotic region depend only on the power law and the known cross sections at 18 BeV/c, but contribute very little to g_Y^2 in any case.

The results for various values of β are given in Table I. The error quoted is solely due to variations of the EFR parameters[1] within their quoted errors. Errors in the reported cross sections give rise to much smaller variations. The coupling constants obtained using any combination of real and imaginary parts of the unphysical amplitude agree with each other within these errors. Table I also contains representative values of the integrals over the unphysical and physical regions which appear in Eq. (1) for the $KN\Lambda$ case. These contributions vary individually by a factor of 4 or 5 as β changes, showing that stability of the resultant coupling constant is by no means trivial.

The whole calculation was repeated using the CSL extrapolation with the parameters also given by Kim.[13] The results are given in Table II, the errors again representing variations in the CSL parameters. The values for the coupling constants obtained for different β disagree quite badly with each other.

We assert that the stability of g_Y^2 against variations in β is a measure of the self-consistency of such a determination, or more specifically, provides a criterion for the compatibility of the extrapolated amplitude in the unphysical region with the known total cross sec-

Table II. *KNY* coupling constants using CSL parametrization.

β	$g_{KN\Lambda}^2$	$g_{KN\Sigma}^2$
0.1	4.7 ± 0.08	3.4 ± 0.4
0.2	5.2 ± 0.04	2.4 ± 0.1
0.3	5.2 ± 0.04	2.0 ± 0.1
0.4	5.0 ± 0.05	1.6 ± 0.1
0.5	4.6 ± 0.07	1.2 ± 0.1
0.6	4.0 ± 0.09	0.8 ± 0.1
0.7	3.0 ± 0.12	0.2 ± 0.1
0.8	1.1 ± 0.20	−0.8 ± 0.2
0.9	−4.4 ± 0.44	−3.3 ± 0.4

tions in the physical region. Use of the multichannel effective range extrapolation with the parameters recently determined by Kim[1] leads to a determination of $g_Y{}^2$ which is self-consistent, within the errors quoted for the parameters, while use of the constant-scattering-length approximation does not. Furthermore, the results in Table I can be summarized as follows:

$$g_{KN\Lambda}{}^2 = 13 \pm 3, \qquad (2)$$

$$g_{KN\Sigma}{}^2 = 0 \pm 1. \qquad (3)$$

These results are compatible with pure SU(3) invariance, giving the mixing parameter $\alpha \equiv F/(F+D)$ as

$$0.37 \leqslant \alpha \leqslant 0.41. \qquad (4)$$

These values agree quite well with Kim's corrected results of 13.5 ± 2.1 and 0.2 ± 0.4 for $g_{KN\Lambda}{}^2$ and $g_{KN\Sigma}{}^2$, respectively.

*Work supported by the U. S. Atomic Energy Commission, Contract No. AT(11-1)-1428.

[1]Jae Kwan Kim, Phys. Rev. Letters 19, 1074 (1967).

[2]Jae Kwan Kim, Phys. Rev. Letters 19, 1079 (1967).

[3]M. Lusignoli, M. Restignoli, G. A. Snow, and G. Violini, Phys. Letters 21, 229 (1966), and Nuovo Cimento 45A, 792 (1966).

[4]The real part may, however, be more sensitive to errors in these parameters than the imaginary part.

[5]W. Gilbert, Phys. Rev. 108, 1078 (1957).

[6]Yu-Chien Liu and S. Okubo, Phys. Rev. Letters 19, 190 (1967).

[7]We use the normalization $\sigma_{\overline{K}N}(\nu) = (4\pi/k)\,\mathrm{Im}T(\nu)$, where k is the laboratory momentum of the kaon.

[8]In fact, the high-energy convergence of our integrals is exactly the same as in the conventional case.

[9]Our coupling constants correspond to an interaction Lagrangian $(4\pi)^{1/2}(g_{\pi NN}\overline{N}\gamma_5\vec{\tau}\cdot\vec{\pi}N + g_{KN\Sigma}\overline{N}\gamma_5 K\Lambda + \mathrm{H.c.} + g_{KN\Sigma}\overline{N}\gamma_5\vec{\tau}\cdot\Sigma K + \mathrm{H.c.})$ and $g_{\pi NN}{}^2 \cong 14.5$.

[10]This error seems to occur quite often in previous determinations of $g_Y{}^2$, but only recently has the accuracy increased to the point where this error (20%) becomes important. The correct form of the residue function is given in the original literature [P. T. Mathews and A. Salam, Phys. Rev. 110, 569 (1958)], or more recently by J. D. Jackson, in Dispersion Relations, edited by G. R. Screaton (Oliver and Boyd, Edinburgh, Scotland, 1961), p. 46.

[11]S. Goldhaber, W. Chinowsky, G. Goldhaber, W. Lee, T. O'Halloran, T. F. Stubbs, G. M. Pjerrow, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters 9, 135 (1962); V. J. Stenger, W. E. Slater, D. H. Stork, H. K. Ticho, G. Goldhaber, and S. Goldhaber, Phys. Rev. 134, B1111 (1964).

[12]R. J. Abrams et al., Phys. Rev. Letters 19, 259, 678 (1967); J. D. Davies et al., Phys. Rev. Letters 18, 62 (1967); R. L. Cool et al., Phys. Rev. Letters 16, 1228 (1966), and 17, 102 (1966); W. Galbraith et al., Phys. Rev. 138, B913 (1965); A. Fridman and A. Michalon, Nuovo Cimento 48A, 344 (1967).

[13]Jae Kwan Kim, Phys. Rev. Letters 14, 29 (1965).

ANGULAR DISTRIBUTION FOR ETA-MESON PHOTOPRODUCTION FROM HYDROGEN AT 775-850 MeV

C. Bacci
Istituto Nazionale di Fisica Nucleare, Sezione di Roma, Roma, Italy

and

R. Baldini-Celio, C. Mencuccini, A. Reale, M. Spinetti, and A. Zallo
Laboratori Nazionali di Frascati del Comitato Nazionale per l'Energia Nucleare, Frascati, Italy
(Received 3 January 1968)

In this paper we report an experiment performed at the Frascati 1.1-GeV electron synchrotron on the photoproduction of the eta meson from hydrogen,

$$\gamma + p \to \eta + p. \qquad (1)$$

The forward differential cross section has been measured at the three energies $K = 775$, 800, and 850 MeV of the incident photons for different eta c.m. angles $\theta_\eta{}^*$. The energy resolution ΔK was typically ±25 MeV. The purpose of this experiment was to investigate, not far from threshold energy, the presence in photoproduction of higher partial waves besides the dominant η-nucleon S-wave resonance.[1]

Our results show that in the reaction $\gamma + p \to \eta + p$ up to $K = 850$ MeV (corresponding to a c.m. total energy $E^* = 1573$ MeV), the differential cross section is not sensibly increasing at forward angles. At backward angles a departure from isotropy could start between $K = 800$ MeV and $K = 850$ MeV. At the correspond-