

INTERPRETATION OF RECURRING MINIMA IN ELASTIC  
SCATTERING AT LARGE MOMENTUM TRANSFERS

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Maxima and minima recently reported in forward elastic scattering at moderately high energy and large momentum transfer are correlated with exceptional points  $\alpha = 0, -1, -2, \dots$  on the  $P'$  and  $\omega$  Regge trajectories. An Ansatz for the Regge residue functions is given that reproduces the qualitative features of the new data. This interpretation leads to further predictions that can be experimentally checked.

Striking new structure has just been reported in  $\pi^-p$ ,  $K^-p$ ,  $\bar{p}p$ , and  $pp$  elastic differential cross sections at moderately high energies and momentum transfers larger than previously studied.<sup>1</sup> In this Letter we remark that the new structure can be correlated with exceptional points  $\alpha(t) = 0, -1, -2, \dots$  on  $I=0$  Regge trajectories. Furthermore, we propose a simple dynamical picture that contains the main features of the data, and leads to further predictions.

The characteristics of the new data<sup>1</sup> are the following:

(i) Dips or inflection points in  $d\sigma/dt$  for the following processes [ $t$  in  $(\text{GeV}/c)^2$ ]:

$$\pi^-p \text{ at } t_1 = -0.8, t_2 = -3,$$

$$\pi^+p \text{ at } t_1 = -0.8, t_2 = ?,$$

$$K^-p \text{ at } t_1 = -1.0, t_2 = ?,$$

$$\bar{p}p \text{ at } t_1 = -0.5, t_2 = -1.8.$$

(ii) The  $pp$  data are relatively smooth except for a slight break in  $d\sigma/dt$  near  $t = -1.6$ . The  $K^+p$  data also appear to be smooth out to large  $|t|$ .

(iii) Differential cross sections at large  $t$  that appear to fall rapidly with increasing energy (with perhaps a fixed lower bound at every  $t$ ).

(iv) Approximate equality of  $d\sigma/dt$  for  $\pi^+p$  and  $\pi^-p$  and also for  $pp$  and  $np$ . This indicates very little isospin dependence.

(v)  $d\sigma/dt$  for  $\bar{p}p$  lies below  $pp$  at large  $t$ , and  $\bar{p}p$  has maxima at  $t = -1$  and  $-2$ . The  $\bar{p}p$  differential cross section rises to meet  $pp$  near the first  $\bar{p}p$  maximum and approaches  $pp$  again near the second maximum but less closely.

Accordingly, we propose a picture with an

essentially energy independent Pomeranchuk amplitude  $P$ ,<sup>2</sup> and with two Regge poles  $P'$  and  $\omega$  associated with the  $f^0$  and  $\omega^0$  mesons. We use only spin-nonflip amplitudes, neglecting the small high-energy polarizations. Then the various scattering amplitudes have the forms

$$f_{\pi^+p} = f_{\pi^-p} = f_{\pi N}(P) + f_{\pi N}(P'),$$

$$f_{pp} = f_{pn} = f_{NN}(P) + f_{NN}(P') + f_{NN}(\omega),$$

$$f_{\bar{p}p} = f_{\bar{p}n} = f_{NN}(P) + f_{NN}(P') - f_{NN}(\omega).$$

The  $KN$  and  $\bar{KN}$  formulas are similar to  $NN$  and  $\bar{NN}$ .

The  $P'$  and  $\omega$  trajectories are assumed linear, as in Fig. 1. We approximate them, in the following argument, by a single degenerate trajectory  $\alpha(t)$ . The individual contribu-

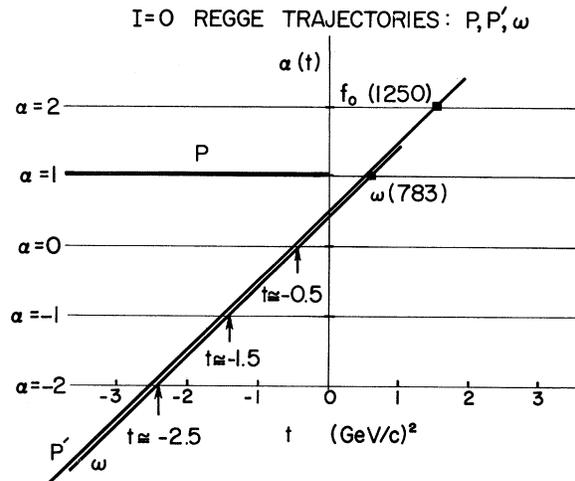


FIG. 1. Straight-line  $P$ ,  $P'$ , and  $\omega$  Regge trajectories for the proposed interpretation of maxima and minima in elastic-scattering differential cross sections.

tions (suppressing subscripts  $\pi N$ , etc.) then have forms

$$\begin{aligned} f(P) &= \gamma i, \\ f(P') &= -\beta s^{\alpha-1} [1 + \exp(-i\pi\alpha)] / \sin\pi\alpha, \\ f(\omega) &= \bar{\beta} s^{\alpha-1} [1 - \exp(-i\pi\alpha)] / \sin\pi\alpha, \end{aligned}$$

where  $\gamma(t)$ ,  $\beta(t)$ , and  $\bar{\beta}(t)$  are the residue functions and  $d\sigma/dt = |f|^2$ .

As  $\alpha(t)$  goes through the exceptional points  $0, -1, -2, \dots$ , etc., the  $P'$  and  $\omega$  amplitudes come alternately in and out of phase with  $P$ , so that the interference terms in  $d\sigma/dt$  tend to oscillate, with maxima or minima near these integers. Furthermore, possible dynamical zeros of the individual Regge amplitudes, at these exceptional points, can either cancel or reinforce the above oscillations.

Thus we immediately expect dips in  $d\sigma/dt$ , associated with some of the exceptional points. It is nontrivial, however, to ensure that dips appear only where required by experiment—and in particular, that they be absent in  $pp$  scattering. A suitable dynamical *Ansatz*, that reproduces the gross features of the data, is the following correlated cyclic residue structure:

$$\begin{aligned} \beta(t) &\approx \lambda(t) \sin^2(\tfrac{1}{2}\pi\alpha), \\ \bar{\beta}(t) &\approx \lambda(t) \cos^2(\tfrac{1}{2}\pi\alpha), \end{aligned}$$

for the range of  $t$  considered. The crucial observation is that  $\beta$  and  $\bar{\beta}$  must have double zeros at right-signature points and no zeros at wrong-signature points<sup>3,4</sup>; also their magnitudes must be correlated. The resulting cross sections are the following:

$$\begin{aligned} d\sigma(NN)/dt &= \gamma^2 + 2\gamma\lambda s^{\alpha-1} + \lambda^2 s^{2\alpha-2}, \\ d\sigma(\bar{N}N)/dt &= \gamma^2 - 2\gamma\lambda s^{\alpha-1} \cos\pi\alpha + \lambda^2 s^{2\alpha-2}, \\ d\sigma(\pi N)/dt &= \tilde{\gamma}^2 \\ &\quad + [2\tilde{\gamma}\tilde{\lambda} s^{\alpha-1} + \tilde{\lambda}^2 s^{2\alpha-2}] \sin^2(\tfrac{1}{2}\pi\alpha). \end{aligned}$$

In this model  $NN$  is smooth; dips occur in  $\pi N$  and  $\bar{N}N$  at  $\alpha = 0, -2, -4, \dots$ ;  $d\sigma(NN)/dt \geq d\sigma(\bar{N}N)/dt$  with equality reached only at  $\alpha = -1, -3, \dots$ . The  $t$  positions of these dips can be read from Fig. 1. The dips come from amplitudes that are falling rapidly as  $s$  increases. Measurements of the  $s$  dependence of  $d\sigma/dt$  at large  $t$  will provide a crucial test of our model. Since

$\lambda s^{\alpha-1}$  is a rapidly decreasing function of  $t$ , the positions of the maxima in  $\pi N$  and  $\bar{N}N$  will be shifted somewhat inward from the  $t$  values at which  $\alpha = -1, -3, \dots$ . A difference in the  $t$  dependence of the residues in  $\pi N$  and  $NN$  scattering may cause some relative differences in structural details.

We have made drastic simplifications, for the sake of clarity. In reality, we expect many small corrections to the above model, including (i) deviations from  $\alpha_{P'}(t) = \alpha_{\omega}(t)$ ; (ii) deviations from the cyclic residue approximation (keeping the same zeros, however); (iii) presence of spin and isospin dependence; (iv) presence of other Regge poles and cuts; (v) deviations from  $\alpha_P(t) = 1$ . Such corrections can be invoked to explain slight shifts in the minima from one process to another (such as the second minima in  $\pi^-p$  and  $\bar{p}p$ ), the departure of  $pp$  from complete smoothness near  $t = -1.6$  (where  $\alpha \approx -1$ ), and the crossover phenomena<sup>5</sup> at  $t \approx -0.15$ . The principal weakness of our conjecture is the noncoincidence at 5.9 GeV/c of the second minima in  $\pi^-p$  and  $\bar{p}p$ . Background effects like those discussed above may explain the shift, but this remains to be quantitatively demonstrated.

Our interpretation of the minima suggests the following predictions:

- (i)  $K^+p$  and  $K^-p$  scattering should qualitatively resemble  $pp$  and  $\bar{p}p$ .
- (ii) The oscillations must disappear as inverse powers of  $s$  as  $s \rightarrow \infty$  at fixed  $t$ .
- (iii) To the extent that the oscillations depend upon a pure imaginary  $P$  term, the model implies a fixed lower bound on  $d\sigma/dt$  at all  $t$  as  $s \rightarrow \infty$ .
- (iv) Similar dips are likely to occur at large  $t$  in inelastic two-body reactions which are mediated by  $P'$  or  $\omega$  exchanges.

Finally, if the viewpoint suggested here is correct, the new elastic-scattering dips constitute the first evidence for linearly falling trajectories in the region  $\alpha(t) < 0$ . This has important theoretical implications<sup>6</sup> and deserves further experimental confirmation.

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<sup>1</sup>R. Rubinstein, A. Ashmore, C. J. S. Demerell, W. R. Frisken, J. Orear, D. P. Owen, F. C. Peterson, A. L. Read, D. G. Ryan, and D. H. White, in Proceedings of the Topical Conference on High Energy Collisions of Hadrons, CERN, 1968 (to be published); M. L. Perl, *ibid.*; J. V. Allaby, A. N. Diddens, A. Klovning, E. Lillethun, E. J. Sacharidis, K. Schliipmann, and A. M. Wetherall, *ibid.*; M. Longo, *ibid.*; V. Barger, *ibid.*

<sup>2</sup>The Pomernanchuk amplitude is chosen to resemble a fixed pole with  $\alpha_P(t) \approx 1$  in order to give the observed trend towards decreasing shrinkage in  $d\sigma(pp)/dt$  at higher energies and the nonshrinkage in  $d\sigma(\pi^+p)/dt$ . We need no specific hypothesis about the nature of the Pomernanchuk  $J$ -plane singularity (i.e., moving pole,

fixed pole, branch cuts, etc.).

<sup>3</sup>This differs from the traditional assignment of linear amplitude zeros at nonsense wrong-signature points. Such a wrong-signature nonsense zero for  $\omega$  would lead to  $d\sigma(\bar{p}p)/dt > d\sigma(pp)/dt$  in some  $t$  range. We make no pretense of understanding the dynamical origin of the empirically required behavior at right- and wrong-signature exceptional points.

<sup>4</sup>In the special case  $\alpha = 0$  a qualitatively similar prescription has been used by C. B. Chiu, S. Y. Chu, and L. L. Wang, Phys. Rev. **161**, 1563 (1967).

<sup>5</sup>V. Barger and L. Durand, III, Phys. Rev. Letters **19**, 1295 (1967).

<sup>6</sup>According to present theoretical ideas, an indefinitely falling hadron trajectory implies that the hadron is not composed of elementary objects (such as quarks) but is instead a composite of the hadrons. Cf. G. F. Chew, University of California Radiation Laboratory Report No. UCRL-17483, 1967 (unpublished).

## POLARIZATION IN $\pi^-p \rightarrow \eta n$ AND THE RATIO OF $F$ TO $D$ COUPLING OF $N_\gamma(2650)$

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We explain the results of recent polarization measurements in the reaction  $\pi^-p \rightarrow \eta n$  at intermediate energies by using a simple interference model. A possible determination of the ratio of  $F$  to  $D$  coupling of  $N_\gamma(2650)$  is discussed and the conclusion drawn is unaffected in spite of the ambiguity regarding the sign of the  $A_2$  residue associated with the spin-flip amplitude.

The polarization in  $\pi^-p \rightarrow \eta n$  has recently been measured<sup>1</sup> at 3.20, 3.47, and 5.00 GeV/c. The results show a significantly large polarization at 3.20 GeV/c in the region of  $0.14 < |t| < 0.25$ . This paper attempts to explain these results and possibly to deduce the ratio of  $F$  to  $D$  coupling of  $N_\gamma(2650)$ . A Regge-pole model of the  $\pi^-p \rightarrow \eta n$  process has been discussed by several authors<sup>2,3</sup> and we have adopted a similar method.

The polarization at 5.00<sup>1</sup> and 11.9 GeV/c<sup>4</sup> may be consistent with zero although the experimental errors are not as small as those in the  $\pi p$  charge-exchange polarization. At high energy where the direct-channel contributions can be ignored, we assume that the reaction  $\pi^-p \rightarrow \eta n$  is dominated by one Regge-pole exchange, the  $A_2$ , in the  $t$  channel. At lower energies, we assume a simple interference model<sup>5</sup> of the  $A_2$  pole exchange in the  $t$  channel and a resonance in the direct channel.

Scattering amplitudes,  $f^{\eta n}$  (nonflip) and  $g^{\eta n}$

(spin flip), are given, respectively, by

$$f_A^{\eta n} = f_A^{\eta n} + f_{\text{res}}^{\eta n} \quad (1)$$

and

$$g_A^{\eta n} = g_A^{\eta n} + g_{\text{res}}^{\eta n} \quad (2)$$

The amplitudes due to the exchange of the  $A_2$  Regge pole are given by

$$f_A^{\eta n} = -(Mm/4\pi s^{\frac{1}{2}})F(s, t)a_A^{(t)}b_{1A}^{\eta n}(t) \quad (3)$$

and

$$g_A^{\eta n} = (m/16\pi)F(s, t)a_A^{(t)} \times [b_{1A}^{\eta n}(t) - a_A^{(t)}b_{2A}^{\eta n}(t)] \sin \theta \quad (4)$$