TOTAL NEUTRON CROSS SECTION OF ORIENTED ^{165}Ho FROM 2 TO 135 MeV *

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The effect of nuclear orientation on the total neutron cross section of the highly deformed ^{165}Ho nucleus has been measured from 2 to 135 MeV. The data are successfully fitted by adiabatic coupled-channel calculations. The origin of this effect can also be understood by a semiempirical model which makes use of both the black-nucleus model and the nuclear Ramsauer effect.

Several measurements have been made at discrete neutron energies of the effect of nuclear deformation on the total neutron cross section of ^{165}Ho.[1-3] This effect, $\Delta\sigma_{\text{def}}$, defined as σ_t(oriented)-σ_t(unoriented), is the difference in the total neutron cross section due to nuclear orientation. Although the neutron energies, E_n, used for these measurements (0.35,[1] 8,[3] 14,[2] and 15[3] MeV) extended over a relatively large range, the results always showed that the sign of $\Delta\sigma_{\text{def}}$ was that which one would expect from a black nucleus,[2] namely $\Delta\sigma_{\text{def}} > 0$ ($\Delta\sigma_{\text{def}} < 0$) if the prolate holmium nucleus was oriented perpendicular[1,2] (parallel[3]) to the neutron beam. The distorted-wave Born-approximation calculation of Davies et al.[4] at 2, 5, and 14 MeV showed, however, that the sign of $\Delta\sigma_{\text{def}}$ does change. Furthermore, a detailed analysis using the adiabatic coupled-channel calculation (ACC) has been made[3] for the neutron energy range 0.35-15 MeV and showed that $\Delta\sigma_{\text{def}}$ undergoes one oscillation in this energy interval as well as a sign change. Although further oscillations were expected above 15 MeV, no theoretical calculations were made. The present work, which consisted of both the experimental determination of $\Delta\sigma_{\text{def}}$ over the complete energy range 2-135 MeV and the corresponding theoretical analysis, was motivated primarily by the interest in confirming the oscillatory nature and sign change in $\Delta\sigma_{\text{def}}$, and secondly, to see at what energy $\Delta\sigma_{\text{def}}$ would tend, if it ever did, to the value predicted by the black-nucleus model.

These measurements[5] were made with the neutron time-of-flight spectrometer at the Harwell synchrocyclotron.[6] The oriented ^{165}Ho target was produced by cooling a large polycrystalline sample to 0.32°K in a magnetic field of 55 kOe by using the National Bureau of Standards ^{3}He refrigerator.[7] The over-all experimental layout is shown in Fig. 1. Since the neutron time-of-flight spectrometer has been discussed in detail elsewhere,[6] we shall only touch upon those points which are relevant for our particular experimental set-up. The neutron beam, defined by collimators C_1 and C_2, was 1.27 cm square at the holmium sample. Although measurements for σ_t were made in

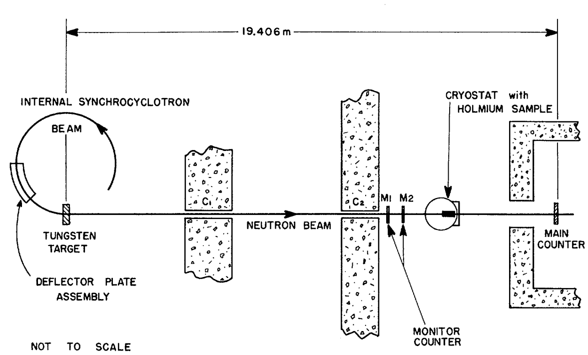

FIG. 1. The experimental arrangement. The direction of nuclear orientation was along the beam direction.

the usual way (sample in–sample out), those for $\Delta\sigma_{\mathrm{def}}$ were made with the sample in the beam at all times, changing just the degree of nuclear orientation. This was achieved by cycling the sample temperature between 4.2 and 0.32°K. Since data were taken at each of those two temperatures for periods up to 12 h, reliable beam monitoring was required. This was achieved by two scintillation counters M_1 and M_2 shown in Fig. 1.

Although the $\Delta\sigma_{\mathrm{def}}$ depends only upon nuclear alignment, and measurements could be made with the holmium-metal single crystal used in Ref. 2, the measured effect would be extremely small at certain neutron energies (e.g., around 8 MeV; see Ref. 3). Since no thicker single crystal of holmium metal was available, a thick (12.0 cm) polycrystalline holmium sample was used in a manner similar to that of Ref. 3. A new cryostat[8] was built for the National Bureau of Standards ^{3}He refrigerator which incorporated a large superconducting solenoid (in a separate ^{4}He bath) mounted at right angles to the vertical axis of the cryostat. Therefore, all the radiation shields at the bottom of the cryostat bend through 90°. This geometry, although technically complicated, enabled us to keep all the cryogenic liquids out of the neutron beam and make use of relatively thin windows in the radiation shields along the beam path. The holmium sample was cooled to 0.32 ± 0.01°K by being in thermal contact with the bottom of the ^{3}He bath.

The calculation of the nuclear orientation parameters (statistical tensors)[9] for a polycrystalline holmium sample has been described previously[3] and will not be reviewed here. Once the temperature and magnetization of the sample are known, the statistical tensors, B_k, can be calculated directly. Since no reliable magnetization data exist above 26 kOe[10] for polycrystalline holmium metal, magnetization measurements were made up to 60 kOe at the National Magnet Laboratory[11] on a spherical sample which was cut from an unused part of our original holmium sample. The results showed that for the values of the magnetic field used in our measurements (50 and 55 kOe), the ratio of the magnetization to saturation magnetization M/M_∞ is 0.80 ± 0.03. Thus the degree of nuclear alignment $B_2/B_2(\mathrm{max})$ achieved was 0.30 ± 0.05.

The total neutron cross section of ^{165}Ho was measured over the energy range 1-135 MeV and is shown in Fig. 2. The error bars are due to counting statistics alone. The agreement with previous measurements[2,3,12] made at discrete neutron energies is very good. Earlier measurements of the total neutron cross sections of Cd and Pb[13,14] over approximately the same energy region are included in Fig. 2 since they are pertinent to the theoretical analysis which is presented later on.

The measurement of $\Delta\sigma_{\mathrm{def}}$ was done in two stages; first a series of measurements was made with the superconducting solenoid turned off and then with it turned on. Each measurement determined the normalized transmitted neutron energy spectrum for a few hours with the sample "cold" (0.32°K) and then for a few hours with the sample "warm" (4.2°K). In zero magnetic field there can, of course, be no nuclear orientation at any temperature in a polycrystalline sample. Thus, the "field-off" measurements serve to indicate any change in transmission associated with changing the sample's temperature from 0.32 to 4.2°K. These measurements showed a small effect which was not strongly energy dependent. Similar measurements were made with the magnetic

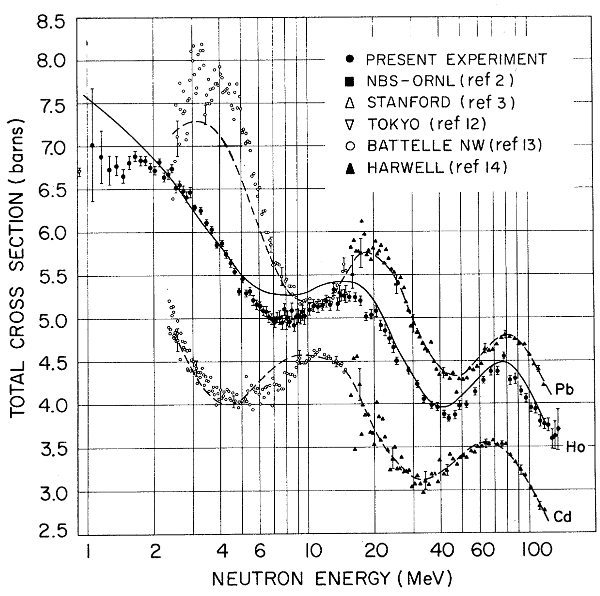

FIG. 2. The total neutron cross section of cadmium, holmium, and lead. The curves for cadmium and lead are the results of optical-model calculations. The optical parameters were obtained from a least-squares search program using both the total-neutron and total-reaction cross sections of cadmium and lead. The curve for holmium was from adiabatic coupled-channel calculations using these same parameters (see text).

field on and the results are shown in Fig. 2, the "field off" contribution to $\Delta\sigma_{\rm def}$ having been subtracted out.

The above experimental results were analyzed using an optical-model potential of the form

$$V(r,\theta,\varphi)=-V(1+e')^{-1}-iW(1+\bar{e}')^{-1}-4iW_D\bar{e}'(1+\bar{e}')^{-2}-V_{s0}(\vec{\sigma}\cdot\vec{l})(r_\pi^{\,2}/ar)e'(1+e')^{-2} \tag{1a}$$

with

$$e'=\exp\{[r-R(\theta,\varphi,\theta_i)]/a\},\quad R(\theta,\varphi,\theta_i)=R_0[1+\beta Y_{20}(\theta')];$$

$$\bar{e}'=\exp\{[r-\bar{R}(\theta,\varphi,\theta_i)]/\bar{a}\},\quad \bar{R}(\theta,\varphi,\theta_i)=\bar{R}_0[1+\beta Y_{20}(\theta')],$$

$$R_0=r_0A^{1/3},\quad R_0=r_0A^{1/3}, \tag{1b}$$

where the angles θ and φ refer to the space-fixed system, θ' to the body-fixed system, and θ_i stands for the Euler angles between these two systems.

In order to fix the parameters in (1), we first fitted the optical-model predictions to the data[13,14] on the total cross section σ_t and the total-reaction cross section σ_R of Cd and Pb. The fit was made over the same range of neutron energies as our present work. This having been done, the calculation of σ_t and $\Delta\sigma_{\rm def}$ for ^{165}Ho could be carried out with no further adjustment of parameters.

In calculating σ_t and σ_R for Cd and Pb, it was assumed that V and W_D decrease linearly with E_n, while W increases linearly; thus a set of three parameters V, W, and W_D is replaced by a set of six adjustable parameters. The other parameters a, $\bar{a}$, and $r_0=\bar{r}_0$ were left as energy-independent adjustable parameters, while V_{s0} was fixed at 7 MeV. Automatic search runs were made[15] to determine the above nine adjustable parameters [β in Eq. (1) is zero for Cd and Pb], and the best set of parameters obtained is the following: $V=47.30-0.227E_n$, $W=0.459+0.111E_n$, $W_D=4.28-0.0414E_n\geqslant 0$ (all in MeV), $r_0=\bar{r}_0=1.211$, $a=0.6812$, and $\bar{a}=0.6448$ (all in fm). The fit to σ_t is shown in Fig. 2 and is seen to be very good.

Having thus fixed the optical-model parameters, we can now proceed to the calculation of σ_t and $\Delta\sigma_{\rm def}$ for ^{165}Ho. It is known,[16] however, that $\beta=0.33$ for ^{165}Ho, and this nonvanishing value of β makes the use of the simple optical-model calculation invalid contrary to the cases for Cd and Pb; we must use the coupled-channel calculations instead. However, since E_n is sufficiently high over most of the range of interest, we can use the ACC[17] which can be carried out much faster than the nonadiabatic coupled-channel calculations. The details of ACC and its use for calculating σ_t and $\Delta\sigma_{\rm def}$ were reported previously,[2,17] and will not be repeated here. We note, however, that in the ACC calculations the values of W and W_D were reduced by 20% compared with their corresponding values for Cd and Pb, since part of the absorption in the elastic channel is now taken into account explicitly in the inelastic scattering processes in the coupled excited states.[17] In the calculation of $\Delta\sigma_{\rm def}$ the degree of nuclear orientation of our sample was taken into account by using the magnetic populations directly.

The ACC was performed by using the computer program JUPITOR-1.[18] The IBM 360/75 computer at Oak Ridge National Laboratory was used for most of the lower E_n, while for several higher values of E_n where the computations take a longer time the CDC 6600 computer at Brookhaven National Laboratory was used.[19] The theoretical σ_t and $\Delta\sigma_{\rm def}$ obtained are compared with experimental data in Figs. 2 and 3 and, as is seen, the agreement with experiment is very good (particularly for $\Delta\sigma_{\rm def}$), although the theoretical σ_t is slightly too large around $E_n=8$ MeV.[20] Because we used no adjustable parameters, these fits are significant and we may conclude that our data on σ_t and $\Delta\sigma_{\rm def}$ can be accounted for very well by ACC.

Although we thus have a good theoretical understanding of our data, it would be instructive to show that at least $\Delta\sigma_{\rm def}$ can also be fitted nicely without any involved numerical calculations. The idea is to use some concepts of the black-nucleus model and of the nuclear Ramsauer effect,[21] and the experimental σ_t itself. If we consider an aligned target with the symmetry axes of the nuclei parallel to the beam and contrast it with an unaligned target, we see that (1) there is a decrease in the effective area of the nuclei seen by the beam, and

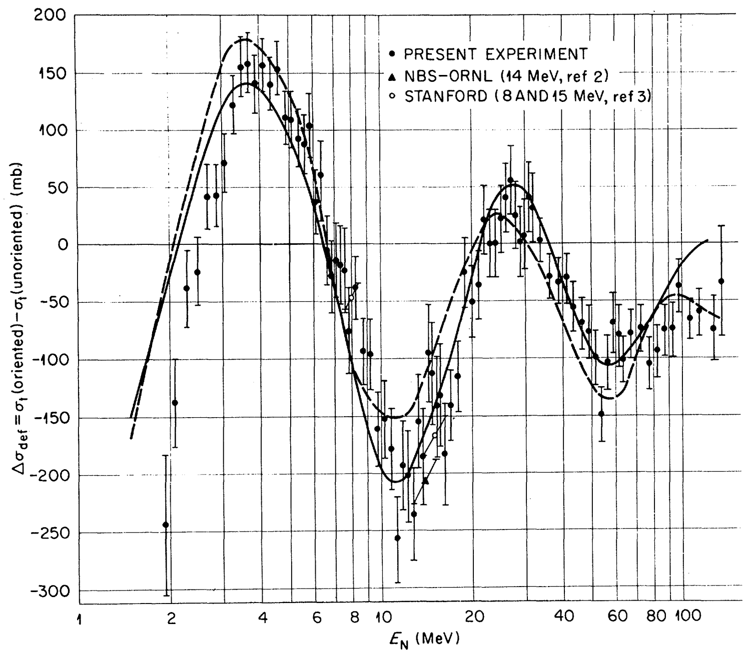

FIG. 3. $\Delta\sigma_{def}$ for holmium. The solid curve is the result of adiabatic coupled-channel calculations using the same optical-model parameters as those used in Fig. 2 and for our degree of nuclear alignment. The dashed curve was obtained from a semiempirical model (see text). The last energy point is 135 MeV.

(2) there is a corresponding increase in the effective path length through the nuclei. The first effect serves to reduce the total cross section by the value calculated in the black-nucleus model, namely $\Delta\sigma_{def} = 0.0242\pi(R+\lambda\!\!\!^{-})^2$, the value of the numerical constant being given by the value of $\beta = 0.33$ and our degree of nuclear alignment. The second effect causes a shift in the energies at which the maxima and minima in the values of σ_t occur. The shift in energy $\Delta E(E)$ of the maxima and minima of the cross section can be calculated knowing the change in path length. Assuming that $\Delta E(E)$ can also be calculated at intermediate energies by interpolation, an approximate value of $\Delta\sigma_{def}(E)$ is given by $\Delta\sigma_{def}(E) = -[d\sigma_t(\text{unoriented})/dE]\Delta E(E) - 0.0242\pi(R+\lambda\!\!\!^{-})^2$. The result of such a calculation is given as the dashed curve in Fig. 3. The general features of the shape and oscillations of $\Delta\sigma_{def}$ are well reproduced over the whole energy range, indicating that the physical picture presented by this simple model is valid.

As seen in Fig. 3, the experimental values of $\Delta\sigma_{def}$ are not inconsistent within the experimental error with the black-nucleus value of -95 mb, for $E_n \gtrsim 70$ MeV. However, the general tendency of the data for these energies indicates that $\Delta\sigma_{def}$ is still oscillating, and therefore, the black-nucleus limit has not yet been reached. This conclusion has support from the data on σ_t which, as is seen in Fig. 2, is still oscillating at these high energies.

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[20]Actually Perey's search run, Ref. 15, was made for $E_n = 10$ to 120 MeV, and we extrapolated his parameters to $E_n < 10$ MeV. This is the reason why the fit is poorer for these lower energies. An extension of the search run to $E_n < 10$ MeV is underway and it is hoped that a better fit will be obtained over the entire range of E_n.

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UNITARITY CORRECTIONS TO LOW-ENERGY PARAMETERS IN SOFT-PION CALCULATIONS

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We unitarize the results of soft-pion current-algebra calculations of low-energy parameters for s-wave pion-nucleon scattering in the $I=\frac{1}{2}$ channel, using the N/D formalism. We isolate clearly corrections arising from unitarity, off-shell kinematics, and off-shell dynamics.

Considerable success has been achieved in recent years in understanding a wide range of scattering and decay phenomena by use of the chiral algebra of currents[1] in conjunction with the hypothesis of partial conservation of axial-vector currents (PCAC).[2] We recall in particular the calculations of pion-nucleon scattering lengths by Weinberg,[3] Tomozawa,[4] and others.[5]

Such calculations of low-energy parameters in elastic scattering have several characteristic features. Use of the PCAC hypothesis necessitates a consideration of amplitudes with both initial and final pions off their mass shells. In each partial wave and isospin channel one is thus led to consider functions of the form $f_{l\pm}{}^{I}(s,q^2)$, where s is the square of the total center-of-mass energy in the direct channel and q^2 is the variable mass of the off-shell pions (initial and final pions are assumed to have the same mass). For comparison with experiment, the amplitude is evaluated at the point $q^2=0$. Weinberg combined the off-shell limit with the soft-pion limit $q_\mu=0$. Schnitzer[6] subsequently investigated the amplitude in the same off-shell limit without making the pions soft. In both cases, smooth extrapolation to $q^2=m_\pi{}^2$ is assumed, which allows comparison of threshold parameters evaluated at $s=m_N{}^2$ against experimental values.

We adopt the viewpoint that the extrapolation of low-energy parameters in pion-nucleon scattering evaluated at the unphysical points $s=m_N{}^2$ and $q^2=0$ to the physical points $s=(m_N+m_\pi)^2$