J. M. Wang, Phys. Rev. 160, 1560 (1967).

[9]S. Mandelstam, in the Conference on πN Scattering, at Irvine, California, 1-2 December 1967 (unpublished).

[10]The pion and π_c trajectories have been taken to be degenerate, passing through μ_π^2 and having the canonical slope 1 GeV. Since the fit presented covers a very narrow range of t, it is insensitive to the slopes of these trajectories. Moreover, we have neglected the possible contribution of π_c to F_1, since it is not needed to fit the near-forward data.

[11]B. Richter, Stanford Linear Accelerator Report No. SLAC-PUB-353, 1965 (unpublished).

[12]Our conclusion differs from that of J. P. Ader, M. Capdeville, and Ph. Salin [CERN Report No. TH. 803 (unpublished)], who ignored the $t=0$ condition. Independent confirmation of this condition can easily be deduced from the method used by E. Leader, to be published.

[13]J. K. Kim, Phys. Rev. Letters 19, 1074 (1967).

[14]The details of the K^* exchange are unimportant here. We chose $K^*(1420)$, a member of the A_2 octet, because the A_2 seems to have a rather flat trajectory [R. Phillips and W. Rarita, Phys. Letters 19, 598 (1965)] which avoids a dip in the region explored.

DECAY RATES OF $\Delta S=-\Delta Q$ TRANSITIONS AND POSSIBLE $\Delta S=2$ LEPTONIC DECAYS*

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Relations between possible $\Delta S=-\Delta Q$ decay rates of K^0-$\bar{K}^0$, K^+, Σ^+, and Ξ^0 are given. A connection between $\Delta S=-\Delta Q$ and $\Delta S=2$ leptonic decays is also suggested.

The degree of validity of the selection rule $\Delta S=\Delta Q$ remains one of the most interesting questions in weak-interaction physics. Much effort has been dedicated to the search for possible decays which violate this rule, e.g., $K^0 \to \pi^+ e^- \bar{\nu}_e$; $K^+ \to \pi^+\pi^+ e^- \bar{\nu}_e$; $\Sigma^+ \to n e^+ \nu_e$. The experimental information, summarized below, is becoming increasingly significant, and it is likely to improve considerably in the near future. In this note we present some possible relations between the $\Delta S=-\Delta Q$ decay rates of K^0-$\bar{K}^0$ and Σ^+, and the rate of the still undiscovered $\Xi^0 \to \Sigma^- e^+ \nu_e$ decay. Also, we discuss a possible connection between $\Delta S=-\Delta Q$ and $\Delta S=2$ leptonic decays. As a result, we find that decays of the type[1] $\Xi^0 \to p l^- \bar{\nu}_l$ and $\Xi^- \to n l^- \bar{\nu}_l$ might exist with detectable branching ratios.

We shall first summarize the present experimental knowledge on $\Delta S=-\Delta Q$ decays:

(i) Results on K_{l3}^0 decays are usually given in terms of a parameter which is the ratio of the $\Delta S=-\Delta Q$ amplitude $A(\bar{K}^0 \to \pi^- l^+ \nu_l) \equiv g$ to the $\Delta S=\Delta Q$ amplitude $A(K^0 \to \pi^- l^+ \nu_l) \equiv f$; i.e., $g/f=|x|e^{i\varphi}$. A weighted average of world data[2] gives the result

$$|x|=0.22\pm0.08, \quad \varphi\sim60^\circ. \tag{1}$$

(ii) No events of the type $K^+ \to \pi^+\pi^+ e^- \bar{\nu}_e$ have been observed to date. Based on 208 $K^+ \to \pi^+\pi^- \times e^+ \nu_e$ events which have been reported,[3] one can set the following upper limit:

$$\Gamma(K^+ \to \pi^+\pi^+ e^- \bar{\nu}_e)/\Gamma(K^+ \to \pi^+\pi^- e^+ \nu_e) < 2\,\%. \tag{2}$$

(iii) There are two reported events which are candidates[4] for $\Sigma^+ \to n\mu^+\nu_\mu$ and one reported event which is a candidate for $\Sigma^+ \to ne^+\nu_e$.[5] If these three candidates are taken as certain, then we have the following branching ratio[6]:

$$\frac{\Gamma(\Sigma^+ \to ne^+\nu_e)+\Gamma(\Sigma^+ \to n\mu^+\nu_\mu)}{\Gamma(\Sigma^+ \to \text{all modes})} \sim (4\pm3)\times10^{-5}, \tag{3}$$

consistent with an independent upper limit recently published[7]:

$$\frac{\Gamma(\Sigma^+ \to ne^+\nu_e)+\Gamma(\Sigma^+ \to n\mu^+\nu_\mu)}{\Gamma(\Sigma^- \to ne^-\bar{\nu}_e)+\Gamma(\Sigma^- \to n\mu^-\bar{\nu}_\mu)} \lesssim 3.4\,\% \text{ (90\% confidence level)}. \tag{4}$$

We shall now discuss some theoretical considerations concerning these processes. By analogy to the usual weak processes, we shall assume that $\Delta S=-\Delta Q$ decays are described by an effective Hamiltonian of the current×current type,[8]

$$H=\frac{G}{\sqrt{2}}\sum_{l=e,\,\mu} J^{\mu}(\Delta S=-\Delta Q)\bar{\psi}_l \times\gamma^{\mu}(1-\gamma_5)\psi_{\nu_l}+\text{h.c.}, \quad (5)$$

where G is the Fermi coupling constant, $G = 1.02\times10^{-5}/m_p^2$. We shall also assume that $J^{\mu}(\Delta S=-\Delta Q)$ consists of a vector part and an axial-vector part: $J^{\mu}(\Delta S=-\Delta Q)=V^{\mu}(\Delta S=-\Delta Q) + A^{\mu}(\Delta S=-\Delta Q)$. Decays of the type $K^0\to\pi^+l^-\bar{\nu}_l$ and $\bar{K}^0\to\pi^-l^+\nu_l$ can then be implemented by the vector component $V^{\mu}(\Delta S=-\Delta Q)$, while $K^+ \to\pi^+\pi^+l^-\bar{\nu}_l$ decays can only proceed through the axial component $A^{\mu}(\Delta S=-\Delta Q)$.[9] A priori, both the axial and vector components can contribute to the decays $\Sigma^+\to nl^+\nu_l$ and $\Xi^0\to\Sigma^-l^+\nu_l$. More specifically, the matrix element of the hadronic current for these hyperon leptonic decays has the following structure[10]:

$$\langle p'|J^{\mu}|p\rangle=\bar{u}(p')[T_V\gamma^{\mu}-T_A\gamma^{\mu}\gamma_5]u(p), \quad (6)$$

where p and p' denote the energy momenta of the initial and final hyperon; T_V and T_A are unknown coupling constants, in principle different for each decay:

$$T_V=a \text{ and } T_A=b, \quad \text{for } \Sigma^+\to nl^+\nu_l; \quad (7)$$

$$T_V=a' \text{ and } T_A=b', \quad \text{for } \Xi^0\to\Sigma^-l^+\nu_l. \quad (8)$$

For comparison, we shall recall that in the Cabibbo theory,[11] the $\Delta S=\Delta Q$ decays corresponding to those in Eqs. (7) and (8), i.e., $\Sigma^-\to nl^-\bar{\nu}_l$ and $\Xi^0\to\Sigma^+l^-\bar{\nu}_l$, are similarly described by matrix elements as in Eq. (6) with

$$T_V=\sin\theta_V \text{ and } T_A=0.38\sin\theta_A, \quad \text{for } \Sigma^-\to nl^-\bar{\nu}_l;$$

$$T_V=-\sin\theta_V \text{ and } T_A=1.18\sin\theta_A, \quad \text{for } \Xi^0\to\Sigma^+l^-\bar{\nu}_l;$$

where[12] $\theta_V=0.21$ and $\theta_A=0.27$ are the vector and axial-vector Cabibbo angles. In terms of the parameters a, b and a', b' we obtain the decay rates listed in the second column of Table I. Comparison between the rates for $\Sigma^+\to nl^+\nu_l$ and the upper limit quoted in Eq. (4) leads to the result

$$|a|^2+3|b|^2\lesssim3.5\times10^{-3} \text{ (90\% confidence level).}$$

Let us now make the assumption that the ratios of $\Delta S=-\Delta Q$ vector coupling constants to the corresponding $\Delta S=\Delta Q$ are of the same order magnitude for meson and hyperon decays; i.e.,

$$|a/\sin\theta_V|\sim|a'/\sin\theta_V|\sim|x| \ (=0.22\pm0.08). \quad (9)$$

This implies a lower limit for the various $\Delta S = -\Delta Q$ hyperon decay rates. Using the experimental value quoted in Eq. (1) we obtain the results given in the third column of Table I. It is interesting that the limits obtained for the rates of $\Sigma^+\to nl^+\nu_l$ decays are compatible with the experimental values quoted in Eqs. (3) and (4). The rates predicted for $\Xi^0\to\Sigma^-l^+\nu_l$ decays are smaller than those for $\Sigma^+\to nl^+\nu_l$ because less phase space is available. We must emphasize, however, that events of the type $\Xi^0\to\Sigma^-l^+\nu_l$ should be less ambiguous than $\Sigma^+\to nl^+\nu_l$ decay, because no intermediate π^+ decay could simulate such a decay.

The existence of $\Delta S=-\Delta Q$ transitions can also be tested in neutrino experiments. Detection of events of the type $\nu_l n\to\Sigma^+l^-$ would be an unambiguous proof of the existence of $\Delta S = -\Delta Q$ transitions. From the previous considerations on decay processes, cross sections for $\nu_l n\to\Sigma^+l^-$ $(\Delta S=-\Delta Q)$ are expected to be two orders of magnitude smaller than those

Table I. Results on $\Delta S=-\Delta Q$ hyperon decays.

Decay	Rate (sec^{-1})	Lower limit for total branching ratio assuming Eq. (9)
$\Sigma^+\to ne^+\nu_e$	$(\lvert a\rvert^2+3\lvert b\rvert^2)\ 7.7\times10^7$	1.3×10^{-5}
$\Sigma^+\to n\mu^+\nu_\mu$	$(\lvert a\rvert^2+3\lvert b\rvert^2)\ 3.2\times10^7$	5.6×10^{-6}
$\Xi^0\to\Sigma^-e^+\nu_e$	$(\lvert a'\rvert^2+3\lvert b'\rvert^2)\ 2.1\times10^6$	1.4×10^{-6}
$\Xi^0\to\Sigma^-\mu^+\nu_\mu$	$(\lvert a'\rvert^2+3\lvert b'\rvert^2)\ 3.7\times10^3$	0.2×10^{-8}

for $\bar{\nu}_l n \to \Sigma^- l^+$ ($\Delta S = \Delta Q$).

From the point of view of unitary symmetry of hadronic currents,[13] the lowest SU(3) representation which can accommodate currents with $S=1$, $Q=-1$ and $S=-1$, $Q=1$ are, respectively, $\underline{10}$ and $\underline{10}^*$. In fact, at the limit of exact SU(3) symmetry, these are the only representations in which the vector current $V^\mu(\Delta S = -\Delta Q)$ can induce the decays $\bar{K}^0 \to \pi^- l^+ \nu_l$ and $K^0 \to \pi^+ l^- \bar{\nu}_l$.

If $J^\mu(\Delta S = -\Delta Q)$ belongs to the $\underline{10}$ and $\underline{10}^*$ representations only, then at the limit of exact SU(3) symmetry, the coupling constants in Eqs. (7) and (8) are related thus: $a = a'$ and $b = b'$. This leads to the following prediction[14]:

$$\Gamma(\Xi^0 \to \Sigma^- e^+ \nu_e) = 2.7 \times 10^{-2} \times \Gamma(\Sigma^+ \to n e^+ \nu_e).$$

We remark that $\underline{10}^*$ is also the lowest SU(3) representation which can accommodate currents with $S=2$ and $Q=1$, i.e., currents $J^\mu(\Delta S=2)$ which can implement decays such as $\Xi^0 \to p l^- \bar{\nu}_l$ and $\Xi^- \to n l^- \bar{\nu}_l$ through a current×current Hamiltonian as in Eq. (5). Predictions can be made if both the $J^\mu(\Delta S = -1, \Delta Q = 1)$ and $J^\mu(\Delta S = 2)$ currents belong to the same $\underline{10}^*$ representation of SU(3). Assuming that these currents are coupled to the leptonic current with equal weight, we get

$$\Gamma(\Xi^0 \to p e^- \bar{\nu}_e) = 6.9 \times \Gamma(\Sigma^+ \to n e^+ \nu_e); \qquad (10a)$$

$$\Gamma(\Xi^0 \to p \mu^- \bar{\nu}_\mu) = 11 \times \Gamma(\Sigma^+ \to n \mu^+ \nu_\mu). \qquad (10b)$$

Also, from isospin invariance alone, we have

$$\Gamma(\Xi^0 \to p l^- \bar{\nu}_l) = 0.9\, \Gamma(\Xi^- \to n l^- \bar{\nu}_l).$$

The predictions given in Eqs. (10a) and (10b) depend critically on the assumption of "equal-weight coupling." Indeed, if a rotation of the Cabibbo type (i.e., a rotation of angle $2\theta'$ around the second axis of U spin) is applied to the $\underline{10}^*$ currents, then it is the combination $(\cos^3\theta') \times J^\mu(\Delta S = -\Delta Q) + (\sin^3\theta') J^\mu(\Delta S = 2)$ which, correspondingly, should couple to the leptonic current.[15] A priori, nothing is known about the value of θ'. For $\theta' = 45°$, we get the results given in Eqs. (10a) and (10b). If instead, we take θ' equal to the Cabibbo angle for the octet $\theta \sim 15°$, the predicted rates for $\Xi^0 \to p l^- \bar{\nu}_l$ decays would be roughly three orders of magnitude smaller than those for the corresponding $\Sigma^+ \to n l^+ \nu_l$ decays. Experimental information concerning the rates of $\Xi^0 \to p l^- \bar{\nu}_l$ and $\Xi^- \to n l^- \bar{\nu}_l$ seems very desirable in order to decide this question.

Finally, we wish to comment on $K^+ \to \pi^+ \pi^+ e^- \bar{\nu}_e$ decays. As we have seen,[9] only the axial-vector current $A^\mu(\Delta S = -\Delta Q)$ contributes to these decays. Since the two pions are in a pure isospin state $I=2$, they are emitted in even orbital angular momentum states. Thus, to a very good approximation, the matrix element of the hadronic current describing $K^+ \to \pi^+ \pi^+ e^- \bar{\nu}_e$ decay has the following structure:

$$\langle \pi^+(p_1), \pi^+(p_2) | A^\mu | K^+(p) \rangle = M^{-1} F(s) (p_1 + p_2)^\mu,$$

where $s = (p_1 + p_2)^2$, and M is the K^+ mass.[16] From this, we obtain for the decay rate the following expression:

$$\Gamma(K^+ \to \pi^+ \pi^+ e^- \bar{\nu}_e) = \frac{G^2}{96}\left(\frac{M}{4\pi}\right)^5 \int_{4m^2/M^2}^{1} dx\, |F(x)|^2 \times \left(1 - \frac{4m^2}{M^2 x}\right)^{1/2} [1 - 8x + 8x^3 - x^4 - 12x^2 \ln x],$$

where m is the π^+ mass and $x = s/M^2$. The simplest assumption which we can make is to take $F(x)$ constant. Then, from the upper limit quoted in Eq. (2), we obtain $|F| < 0.28$. To get a feeling for this value, let us recall that using current-algebra techniques, the predicted value of the corresponding form factor F_1 in $K^+ \to \pi^+ \pi^- e^+ \nu_e$ decays is[17] $|F_1| = 0.97 \pm 0.03$. If we extend to axial-vector coupling constants the assumption of the same order of magnitude for ratios $\Delta S = -\Delta Q$ over $\Delta S = \Delta Q$, then we would expect $|F| \sim 0.21$. In fact, the expected value for $|F|$ is likely to be still smaller if energy dependence effects due to final-state interactions are taken into account.[18] Indeed, the possibility of a strong, low-energy, π-π interaction in the $I=0, l=0$ state, compared with the interaction in the $I=2, l=0$ state, might enhance significantly the effect of $F_1(x)$ with respect to that of $F(x)$ in the corresponding decay rates.

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[1]Throughout this note, the symbol l denotes electron or muon.

[2]V. L. Fitch, Lectures at the Second Hawaii Topical Conference, 1967 (unpublished). The latest, published, experimental result (included in the weighted average) gives $|x| = 0.26^{+0.08}_{-0.11}$ and $\varphi = 50°^{+25°}_{-27°}$. [See D. G. Hill et al., Phys. Rev. Letters 19, 668 (1967).]

[3]R. W. Birge et al. (Berkeley-UCLA-Wisconsin Collaboration), University of California Radiation Laboratory Report No. UCRL 17088 (unpublished), and in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, 1966 (University of California Press, Berkeley, California, 1967).

[4]A. Barbaro-Galtieri et al., Phys. Rev. Letters 9, 26 (1962); F. Eisele et al., Z. Physik 205, 405 (1967).

[5]U. Nauenberg et al., Phys. Rev. Letters 12, 679 (1964).

[6]W. Willis, rapporteur's talk at the Proceedings of the International Conference on Elementary Particles, Heidelberg, Germany, 1967 (unpublished).

[7]N. Baggett et al., Phys. Rev. Letters 19, 1458 (1967).

[8]It is well known that, in a universal current×current interaction, the existence of $\Delta S = -\Delta Q$ currents leads in general to $\Delta S = 2$ terms in the nonleptonic Hamiltonian. Yet, as was pointed out by Okun' and Pontecorvo [L. Okun' and B. Pontecorvo, Soviet Phys.-JETP 5, 1297 (1957)] the K_1^0-K_2^0 mass difference places a very severe limit on the magnitude of $\Delta S = 2$ nonleptonic matrix elements. However, it has been shown by B. d'Espagnat [Orsay Report No. Th-115, 1965 (unpublished)] that there exist possible coupling schemes of weak currents which give no $\Delta S = 2$ nonleptonic interactions, while admitting currents with $\Delta S = -\Delta Q$. See also L. Wolfenstein, Nuovo Cimento 24, 859 (1963), and Phys. Rev. 135, B1436 (1964).

[9]Because of the identity of the two final pions there is no contribution from the matrix element of the vector current.

[10]This neglects possible contributions from terms proportional to the momentum transfer.

[11]N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).

[12]N. Brene, L. Veje, M. Roos, and C. Cronström, Phys. Rev. 149, 1288 (1966).

[13]M. Gell-Mann, Phys. Rev. 125, 1067 (1962), and Physics 1, 63 (1964).

[14]This prediction also holds if $J^\mu(\Delta S = -\Delta Q)$ belongs to the 27 representation only. In this case, however, $\langle\pi|J^\mu(\Delta S = -\Delta Q)|K\rangle \to 0$ at the limit of exact SU(3) symmetry.

[15]Here we have assumed that only the U-spin components of the 10* with $U_Z = \pm\frac{3}{2}$ are coupled to the leptonic current. Another possibility is that all the four $U = \frac{3}{2}$ components are coupled to the leptonic current. In the latter case, the 10* currents appear in the combination: $\cos^3\theta' J^\mu(\Delta S = -\Delta Q) + \sqrt{3}\cos^2\theta' \sin\theta' G^\mu(\Delta S = 0) + \sqrt{3}\sin^2\theta' \cos\theta' G^\mu(\Delta S = \Delta Q) + \sin^3\theta' J^\mu(\Delta S = 2)$. The presence of currents $G^\mu(\Delta S = 0)$ and $G^\mu(\Delta S = \Delta Q)$ would lead to corrections for the rates of the usual semileptonic decays (except for the decays $K \to l\bar{\nu}_l$ and $\pi \to l\bar{\nu}_l$). These corrections are proportional to $2\sqrt{3}|x| \sin\theta' \tan\theta'$ for $\Delta S = 0$ transitions and to $2\sqrt{3}|x|\tan^2\theta'$ for $\Delta S = \Delta Q$ transitions.

[16]We recall that the assumption that form factors in K_{l4} decays depend only on the variable s implies that the two final pions are emitted in s and p waves only. See, e.g., N. van Hieu, Zh. Eksperim. i Teor. Fiz. 44, 162 (1963) [translation: Soviet Phys.-JETP 17, 113 (1963)].

[17]S. Weinberg, Phys. Rev. Letters 17, 336 (1966), and 18, 1178 (1967).

[18]For a recent discussion of these effects in the $\Delta S = \Delta Q$ K_{e4} decays, see F. A. Berends, A. Donnachie, and G. C. Oades, Nucl. Phys. B3, 569 (1967); A. Pais and S. B. Treiman, "Pion Phase Shift Information from K_{l4} Decays" (unpublished).

ON INFINITIES IN ELECTROMAGNETIC MASS DIFFERENCES TO ANY ORDER IN α†

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We show to any order in α that the electromagnetic mass difference between the charged and the neutral pion is at most logarithmically divergent and derive a necessary and sufficient condition that the pion mass difference times the isovector form factor is finite to any order in α. All results are valid whether or not perturbation theory makes sense.

Recently, it has been shown by Wick and Zumino[1] and by Gerstein et al.[2] that the electromagnetic mass difference between the charged and the neutral pion is logarithmically divergent in the lowest order of α (for pions on the mass shell). Since the pion mass difference is a physical quantity, this is clearly a highly undesirable situation. From the point of view of principles there is, however, the possibility that perturbation theory might be wrong for some reason, at least so far as the electromagnetic mass differences are concerned. Since we have no direct way of treating the "exact" theory, it is not possible to answer this