of the 30-in. Brookhaven hydrogen bubble chamber. This film is currently being analyzed for other purposes as part of a Columbia-Rutgers-Stony Brook collaboration.

[12]This number is consistent with the number of K^- decays expected from 350 000 incident K^- mesons with an average momentum 220 MeV/c in hydrogen.

[13]One conceivable source of bias would be the fact that in testing for momentum conservation, we used the momentum and angles of each track assuming it to be a pion. That this is not a serious source of bias can be inferred from the fact that the transverse missing momentum distribution for all events had a width of 10 MeV/c when all tracks were interpreted as pions, and a width of 20 MeV/c when all tracks were interpreted as electrons. (The cut on allowed transverse missing momentum was 30 MeV/c.)

[14]This could conceivably arise because of differences in scanning efficiency, differences in the reconstruction-program reject rate for electrons and pions, and in other ways.

[15]We assume that the transition matrix element for (2) is the same as that for ordinary $K_{e3}^{\pm}$ decays except that the Fermi coupling constant G_β has been replaced by $G_{\beta\beta}$. Because (2) can only proceed through $|\Delta I| = \frac{3}{2}$, $|\Delta Q| = 2$ transition, the suppression of $|\Delta I| = \frac{3}{2}$ relative to that of $|\Delta I| = \frac{1}{2}$ amplitude in ordinary semileptonic K decays has been ignored in this estimation.

[16]We have also estimated ξ from the data of the $K^+ \to \pi^+e^+e^-$ experiment [U. Camerini et al., Phys. Rev. Letters 13, 318 (1964)]. Since one cannot differentiate $K^+ \to \pi^-e^+e^+$ from $K^+ \to \pi^+e^+e^-$ by a 1C fit kinematically (when the measured momenta were ignored), we can therefore estimate that $R < (8/9.4)\times 10^5 = 0.85\times 10^{-5}$, which is in agreement with our result, $R < 1.5\times 10^{-5}$.

SYMMETRY BREAKING IN AXIAL-VECTOR SPECTRAL-FUNCTION SUM RULES*

R. J. Oakes
The Institute for Advanced Study, Princeton, New Jersey
(Received 1 December 1967)

Weinberg's first sum rule applied to the axial-vector currents is shown to split into two separately valid sum rules if the usual assumptions concerning SU(3) mixing are adopted. New relations among masses, coupling constants, and mixing angles are obtained in the meson-dominance approximation, e.g., $F_K/F_\pi = 1$, and the implications are discussed.

Weinberg's first sum rule[1] extended to the nonet of axial-vector currents $A_\mu^{\alpha}(x)$ $(\alpha = 0, 1, \cdots, 8)$ reads

$$\int dm^2[m^{-2}\rho_{\alpha\beta}^{(1)}(m^2) + \rho_{\alpha\beta}^{(0)}(m^2)] = s\delta_{\alpha\beta} + s'\delta_{\alpha 0}\delta_{\beta 0}, \tag{1}$$

where s and s' are constants independent of α and β. The spin-0 and spin-1 spectral functions $\rho^{(0)}(m^2)$ and $\rho^{(1)}(m^2)$ are defined by the representation of the axial-vector-current propagator

$$\Delta_{\mu\nu}^{\alpha\beta}(q) = -i\int d^4x\, e^{iqx} \langle 0|TA_\mu^{\alpha}(x)A_\nu^{\beta}(0)|0\rangle = \int dm^2[\rho_{\alpha\beta}^{(1)}(m^2)(-g_{\mu\nu} + m^{-2}q_\mu q_\nu) + \rho_{\alpha\beta}^{(0)}(m^2)q_\mu q_\nu]/(q^2 - m^2 + i\epsilon) + \text{Schwinger terms}. \tag{2}$$

To discuss SU(3) symmetry breaking[2] it is convenient to define the (9×9) matrices

$$\Delta_{\alpha\beta}^{(i)}(q^2) = \int dm^2 \rho_{\alpha\beta}^{(i)}(m^2)/(q^2 - m^2 + i\epsilon) \quad (i = 0, 1) \tag{3}$$

and to write their inverses, suppressing indices, as

$$[\Delta^{(i)}(q^2)]^{-1} = M^{(i)} + \Pi^{(i)}(q^2). \tag{4}$$

Here $M^{(i)} = [\Delta^{(i)}(0)]^{-1}$ is a constant matrix and $\Pi^{(i)}(0) = 0$.

There are two basically different models for introducing first-order SU(3) violations into the inverse propagator matrix Δ^{-1}: (i) the "mass-mixing" (or "particle-mixing") model[3] in which the mass matrix M is not SU(3) symmetric but contains an asymmetric part of the octet type, while $\Pi(q^2)$ is SU(3) symmetric and

(ii) the "current-mixing" (or "vector-mixing") model[4] in which the mass matrix M is SU(3) symmetric, but $\Pi(q^2)$ contains a symmetry-breaking part of the octet type. Although more complicated schemes are possible, it is usually argued[4] that "mass mixing" is appropriate for spin-0 fields while "current mixing" is appropriate for spin-1 fields. Indeed, it has been shown[5] that Weinberg's sum rule (1) requires the "current-mixing" description for the conserved vector currents.

Adopting this conventional view, we take $\Pi^{(0)}(q^2)$ and $M^{(1)}$ to be SU(3) symmetric[6] but allow $M^{(0)}$ and $\Pi^{(1)}(q^2)$ to contain SU(3) nonconserving parts of the octet type. From the SU(3) symmetry of $\Pi^{(0)}(q^2)$ and $M^{(1)}$ one immediately obtains the two axial-vector spectral sum rules,[7]

$$\int dm^2 \rho_{\alpha\beta}^{(0)}(m^2) = s_0\delta_{\alpha\beta} + s_0'\delta_{\alpha 0}\delta_{\beta 0} \tag{5}$$

and

$$\int dm^2 m^{-2}\rho_{\alpha\beta}^{(1)}(m^2) = s_1\delta_{\alpha\beta} + s_1'\delta_{\alpha 0}\delta_{\beta 0}, \tag{6}$$

where the constants $S_0 \cdots s_1'$ are independent of α and β. Clearly, the sum of Eqs. (5) and (6) is just Weinberg's sum rule (1) which actually holds for SU(3) ⊗ SU(3) although sum rules (5) and (6) are separately valid only within SU(3). In addition, to first order in the assumed octet-type symmetry breaking in $M^{(0)}$ and $\Pi^{(1)}(q^2)$, one finds that

$$\int dm^2 m^2 \rho_{\alpha\beta}^{(0)}(m^2) = z_0\delta_{\alpha\beta} + z_0' d_{\alpha\beta 8} \tag{7}$$

and

$$\int dm^2 m^{-4}\rho_{\alpha\beta}^{(1)}(m^2) = z_1\delta_{\alpha\beta} + z_1' d_{\alpha\beta 8}. \tag{8}$$

That is, these moments of the spectral functions satisfy the Gell-Mann–Okubo formula.

In the approximation that the spectral functions $\rho^{(0)}$ and $\rho^{(1)}$ are dominated by the $0^-(\pi, K, \eta, \eta')$ and $1^+(A_1, K_A, D, E)$ mesons,[8] respectively, Eqs. (5)-(8) yield the following relations:

$$F_\pi^{\;2} = F_K^{\;2} = \tfrac{3}{4}F_Y^{\;2}, \tag{9}$$

$$\sin(\chi_Y - \chi_B) = 0, \tag{10}$$

$$4m_K^{\;2} = m_\pi^{\;2} + 3(\cos^2\chi m_\eta^{\;2} + \sin^2\chi m_{\eta'}^{\;2}), \tag{11}$$

$$\frac{m_{A_1}^{\;2}}{g_{A_1}} = \frac{m_{K_A}^{\;2}}{g_{K_A}} = \frac{\tfrac{3}{4}(\cos^2\psi_Y m_E^{\;2} + \sin^2\psi_Y m_D^{\;2})}{g_Y^{\;2}}, \tag{12}$$

$$m_D^{\;2}\tan\psi_Y = m_E^{\;2}\tan\psi_B, \tag{13}$$

$$4m_{K_A}^{\;-2} = m_{A_1}^{\;-2} + 3(\cos^2\psi m_E^{\;-2} + \sin^2\psi m_D^{\;-2}). \tag{14}$$

Here the coupling constants and mixing angles are defined by the following matrix elements of the nonet of axial-vector currents[9]:

$$\begin{aligned}
&\langle 0|A_\mu^{\;3}|A_1^{\;0}\rangle = m_{A_1}^{\;2} g_{A_1}^{\;-1}\epsilon_\mu, \\
&\sqrt{2}\langle 0|A_\mu^{\;4}|K_A^{\;-}\rangle = m_{K_A}^{\;2} g_{K_A}^{\;-1}\epsilon_\mu, \\
&(2/\sqrt{3})\langle 0|A_\mu^{\;8}|E\rangle = \cos\psi_Y m_E^{\;2} g_Y^{\;-1}\epsilon_\mu, \\
&(2/\sqrt{3})\langle 0|A_\mu^{\;8}|D\rangle = -\sin\psi_Y m_D^{\;2} g_Y^{\;-1}\epsilon_\mu, \\
&\sqrt{\tfrac{2}{3}}\langle 0|A_\mu^{\;0}|E\rangle = \sin\psi_B m_E^{\;2} g_B^{\;-1}\epsilon_\mu, \\
&\sqrt{\tfrac{2}{3}}\langle 0|A_\mu^{\;0}|D\rangle = \cos\psi_B m_D^{\;2} g_B^{\;-1}\epsilon_\mu,
\end{aligned} \tag{15}$$

$$\begin{aligned}
&\langle 0|A_\mu^{\;3}|\pi^0\rangle = F_\pi p_\mu, \\
&\sqrt{2}\langle 0|A_\mu^{\;4}|K^-\rangle = F_K p_\mu, \\
&(2/\sqrt{3})\langle 0|A_\mu^{\;8}|\eta\rangle = \cos\chi_Y F_Y p_\mu, \\
&(2/\sqrt{3})\langle 0|A_\mu^{\;8}|\eta'\rangle = -\sin\chi_Y F_Y p_\mu, \\
&\sqrt{\tfrac{2}{3}}\langle 0|A_\mu^{\;0}|\eta\rangle = \sin\chi_B F_B p_\mu, \\
&\sqrt{\tfrac{2}{3}}\langle 0|A_\mu^{\;0}|\eta'\rangle = \cos\chi_B F_B p_\mu.
\end{aligned} \tag{16}$$

For convenience, the angles χ and ψ have been introduced such that[10]

$$\chi = \chi_Y = \chi_B \tag{17}$$

and

$$\tan\psi = \frac{m_D}{m_E}\tan\psi_Y = \frac{m_E}{m_D}\tan\psi_B. \tag{18}$$

Finally, some remarks on the result $F_K/F_\pi = 1$ are in order:

(i) Previous calculations[11] of F_K/F_π based on the Weinberg sum rules[1] have given

$$\frac{F_K}{F_\pi} = \left[\frac{1-m_\rho^{\;2}/m_{K_A}^{\;2}}{1-m_\rho^{\;2}/m_{A_1}^{\;2}}\right]^{1/2} \simeq 1.17. \tag{19}$$

However, in obtaining Eq. (19) one uses the extension of Weinberg's second sum rule[1] to SU(3) ⊗ SU(3) which has been shown by several authors[12] to be in disagreement with experiment.

(ii) Assuming that Weinberg's first sum rule (1) holds for SU(3) ⊗ SU(3) but that the second sum rule is valid only for SU(2) ⊗ SU(2), and neglecting the contribution of the spin-0 spectral functions of the strangeness-changing vector current which is of second order in SU(3)-symmetry breaking, one obtains[13]

$$\frac{F_K}{F_\pi}=\left[\frac{1-m_{K^*}^2/m_{K_A}^2}{1-m_\rho^2/m_{A_1}^2}\right]^{1/2}. \qquad (20)$$

Combining Eq. (20) with Weinberg's result[1] $m_{A_1}=\sqrt{2}m_\rho$ and the present result $F_K/F_\pi=1$, we conclude that

$$m_{K_A}=\sqrt{2}m_{K^*}\simeq 1260 \text{ MeV},$$

in close agreement with the recently reported meson[14] $K^*(1250)$.

(iii) From the ratio of the $K_{\mu 2}$ and $\pi_{\mu 2}$ decay rates[15] one determines $|F_K/F_\pi \tan\theta_A|^2 = 0.075$, where θ_A is the axial-vector Cabibbo angle.[16] With $F_K/F_\pi=1$ this yields $\tan\theta_A\simeq 0.27$ in excellent agreement with the value determined from the hyperon β-decay rates. However, from the K_{e3} decay rate one finds $\tan\theta_V\simeq 0.23$, where θ_V is the vector Cabibbo angle,[16] if the SU(3) breaking in the K_{e3} form factor $f_+(q^2)$ is neglected. If indeed $\theta_V\neq\theta_A$, one looses the elegance of Cabibbo's universality hypothesis.[16] However, more attractive alternatives remain open. For example,[17] the SU(3)-symmetry breaking in $f_+(q^2)$ might be significant (~15% is required if $\theta_V=\theta_A$) even though it is of second order.[18]

I wish to thank Dr. L. Chan and Dr. B. Renner for helpful discussions.

---

*Research sponsored by the Air Force Office of Scientific Research, U. S. Air Force, under Grant No. Nr 68-1365.

[1]S. Weinberg, Phys. Rev. Letters 18, 507 (1967).

[2]We shall discuss the symmetry breaking in the axial-vector current propagator directly. One could arrive at the same conclusions starting from the symmetry breaking in the pseudoscalar and axial-vector meson propagators and either using field-current identities analogous to those discussed by N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. 157, 1376 (1967) for the vector currents, or using the meson-dominance approximation for the spectral functions.

[3]S. Okubo, Phys. Rev. Letters 5, 165 (1963); S. L. Glashow, Phys. Rev. Letters 11, 48 (1963); J. J. Sakurai, Phys. Rev. 132, 434 (1963).

[4]S. Coleman and H. J. Schnitzer, Phys. Rev. 134, B863 (1964); N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. 157, 1376 (1967).

[5]R. J. Oakes and J. J. Sakurai, Phys. Rev. Letters 19, 1266 (1967).

[6]In fact Weinberg's sum rule (1) requires $M^{(1)}$ to be SU(3) symmetric if $\Pi^{(0)}(q^2)$ is SU(3) symmetric. The converse also holds in the meson-dominance approximation, where $\Pi(q^2)$ is a linear function of q^2.

[7]Observe that Eqs. (5) and (6) imply

$$\int dm^2\rho_{\alpha\beta}^{(0)}(m^2)=\left(\frac{s_1}{s_1}\right)\int dm^2 m^{-2}\rho_{\alpha\beta}^{(1)}(m^2)$$

$$(\alpha,\beta=1,\cdots,8).$$

It is tempting to conjecture further that $s_0=s_1$. Then the somewhat controversial current-algebra results of K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters 16, 255 (1966), and Riazuddin and Fayyazuddin, Phys. Rev. 147, 1071 (1966), could be derived directly from the sum rules in the meson-dominance approximation. Unfortunately, we have so far been unable to prove this conjecture.

[8]A. H. Rosenfeld et al., Rev. Mod. Phys. 39, 1 (1967).

[9]The currents are normalized in the conventional manner, so that A_μ^3, $A_\mu^4+iA_\mu^5$, $(2/\sqrt{3})A_\mu^8$, and $\sqrt{\frac{2}{3}}A_\mu^0$ are the usual axial-vector isospin, strangeness-changing, hypercharge and baryon currents, respectively.

[10]It should be emphasized that while χ is the mixing angle in the orthogonal transformation relating the pseudoscalar mesons η and η' to the pure singlet and octet states, ψ does not have the corresponding interpretation since the transformation relating the axial-vector mesons D and E to the pure singlet and octet states is not orthogonal due to the different mixing ($\psi_Y\neq\psi_B$) in the axial-vector hypercharge and baryon currents.

[11]H. T. Nieh, Phys. Rev. Letters 19, 43 (1967); S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Letters 19, 139 (1967).

[12]S. P. DeAlwis, Nuovo Cimento 51A, 846 (1967); T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters 19, 470 (1967); J. J. Sakurai, Phys. Rev. Letters 19, 803 (1967).

[13]L. Chan, private communication; P. P. Srivastava, CERN Report No. TH 848 (unpublished); J. W. Moffat and P. J. O'Donnell, Can. J. Phys. 45, 3901 (1967).

[14]G. Goldhaber et al., Phys. Rev. Letters 19, 972 (1967).

[15]For a review of the experimental data, see N. Cabibbo, in Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, 1966 (University of California Press, Berkeley, California, 1967).

[16]N. Cabibbo, Phys. Rev. 10, 531 (1963).

[17]The validity of the conventional mixing model might also be questioned in spite of its simplicity. A priori, the possibility cannot be excluded that the symmetry breaking among the axial-vector currents, and therefore also the 0^- and 1^+ mesons if the meson dominance approximation is valid, is much more complicated than is usually assumed.

[18]M. Ademollo and R. Gatto, Phys. Rev. Letters 13, 264 (1964).