tity and Eq. (9) leads to

$$m_{A_1}^{\ 2}-4m_{K_A}^{\ 2}+3m_{E_8}^{\ 2}=0,$$

$$m_{A_1}^{\ 2}F_\pi^{\ 2}-4m_{K_A}^{\ 2}F_K^{\ 2}+3m_{E_8}^{\ 2}F_{\eta_8}^{\ 2}=0. \qquad (10)$$

Solving Eqs. (5) and (10) together, we find the solutions to be either

$$F_\pi^{\ 2}=F_K^{\ 2}=F_{\eta_8}^{\ 2} \qquad (11)$$

or

$$m_{A_1}^{\ 2}=m_{K_A}^{\ 2}=m_{E_8}^{\ 2} \qquad (12)$$

Since the masses of mesons are nondegenerate, we thus conclude that the leptonic-decay amplitudes F_β are nonrenormalizable to the first order in the SU(3)-symmetry breaking.

In summary, we show that, by solving the spectral-function sum rules consistently, the ratio F_K/F_π is found still to be unity in the first order of SU(3) breaking.

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[10]We have not written down a term $F_\kappa^{\ 2}$ due to the scalar κ excitations for the case of the strangeness-changing vector currents. From Eq. (8), we see immediately that it is of the second order in SU(3) breaking.

SEARCH FOR DOUBLE BETA DECAY OF K^- MESON

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In weak interactions, the leptonic currents J_λ are considered to be well described by exact $V-A$ theory,[1] where

$$J_\lambda=\sum_{l=\mu,e}[\psi_l^{\ \dagger}\gamma_4\gamma_\lambda(1+\gamma_5)\psi_{\nu_l}]. \qquad (1)$$

This form implies, in particular, (a) conservation of leptons and (b) absence of neutral and doubly charged leptonic currents. The experimental evidence to date which corroborates these requirements has involved the study of processes which involve (i) $\Delta S=0$, $|\Delta Q|=1$, and $|\Delta L|=2$, such as nuclear double-beta decays,[2,3] neutrino experiments at nuclear reactors,[4] and μ^+ production from high energy ν_μ-nucleon reactions[5]; (ii) $\Delta S=0$ and $|\Delta L|=1$, such as neutrinoless muon beta decays,[6] muon-nucleon collisions,[7] and e^- production from high energy ν_μ-nucleon reactions[8]; and (iii) $|\Delta S|=1$ and $\Delta Q=\Delta L=0$, such as leptonic or semileptonic kaon decays.[6] The experimental results[2-8] are consistent with the absolute forbiddenness of $|\Delta L|=1$ and 2, where

$$\Delta L=\Delta(\Sigma L_\mu) \text{ or } \Delta(\Sigma L_e)$$

and

$$L_e=+1 \text{ for } e^-,\ \nu_e,$$
$$=-1 \text{ for } e^+,\ \bar{\nu}_e,$$
$$=0 \text{ for other particles;}$$

$$L_\mu = +1 \text{ for } \mu^-, \nu_\mu,$$
$$= -1 \text{ for } \mu^+, \bar{\nu}_\mu,$$
$$= 0 \text{ for other particles}$$

are the lepton numbers.[9] However, the experimental limits on the absence of $|\Delta L| = 2$ are much less stringent than those for $|\Delta L| = 1$.

If weak reactions involving $|\Delta S| = 1$, $|\Delta I| = \frac{3}{2}$, $|\Delta Q| = 2$, and $|\Delta L| = 2$ such as

$$K^\pm \to \pi^\mp l^\pm l^\pm \tag{2}$$

exist, then they would lead to violation of the lepton conservation law with additive quantum numbers, and doubly charged lepton currents would be allowed independent of any detailed theory on the nature of the neutrino.[3,10]

In this paper, we report an upper limit for the branching ratio

$$R \equiv \Gamma(K^- \to \pi^+ e^- e^-)/\Gamma(K^- \to \text{all}) \tag{3}$$

on the basis of a systematic analysis of 65 000 decaying K^- mesons in the 30-in. Columbia-Brookhaven National Laboratory hydrogen bubble chamber.[11] As no examples of $K^- \to \pi^+ e^- e^-$ were found, we estimate that $R < 1.5 \times 10^{-5}$ (with 65% confidence). This number is obtained using the number[12] of tau decays observed (3600) and the tau-decay branching ratio.[6]

As possible candidates for the decay $K^- \to \pi^+ e^- e^-$, we considered all three-prong events that are consistent with momentum conservation. This requirement essentially eliminated all events with neutrals in the final state and spurious events, without causing any serious bias against the real $K^- \to \pi^+ e^- e^-$ events.[13] All three-prong events having a momentum imbalance transverse to the K^- direction of less than 30 MeV/c were assumed to correspond to decays of the type $K^- \to \pi^+ x^- x^-$. Energy conservation permitted us calculate the mass of the x^- particle for each event. The spectrum in the square of the x^- mass shown in Fig. 1 gives no clear indication of any decay mode other than $K^- \to \pi^+\pi^-\pi^-$. All events having an x^--mass squared in the vicinity of the square of the electron mass were kinematically fitted to a number of hypotheses using the program SQUAW. The hypotheses tried include the following:

$$K^- \to \pi^+\pi^-\pi^-, \tag{4}$$

$$K^- \to \pi^+ e^- e^-, \tag{5}$$

$$K^- \to \pi^-\pi^0 \text{ followed by } \pi^0 \to \gamma e^+ e^-. \tag{6}$$

For two events, good fits were obtained to Reaction (5); however, in both cases Reaction (6) also gave a good fit. Both of these events were unambiguously identified as corresponding to Reaction (6) on the basis of ionization, thus leaving no candidates for the decay $K^- \to \pi^+ e^- e^-$.

We believe that our overall efficiency for detecting such events (if they existed) is the same as our overall detection efficiency for tau decays. Our quoted upper limit for the $K^- \to \pi^+ e^- e^-$ branching ratio is then independent of this efficiency. The same factors that might lead to a difference in the detection efficiencies between Reactions (4) and (5) would also presumably cause a similar difference in detection efficiencies between Reactions (4) and (6).[14] We have used a sample of the data to determine the ratio of the detection efficiencies between Reactions (4) and (6), using the number of tau decays in the sample (820) and the number of $K_{\pi 2}$ decays with internal conversions (34), as determined from kinematical fits. Using the published branching ratios[6] for Reactions (4) and (6), we obtain $\gamma \equiv \epsilon_4/\epsilon_6 = 1.11 \pm 0.18$ for the ratio of the detection efficiencies

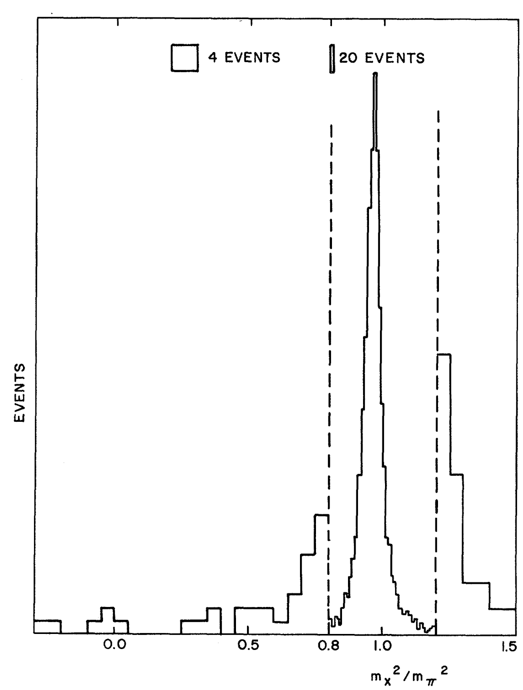

FIG. 1. The mass-squared distribution for x^- calculated, in units of pion-mass squared, from the reaction $K^- \to \pi^+ x^- x^-$. This plot contains 4783 three-prong events having a momentum imbalance transverse to the K^- direction of motion less than 30 MeV/c.

for these reactions. Assuming that the ratio of detection efficiencies between Reactions (4) and (5) has the same value, then the observation of one real $K^- \to \pi^+ e^- e^-$ event would give 1.6×10^{-5} for this branching ratio. As no events were found, we do not feel that it is necessary to make this correction, and with 65% confidence we give the original figure of 1.5×10^{-5} for the upper limit on the $K^- \to \pi^+ e^- e^-$ branching ratio.

To estimate the significance of this upper limit, we express the decay rate for Reaction (2) in terms of that for $K_{e3}^{\pm}$ decays, giving[15]

$$\Gamma(K^{\pm} \to \pi^{\mp} e^{\pm} e^{\pm}) = 2\xi\rho\Gamma(K^{\pm} \to \pi^0 e^{\pm} \nu_e), \quad (7)$$

$$= 7.2\xi \times 10^6 \text{ sec}^{-1}, \quad (8)$$

where $\xi = (G_{\beta\beta}/G_\beta)^2$, ρ = ratio of the phase spaces for the two decay processes, the factor 2 represents the indistinguishability of the two identical leptons in the final states of (2), G_β = the Fermi coupling constant, and $G_{\beta\beta}$ = the constant characterizing the coupling strength between a hardronic current with $|\Delta S| = 1$, $|\Delta Q| = 2$, $|\Delta I| = \frac{3}{2}$, and a doubly charged leptonic current.[15] Therefore, from (3), (8), and total decay rate of ($K^{\pm} \to$ all), we find[16]

$$\xi < 1.69\times10^{-3}. \quad (9)$$

This result shows that the present experimental limit of $\xi < 1.69\times10^{-3}$ is compatible with, and better by an order of magnitude than, that obtained from the neutrino reactions which are classified as Processes (i)[4,5] in the first paragraph. There are also other processes, (ii),[6-8] with even better upper limits than the present one; however, those reactions involve both muons and electrons, and where both L_e and L_μ conservations are violated ($\Delta L = 1$), the mechanism could be different.

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[11]This represents approximately 10% of an exposure

of the 30-in. Brookhaven hydrogen bubble chamber. This film is currently being analyzed for other purposes as part of a Columbia-Rutgers-Stony Brook collaboration.

[12]This number is consistent with the number of K^- decays expected from 350 000 incident K^- mesons with an average momentum 220 MeV/c in hydrogen.

[13]One conceivable source of bias would be the fact that in testing for momentum conservation, we used the momentum and angles of each track assuming it to be a pion. That this is not a serious source of bias can be inferred from the fact that the transverse missing momentum distribution for all events had a width of 10 MeV/c when all tracks were interpreted as pions, and a width of 20 MeV/c when all tracks were interpreted as electrons. (The cut on allowed transverse missing momentum was 30 MeV/c.)

[14]This could conceivably arise because of differences in scanning efficiency, differences in the reconstruction-program reject rate for electrons and pions, and in other ways.

[15]We assume that the transition matrix element for (2) is the same as that for ordinary $K_{e3}^{\pm}$ decays except that the Fermi coupling constant G_β has been replaced by $G_{\beta\beta}$. Because (2) can only proceed through $|\Delta I| = \frac{3}{2}$, $|\Delta Q| = 2$ transition, the suppression of $|\Delta I| = \frac{3}{2}$ relative to that of $|\Delta I| = \frac{1}{2}$ amplitude in ordinary semileptonic K decays has been ignored in this estimation.

[16]We have also estimated ξ from the data of the $K^+ \to \pi^+ e^+ e^-$ experiment [U. Camerini et al., Phys. Rev. Letters 13, 318 (1964)]. Since one cannot differentiate $K^+ \to \pi^- e^+ e^+$ from $K^+ \to \pi^+ e^+ e^-$ by a 1C fit kinematically (when the measured momenta were ignored), we can therefore estimate that $R < (8/9.4) \times 10^5 = 0.85 \times 10^{-5}$, which is in agreement with our result, $R < 1.5 \times 10^{-5}$.

SYMMETRY BREAKING IN AXIAL-VECTOR SPECTRAL-FUNCTION SUM RULES*

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Weinberg's first sum rule applied to the axial-vector currents is shown to split into two separately valid sum rules if the usual assumptions concerning SU(3) mixing are adopted. New relations among masses, coupling constants, and mixing angles are obtained in the meson-dominance approximation, e.g., $F_K/F_\pi = 1$, and the implications are discussed.

Weinberg's first sum rule[1] extended to the nonet of axial-vector currents $A_\mu^\alpha(x)$ $(\alpha = 0, 1, \cdots, 8)$ reads

$$\int dm^2 [m^{-2} \rho_{\alpha\beta}^{(1)}(m^2) + \rho_{\alpha\beta}^{(0)}(m^2)] = s\delta_{\alpha\beta} + s'\delta_{\alpha 0}\delta_{\beta 0}, \tag{1}$$

where s and s' are constants independent of α and β. The spin-0 and spin-1 spectral functions $\rho^{(0)}(m^2)$ and $\rho^{(1)}(m^2)$ are defined by the representation of the axial-vector-current propagator

$$\Delta_{\mu\nu}^{\alpha\beta}(q) = -i\int d^4x\, e^{iqx} \langle 0|TA_\mu^\alpha(x)A_\nu^\beta(0)|0\rangle = \int dm^2 [\rho_{\alpha\beta}^{(1)}(m^2)(-g_{\mu\nu} + m^{-2}q_\mu q_\nu) + \rho_{\alpha\beta}^{(0)}(m^2) q_\mu q_\nu]/(q^2 - m^2 + i\epsilon) + \text{Schwinger terms}. \tag{2}$$

To discuss SU(3) symmetry breaking[2] it is convenient to define the (9×9) matrices

$$\Delta_{\alpha\beta}^{(i)}(q^2) = \int dm^2 \rho_{\alpha\beta}^{(i)}(m^2)/(q^2 - m^2 + i\epsilon) \quad (i = 0, 1) \tag{3}$$

and to write their inverses, suppressing indices, as

$$[\Delta^{(i)}(q^2)]^{-1} = M^{(i)} + \Pi^{(i)}(q^2). \tag{4}$$

Here $M^{(i)} = [\Delta^{(i)}(0)]^{-1}$ is a constant matrix and $\Pi^{(i)}(0) = 0$.

There are two basically different models for introducing first-order SU(3) violations into the inverse propagator matrix Δ^{-1}: (i) the "mass-mixing" (or "particle-mixing") model[3] in which the mass matrix M is not SU(3) symmetric but contains an asymmetric part of the octet type, while $\Pi(q^2)$ is SU(3) symmetric and