ENERGY AND WIDTH MEASUREMENTS OF LOW-Z PIONIC X-RAY TRANSITIONS*

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New measurements have been performed of the energies and natural linewidths of $2p$-$1s$ pionic x-ray transitions, as well as muonic transition energies, in Li^6, Li^7, Be^9, B^{10}, B^{11}, and C^{12}. High-resolution Si(Li) and Ge(Li) spectrometers were used, resulting in a more precise determination than those previously reported of the transition energies and natural linewidths. Comparison of the measured complex energy shifts in the pionic transitions has been made with the theory of the Ericsons and good agreement found.

Several measurements[1,2] of the energies and natural linewidths of pionic $2p$-$1s$ x-ray transitions have recently been given in the literature. We wish to report new and more precise measurements, in addition to theoretical calculations, for pionic transitions in Li^6, Li^7, Be^9, B^{10}, B^{11}, and C^{12}.

Most of the data were obtained using a 75-MeV negative pion beam (with approximately 15% muon contamination) at the Carnegie-Mellon synchrocyclotron, a small portion being obtained with a 100-MeV beam at the Space Radiation Effects Laboratory (SREL). Each of these pion beams was focused into a scintillation-counter array and standard coincidence techniques insured that the 1600-channel analyzer used to sort and store pulses from the Si(Li) or Ge(Li) detector did so only when a fast timing coincidence existed between the π-stop signal and the detector signal. Some of the more important experimental considerations were the following:

(1) The detector used with each of the elements was chosen on the bases of both resolution and efficiency requirements. For the Li^6, Li^7, and Be^9 targets, the $80\text{-mm}^2 \times 3$-mm Si(Li) detector was used; for B^{10}, B^{11}, and C^{12}, the $3.5\text{-cm}^2 \times 5$-mm Ge(Li) detector.

(2) The detectors were operated out of the direct beam so as to minimize pulse pile-up problems.

(3) Calibration runs of the detection system, using well-known lines of Am^{241} and Ta^{182}, were taken before and after each x-ray run.

(4) For runs with the Ge(Li) spectrometer, which was used for the energy range of 50-100 keV, gain drifts were minimized by use of a digital gain stabilizer whose reference peak was the 59.57-keV γ-ray line of Am^{241}.

The instrumental resolutions of the cooled, field-effect transistor, Si(Li) and Ge(Li) spectrometers under beam conditions were, respectively, 0.62 keV [full width at half-maximum (FWHM)] at 33 keV and 1.1 keV (FWHM) at 75 keV, as determined from muonic x rays. These figures represent less than a 10% degradation in resolution as previously determined with radioactive sources under laboratory conditions. The operation and characteristics of these spectrometers have been described elsewhere.[3,4]

Muonic x-ray lines were present in the π runs because of the muon contamination in the beam and constituted a substantial background because of the relatively low yield of pionic $2p$-$1s$ transitions. However, in most cases, spectrometer resolution permitted unambiguous separation of the pionic $2p$-$1s$ peaks and the more intense of the contaminating muonic lines. In order to account properly for those muonic lines which appeared directly under the pionic lines, separate μ-beam data were accumulated. This permitted an independent determination of the relative intensities of the μ-K_α, μ-K_β, μ-K_γ, and μ-K_δ lines which were used in the background subtraction analysis for the pionic x-ray data.

Briefly, the major features of the analysis for the $2p$-$1s$ pionic peaks were the following: (1) subtraction of the muon contaminating lines under the pionic x-ray peaks; (2) determination of the center channels of the pionic peaks by fitting the contaminant-free pionic data to both Gaussian and Lorentzian functions and a linear background term (for a given peak, the center channel value obtained from each of these two fits was the same within the statistical error); (3) determination of the natural linewidths by using Voigt profiles (convolution integrals of a Gaussian and a Lorentzian function) to fit the pionic data (it should be noted that, in contrast to extracting a center channel value, the determination of the natural linewidth was very shape dependent, as verified by the differences in widths obtained from separate Gaussian and Lorentzian fits); and (4) use of a precision pulser to determine uncertainties introduced into the energy measurements as a result of system nonlinearities.

The experimental results for the measured transition energies and natural linewidths are given in Tables I and II. The errors quoted include statistical uncertainties as well as the uncertainties in background subtraction and system linearity. The agreement between the corresponding measured energies given in Table I(A) is, in general, quite satisfactory. The values of E_{calc} quoted for the $2p$-$1s$ pionic transition energies were obtained by correcting the Klein-Gordon value for (a) vacuum polarization effects (including finite size) according to the method of Mickelwait and Corben[5] and (b) Coulomb effects using Pustovalov's technique[6] with nuclear radii determined from electron scattering data.[7-13] The difference between E_{calc} and E_{exp} constitutes the mea-

Table I. (A) $2p$-$1s$ pionic x-ray energies (in keV). (B) $2p$-$1s$ muonic x-ray energies (in keV) and nuclear radii.

(A)

Element	E_{exp} This Work	E_{exp} Other	E_{calc}[a]	E_{nuc} (measured)	E_{nuc}[a] (theoretical)
Li^6	24.18±0.06	23.9 ±0.2[b]	24.53	0.35±0.06	0.47
Li^7	24.06±0.06	23.8 ±0.2[b]	24.63	0.57±0.06	0.79
Be^9	42.32±0.05	42.38±0.20[c]	43.95±0.06	1.63±0.08	2.12
B^{10}	65.79±0.11	65.94±0.18[c]	68.75±0.04	2.96±0.12	3.29
B^{11}	65.00±0.11	64.98±0.18[c]	68.85±0.04	3.85±0.12	4.56
C^{12}	93.19±0.12	92.94±0.15[c]	99.15±0.03	5.96±0.12	6.50

(B)

Element	E_{exp} This Work	E_{exp} Other	Radius (fm) - Equivalent Uniform Charge: This Work	Radius (fm) - Equivalent Uniform Charge: Electron Scattering
Li^6	18.64±0.07	18.1 ±0.4[b]	4.96±6.0	3.28±0.06[e]
Li^7	18.69±0.06	18.1 ±0.4[b]	4.94±5.0	3.09±0.04[e]
Be^9	33.39±0.05	33.0 ±0.2[b]	3.38±1.16	3.25±0.70[f]
B^{10}	52.18±0.10	52.23±0.15[d]	3.56±0.8	3.16±0.15[g]
B^{11}	52.23±0.09	52.31±0.15[d]	3.56±0.78	3.12±0.15[g]
C^{12}	75.23±0.08	75.25±0.15[d]	3.36±0.34	3.11±0.05[h]

[a]See text.
[b]See Ref. 1.
[c]See Ref. 2.
[d]G. Backenstoss et al., Phys. Rev. Letters 25B, 547 (1967).
[e]See Ref. 7.
[f]See Refs. 8-10.
[g]See Ref. 11.
[h]See Refs. 12 and 13.

Table II. $2p$-$1s$ pionic natural linewidths Γ_n (in keV).

Element	Γ_n		$\Gamma_n{}^a$
	This Work	Other	(theoretical)
Li^6	0.15 ± 0.05	0.39 ± 0.36^b	0.11
Li^7	0.19 ± 0.05	0.59 ± 0.30^b	0.18
Be^9	0.58 ± 0.05	1.07 ± 0.30^c	0.56
B^{10}	1.68 ± 0.12	1.27 ± 0.25^c	1.48
B^{11}	1.72 ± 0.15	1.87 ± 0.25^c	1.75
C^{12}	3.25 ± 0.15	2.96 ± 0.25^c	3.41

[a]See text
[b]See Ref. 1.
[c]See Ref. 2.

sured nuclear shift E_{nuc}. The theoretical values of E_{nuc} have been obtained from the work of Ericson and Ericson.[14,15] An expression for the complex energy shift is obtained by them in expansion form by solving the Schrödinger equation for an optical potential with constant parameters representing the pion-nucleus interaction. The constant parameters which specify the single-nucleon potential primarily responsible for the level shifts were obtained from π-N scattering data and have been corrected, as outlined by the Ericsons,[15] for nuclear binding, Fermi motion, finite nuclear correlation lengths, and the shift induced by the real part of the two-nucleon potential. The estimated error on the theoretical values of E_{nuc} is 20-30% and is primarily due to the uncertainty in the two-nucleon correction. Within this uncertainty, agreement between the experimental and theoretical values of E_{nuc} is quite good

The nuclear radii listed in Table I(B) were obtained following the method of Jenkins et al.,[1] in which the measured muonic transition energies are compared with the corresponding Dirac values after correction for vacuum polarization. The radii computed from our muonic x-ray data are compared in Table I(B) with the values obtained from electron scattering data, the latter being more precise for these light nuclei.

Table II consists of experimental results of the natural linewidths Γ_n obtained by us and other workers, in addition to theoretical values for Γ_n. With the exception of the Be^9 and B^{10} widths, the agreement between the experimental results is satisfactory, this work being more precise. The theoretical values of Γ_n have also been calculated from the complex energy shift given by the Ericsons.[14,15] The parameters specifying the two-nucleon potential were obtained from $2N$ pion production data and have been corrected for binding, Fermi motion, and $1/A$ spin-isospin effects. It should be noted that the pion s-wave interaction amplitude β_{11} describing absorption onto two nucleons in a relative triplet state is different in value from that given by the Ericsons. The value used by the Ericsons was obtained from an early measurement[16] of the cross section for the reaction $p+p \rightarrow \pi^+ + d$ in an energy range where the p-wave interaction dominates. The value of β_{11} used in our calculation is presumably more reliable since it was taken from the more recent measurement[17] of the $\pi^+ + d \rightarrow p+p$ cross section at low energies where the s-wave interaction dominates. Use of the older value of β_{11} would result in calculated widths which are approximately 35% lower than those shown in Table II. We estimate a 15% error in the calculated widths and, as is true for the level shifts, there is good agreement between the experimental and theoretical results.

Finally, it should be noted that first-order perturbation theory with hydrogenlike wave functions closely approximates the theoretical values for both the energy shifts and widths listed in Tables I and II, respectively. Perturbation theory results in shifts and widths which are, respectively, 15 and 4% larger than the listed theoretical values. These results contradict the conclusion of Seki and Cromer[18] based on earlier width measurements that first-order perturbation theory is invalid for analysis of low-Z pionic atoms.

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