MAGNETORESISTANCE OF DILUTE MAGNETIC ALLOYS*

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The magnetoresistance of dilute magnetic alloys is calculated for arbitrary magnetic fields and temperatures by S-matrix theory on the basis of the s-d model. The results agree qualitatively with the experiments of Monod on Mn in Cu in the temperature range covered by him, but not with the results of Daybell and Steyert on Cr in Cu.

Since Kondo's theory of the anomalous electrical resistivity due to magnetic impurities,[1] and the subsequent theoretical developments pointing to strong resonance effects between the conduction electrons and the localized moment, the magnetoresistance of such alloys has assumed particular theoretical significance. A magnetic field should not only "freeze out" the local moment (an effect already implicit in the early theory of Yosida[2]) but also, if strong enough, should intrude upon the resonant configuration held responsible for the Kondo effect.

We have calculated the variation of resistivity with temperature and magnetic field by S-matrix theory. Our results are approximate but analytically satisfactory throughout the H-T plane. A detailed perturbation calculation has been done by Béal-Monod and Weiner,[3] valid only in the range $T \gg T_K$ or else $H \gg T_K$ (measuring temperature and field in energy units), where T_K is the temperature below which the Born series for the scattering amplitude diverges

To obtain unrestricted results, we include the magnetic field in the unperturbed Hamiltonian H^0; for the perturbation, we use the "s-d" form:

$$\mathcal{H}' = \int [V(r) + J(r)\vec{\sigma}(r)\cdot\vec{S}]d^3r \tag{1}$$

with J positive, $\vec{S}$ the Pauli matrix vector of the localized spin, and $\frac{1}{2}\vec{\sigma}(r)$ the conduction-electron spin density. For simplicity we consider zero-range potentials, rendering integrals convergent by use of the symmetric "one-band" density of states function

$$\rho(x) = (1-x^2)^{1/2}, \tag{2}$$

where x is the energy measured relative to the Fermi energy $E_F = 1$. This still has sufficient analyticity to permit use of the Ball-Frazer-Froissart method.[4,5]

We label states by quantum numbers appropriate to weak coupling: $\vec{k}$ = conduction-electron wave number ($x = k^2$ = kinetic energy), σ = conduction-electron spin, and S = impurity spin (we consider only spin-$\frac{1}{2}$ impurities). We further assume that the g factors of the conduction electrons and impurity are equal, an assumption self-consistent in our approximate solution of the scattering equations.

The in and out states may then be written as

$$|k,\sigma,S\rangle^{\pm} \equiv C_{k\sigma}^{+}|S\rangle + (\omega_S + \epsilon_{k\sigma} - \mathcal{H} \pm i\delta)^{-1}[\mathcal{H}', C_{k\sigma}^{+}]_{-}|S\rangle.$$

The energy of such a state is

$$\omega_S + \epsilon_{k\sigma} = (\omega_0 + HS) + (x + H\sigma),$$

where the kinetic energy x is conserved in collisions and ω_0 is the field-free ground-state energy.

The scattering equation [analogous to Eq. (13a) of Suhl[6]] has the following form:

$$T_{\sigma'S',\sigma S}(z) = T_{\text{Born}} + \sum[1-f(x+\sigma''H)]\frac{T_{\sigma''S''\sigma'S'}{}^{*}T_{\sigma''S''\sigma S}}{z-x} + \sum f(x+\sigma''H)\frac{T_{\sigma S''\sigma''S'}{}^{*}T_{\sigma'S''\sigma''S}}{z-x}. \tag{3}$$

In this equation, the variables x and z represent kinetic energies; $f(x)$ is the Fermi function and T_{Born} is obtained directly from (1). This scattering equation (with those leading up to it) has satisfactory behavior under time reversal, obeys the unitarity condition and crossing symmetry, and reduces to sensible or known results for large and small H, T.

The invariant decomposition of the scattering amplitude is

$$T = t + \tau\vec{\sigma}\cdot\vec{S} + U\vec{S}\cdot\vec{H} + W(i\vec{\sigma}\cdot\vec{S}\times\vec{H}) + Y(\vec{\sigma}\cdot\vec{H})(\vec{S}\cdot\vec{H}) + Z(\vec{\sigma}\cdot\vec{H}) \tag{4}$$

(the decomposition into only three amplitudes given by Suhl[7] was incomplete).

The first step towards a solution is the extraction of an over-all Froissart factor $R(z)$ defined by the substitutions

$$1-2\pi i\rho t(z)\equiv(1-2\pi i\rho\bar{t})R(z)\equiv S_e(z)R(z),$$

and for any of the other five amplitudes, e.g., τ or U,

$$\tau(z)=S_e(z)R(z)\bar{\tau}(z),\quad U=S_eR\bar{U},\ \text{etc.}$$

$R(z)$ may be found in terms of the barred amplitudes, and $\bar{t}$ is an elastic amplitude. Our approximation consists of replacing the iterative series for the barred amplitudes up to the lowest nontrivial order by Padé approximation displaying the characteristic pole at $T=T_K(H)$. For weak coupling ($J\ll E_F$, $V^2\ll JE_F$) only t, τ, and U contribute significantly to the scattering. The Padé approximations to the other amplitudes have higher order numerators or generate only $J^3\ln T$, etc., singularities. It turns our that $\bar{\tau}$ has a pole in the region P of the H-T plane (see Fig. 1); in that region a Castillejo-Dalitz-Dyson zero must be introduced in S_e.[8] (This procedure is the energy-plane version of the k-plane Blaschke factor used by Suhl and Wong.[9]) All physical results are then continuous across the boundary in Fig. 1.

Our final approximate formulas are (E_F = unit of energy)

$$J[\bar{\tau}(z)]^{-1}=1+2J\int_{-1}^{1}dx\{1-f(x+H)-f(x-H)\}\rho(x)/(z-x),$$

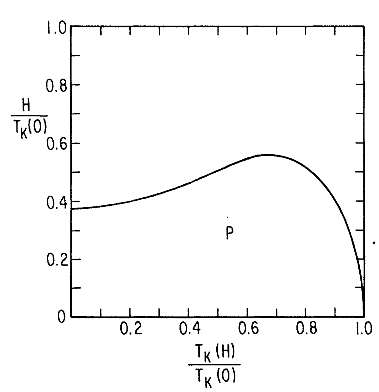

FIG. 1. In the region P of the $T/T_k(0)$-$H/T_k(0)$ plane, the scattering amplitudes calculated by Born series acquire a complex pole. Here $T_k(0)$ is the "Kondo temperature" in the absence of a field.

$$H\bar{U}=2J^2\int_{-1}^{1}dx\{f(x+H)-f(x-H)\}\rho(x)/(z-x),$$

$$R=[1+4\pi^2\rho^2(3|\bar{\tau}|^2+|H\bar{U}|^2)]^{1/2}\exp i\varphi_R,$$

$$\varphi_R(x+i\epsilon)=-\rho(x)\int_{-1}^{1}dx'\{\ln|R(x')|\}/\pi\rho(x')(x'-x),$$

$$V[\bar{t}(z)]^{-1}=1-V\{\pi z-i\pi\rho(z)\},$$

outside the region P (Fig. 1), and

$$V[\bar{t}(z)]^{-1}=1-V\{\pi z-i\pi\rho(z)-A/(x_0-z)\}$$

inside the region P, where A and x_0 are chosen to remove the pole, which can only occur on the imaginary z axis for symmetry reasons.

For $H=0$, $\varphi_R(0)$ is exactly zero by symmetry, and then the results agree exactly with those of Hamann when $V=0$.[10] For various sample cases with $H\neq 0$, φ_R remained very small and was therefore neglected. The Fermi functions $f(x)$ were approximated by straight-line segments in the three ranges $(-\infty,-T)$, $(-T,T)$, and (T,∞). This cannot seriously affect the results but may have been responsible for the nonmonotonic pole boundary in Fig. 1, as well as some of the minute structure of the curves in Fig. 2.

The relaxation times are given (up to a common factor) by

$$(\tau_{\uparrow,\downarrow})^{-1}=\mathrm{Im}t+(\tanh\beta H)\,\mathrm{Im}(HU)\pm(\tanh\beta H)\,\mathrm{Im}\tau,\quad(5)$$

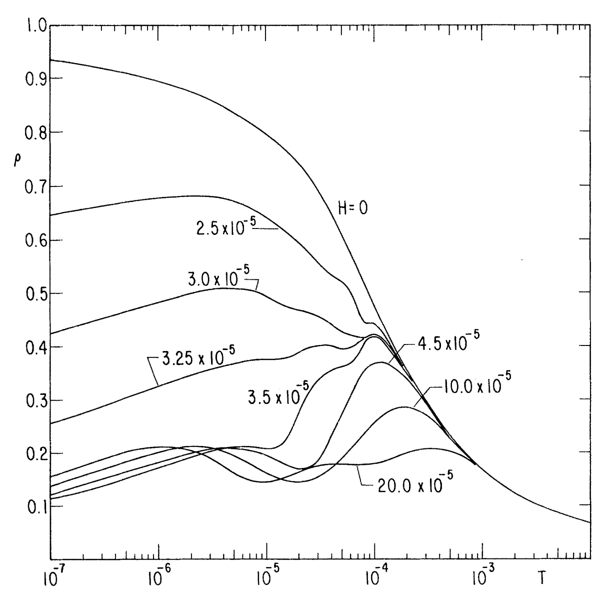

FIG. 2. Resistivity, in units of s-wave unitarity limit, versus temperature, for $J=0.025$ and $V=0.01$, and various H [$T_k(0)=9\times10^{-5}$].

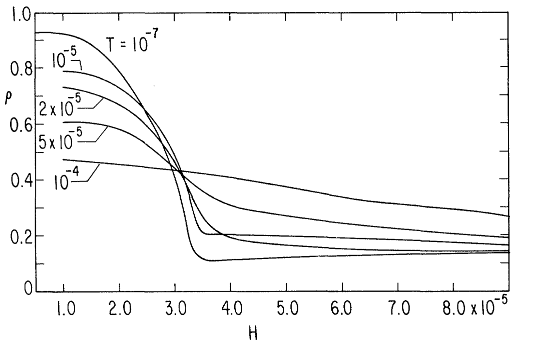

FIG. 3. Resistivity versus field at various temperatures for the same conditions as Fig. 1. Below $H=0.5\times10^{-5}$, the resistance is only weakly field dependent.

evaluated at $x=\mp H$.

Curves of the resistivity versus T for various H are given in Fig. 2. For sufficiently large H, there are two well-separated peaks; the first one occurs at temperatures above T_K, whereas the second (lower temperature) peak is dependent in size and position primarily upon V.

The experiments of Monod[11] on copper manganese agree qualitatively with these calculations insofar as they overlap in the temperature range. (The temperature was low enough, and the fields high enough to check the upper peak in Fig. 2.) On the other hand, Daybell and Steyert[12] working with Cr in Cu, observe no peaks, only a low-temperature plateau region.

In Fig. 3, we show the resistance as function of H for fixed T.

The drop in electrical resistance must be ascribed to four causes: the "freeze out" of the local moment exhibited in Eq. (5); the evaluation of the amplitudes at $x=\pm H$, which for $H>T_K$ is outside the resonance region; the change in $\bar{\tau}$ from its zero-field value; and the appearance of the new amplitude U.

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INVERSION IN THE DEFORMATION EFFECT FOR NEUTRON TRANSMISSION THROUGH ORIENTED Ho^{165} †

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We report measurements of the "deformation effect" in the total cross section for neutrons on oriented Ho^{165} over the energy range 0.330-5.60 MeV. They are in good qualitative agreement with the prediction of a previous theoretical calculation, and the predicted inversion or sign change in the energy range 3-7 MeV is verified.

Measurements of the "deformation effect" in the total cross section for fast neutrons incident on oriented Ho^{165} have been reported at neutron energies of 8, 15, 0.350, and 14 MeV.[1-3] The "deformation effect" is defined by $\Delta\sigma_{def}=\sigma(\text{oriented})-\sigma(\text{unoriented})$, where σ(oriented) and σ(unoriented) are the total cross sections for the oriented and unoriented cases, respectively. The experiments have shown that $\Delta\sigma_{def}$ varies in magnitude with neutron energy, but in all measurements the sign of $\Delta\sigma_{def}$ has corresponded to the change in the geometrical cross section of the Ho^{165} nucleus. A recent calculation,[1] using the coupled-channel formalism in the adiabatic approximation, predicted that $\Delta\sigma_{def}$ would undergo a sign change