## SIGMA HYPERON PRODUCTION IN 2- AND 3-BODY FINAL STATES

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We find the final states  $\Sigma^+K^+$  and  $\Sigma^+K^{*+}$  to be produced peripherally with evidence, in both cases, for a small backward peak. The forward differential cross sections for both reactions are compared with the predictions of a Regge-pole model. For  $\Sigma^+K^{*+}$ we find that the angular correlations in the resonance decay are predicted quite well by an absorptive peripheral model although the model is unable to account for the production angular distribution.

In this Letter we present data on the reactions

$$\pi^+ p \to \Sigma^+ K^+ \tag{1}$$

$$-\Sigma^+ K^+ \pi^0 \tag{2}$$

$$\Sigma^+ \pi^+ K^0 (3)$$

at a momentum of 5.4 GeV/c. Similar experiments have been performed at lower momenta.<sup>1</sup>

The data were obtained from the 30-in. ANL-MURA hydrogen bubble chamber at the zerogradient synchrotron (ZGS) with an exposure of 180 000 pictures which corresponds to 4.1 events/ $\mu$ b within a selected fiducial volume. We observe 74 events of type (1) and 280 events of types (2) and (3) which have a hyperon projected length greater than 0.5 cm and for which both production and decay vertices are within a fiducial volume. In order to compensate for decays too close to the production vertex and for decays outside the fiducial region, each event is weighted by the inverse of its observation probability.<sup>2</sup> The combined scanning efficiency of two scans was 93%.

The  $\Sigma^+K^+$  hypothesis is in a higher constraint class than other possible hypotheses for the two-prong-kink topology; so we have accepted this interpretation over any other fits of comparable confidence level. We have examined the data for possible bias due to small-angle sigma decays missed in scanning by constructing the decay angular distribution in the sigma rest frame. This distribution was isotropic for both decay modes of the  $\Sigma^+$ , as expected. In addition, we observe the correct relative number of both  $\Sigma^+$  decay modes and the time distribution of the observed  $\Sigma$  decays is consistent with the known lifetime.

We first consider Reaction (1). The cross section for this final state is  $29.7 \pm 3.5 \ \mu$ b. We observe four events in the backward direction and assign a cross section of  $1.3 \pm 0.7 \ \mu$ b to events in the backward hemisphere.<sup>3</sup> The squared four-momentum transfer distribution for the forward peak of Reaction (1) is shown in Fig. 1(a). This distribution is adequately



FIG. 1. (a) Distribution of the squared four-momentum transfer at the baryon vertex in the reaction  $\pi^+ p \rightarrow \Sigma^+ K^+$ . The data have been corrected for observation probability. The dashed curve is obtained from the model of Reeder and Sarma. The straight line is obtained from a fit of the data to an exponential function. (b) The  $\Sigma^+$  polarization as a function of squared four-momentum transfer in the reaction  $\pi^+ p \rightarrow \Sigma^+ K^+$ .

fitted by a function of the form

$$\frac{d\sigma}{dt} = \frac{d\sigma}{dt} \Big|_{0} \exp[A(t-t_{0})]$$

in the momentum transfer region  $0 \le t - t_0 \le -0.47$ (GeV/c)<sup>2</sup>, where  $t_0$  is equal to -0.0134 (GeV/c)<sup>2</sup>. A least-squares fit to the data gives a slope of  $6.7 \pm 1.2$  (GeV/c)<sup>-2</sup> and a forward differential cross section of  $148 \pm 37 \ \mu b \ (GeV/c)^{-2}$ .

Within the accuracy of this experiment, there is no evidence for structure in the forward peak such as has been observed at 3.23 GeV/c, where a dip is seen in the differential cross section for a momentum transfer of 0.6  $(\text{GeV}/c)^2$  accompanied by a rapid variation in the  $\Sigma^+$  polarization.<sup>1</sup>

We have investigated the  $\Sigma^+$  polarization in (1) using the subsample of  $\Sigma$ -proton decays. Figure 1(b) shows  $\alpha_{\Sigma}P_{\Sigma}$  as a function of momentum transfer, where  $\alpha_{\Sigma}$ , the asymmetry parameter for the protonic decay, is equal to -0.96;  $P_{\Sigma}$  is the polarization; and the values of  $\alpha_{\Sigma}P_{\Sigma}$  are obtained from the angular distribution of the decay proton with respect to the production plane normal defined by  $\hat{n} = \hat{p}_{\text{beam}}$  $\times \hat{p}_{\Sigma}$ .<sup>4</sup> There is no evidence for any rapid variation of the polarization versus four-momentum transfer.

Reeder and Sarma have explained the variation of the differential cross section and polarization observed at 3.23 GeV/c in terms of a double Regge-pole exchange model involving the exchanges of  $K^*(890)$  and  $K^*(1420)$ .<sup>5</sup> The dip in the momentum transfer distribution and the zero in polarization are then due to the exchanged Regge trajectories passing through  $\alpha(t) = 0$ . In Fig. 1, we show the momentum transfer distribution and polarization predicted by the model of Reeder and Sarma.<sup>6</sup> The agreement is good in the case of the differential cross section, but the limited data make a meaningful comparison difficult in the case of the polarization.

Finally, the forward differential cross section may be compared with the prediction of a double-Regge-pole model proposed by Arnold.<sup>7</sup> The model relates forward cross sections for hypercharge-exchange reaction of the form  $PB \rightarrow P'B'$ , where P and B represent pseudoscalar meson and baryon, respectively. In particular, this model predicts the relationship

$$[d\sigma(\pi^+p \to \Sigma^+K^+)/dt]_0$$
  
=  $\frac{4}{3}[d\sigma(K^-p \to \Lambda^0\pi^0)/dt]_0$  at 5.4 GeV/c. (4)

From Ref. (5) we obtain a value for  $[d\sigma(K^-p \rightarrow \Lambda^0\pi^0)/dt]_0$  at 5.4 GeV/c which, together with (4) above, requires  $[d\sigma(\Sigma^+K^+)/dt_0]$  equal to 150  $\mu$ b (GeV/c)<sup>-2</sup>, in good agreement with our experimental value.

Turning now to Reactions (2) and (3), we find considerable ambiguities between the two hypotheses. Figure 2 shows the Dalitz plot for the  $\Sigma^+K^+, {}^0\pi^0, {}^+$  final states. Events ambiguous between (2) and (3) appear twice. We observe strong  $K^{*+}(890)$  production, but see no clear evidence for other resonance formation. By apportioning ambiguous events in direct proportion to the numbers of unambiguous events, we estimate the cross sections for Reactions (2) and (3) to be  $40.9 \pm 13$  and  $51.3 \pm 14 \ \mu$ b, respectively.<sup>8</sup>

For events ambiguous between the interpretations (2) and (3), a scatter diagram of the two possible  $(K\pi)$  mass combinations indicates no enhancement of the  $K^*(890)$  peak due to double plotting, so that all  $(K\pi)$  combinations falling in the mass range 0.84 to 0.96 GeV/ $c^2$  are used in studying the  $\Sigma^+K^{*+}$  final state. After correcting for background under the  $K^*$  peak,<sup>9</sup> we estimate a cross section of  $42.9 \pm 6.2 \ \mu b$ for  $\Sigma^+K^{*+}$  production.

Figure 3(a) shows the center-of-mass production angular distribution for  $\Sigma^+K^{*+}$ , together with the prediction of an absorptive one-particle-exchange model due to Chilton <u>et al.</u><sup>10</sup> The model fails to account for the shape in the forward direction. The events in the backward hemisphere have a cross section of  $2.2 \pm 0.8 \mu b$ .



FIG. 2. Dalitz plot for the final states  $\Sigma^+ K^{+,0} \pi^{0,+}$ .



FIG. 3. (a) The center-of-mass production angular distribution for the reaction  $\pi^+ p \rightarrow \Sigma^+ K^{*+}$ . The insert shows the prediction of the absorptive peripheral model due to Chilton <u>et al</u>. assuming mixed pseudoscalar- and vectormeson exchange. (b) Spin-space density matrix elements for the  $K^*(890)$  as a function of squared four-momentum transfer in the reaction  $\pi^+ p \rightarrow \Sigma^+ K^{*+}$ . The curves are predictions of the absorptive peripheral model of Chilton <u>et al</u>. The solid curve assumes pure vector exchange while the dashed curve assumes mixed pseudoscalar- and vector-meson exchange.

In Fig. 3(b) we show the spin-space density matrix elements for the  $K^*(890)$  as a function of the four-momentum transfer. The data are uncorrected for background.<sup>11</sup> Similar studies at lower momenta<sup>1</sup> have suggested that in terms of a simple one-meson-exchange model without absorption, both K and  $K^*$  exchange are equally important. The present data are consistent with dominance of  $K^*$  exchange.

The observed density matrix elements are in quite good agreement with calculations using an absorption model.<sup>11</sup> Figure 3(b) shows the results expected for  $K^*$  exchange and also for the case of K and  $K^*$  exchange. The admixture of pseudoscalar exchange clearly produces better agreement with the data.

Using SU(6)<sub>W</sub>, the double Regge-pole model of Arnold<sup>7</sup> is able to relate the forward differential cross section for  $\Sigma^+K^{*+}$  to that for Reaction (1),

$$[d\sigma(\Sigma^{+}K^{+})/dt]_{0} = 6d\sigma(\Sigma^{+}K^{*+})/dt.$$
 (5)

With the present experimental result for the left-hand side of (5), a forward cross section of 22.2  $\mu$ b (GeV/c)<sup>-2</sup> is expected, in poor agreement with the observed value of  $83.8 \pm 17.5 \ \mu$ b (GeV/c)<sup>-2</sup>.

<sup>1</sup>S. S. Yamamoto <u>et al</u>., Phys. Rev. <u>134</u>, B383 (1964); J. Bartsch <u>et al</u>., Nuovo Cimento <u>43A</u>, 1010 (1966); R. R. Kofler, D. D. Reeder, and R. Hartung, Phys. Rev. <u>163</u>, 1479 (1967).

 $^{2}$ The average weight used in this correction was 1.35.

<sup>3</sup>Cosine values for the barycentric angles for these four events are 0.68, 0.82, 0.88, and 0.95. The respective four-momentum transfer t values are -7.11, -7.78, -7.81, and -8.38.

<sup>4</sup>The expected form for the decay distribution is  $\sigma(\theta) \sim 1 + \alpha_{\Sigma P} \cos \theta$ , where  $\theta$  is defined in the  $\Sigma$  rest frame as the angle  $\cos \theta = \hat{p}_{D} \cdot \hat{n}$ .

 ${}^{5}$ K. V. L. Sarma and D. D. Reeder, "Regge Pole Description of Associated Production Reactions," to be published.

<sup>6</sup>Private communication from D. D. Reeder. We are grateful to Dr. Reeder for providing this information.

<sup>7</sup>R. C. Arnold, Phys. Rev. <u>153</u>, 1506 (1967).

<sup>8</sup>About 30% of the  $\Sigma^+ K^+ \pi^0$  and  $\Sigma^+ \pi^+ K^0$  events are also kinematically compatible with the interpretation  $\Sigma^+ \pi^+ K^0 \pi^0$  and  $\Sigma^+ K^+ \pi^0 \pi^0$ , respectively. However, this sample of events does not show a  $K^*(890)$  signal.

<sup>9</sup>The amounts of resonance and phase space are estimated by fitting a product of Breit-Wigner function and phase space to the  $(K\pi)$  effective mass plot.

<sup>10</sup>F. Chilton <u>et al.</u>, Phys. Rev. <u>153</u>, 1610 (1967). <sup>11</sup>We have made the usual assumption that the background may be represented by events taken from regions on the  $(K\pi)$  mass plot of suitable width adjacent to the resonance region. The conclusions are not changed by such a background correction. The spinspace density matrix elements have been computed using the methods of moments, least squares, and also maximum likelihood. Very good agreements were observed between the results of these calculations. In

addition, fitted results for the subsample of events giving a unique fit to the  $\Sigma K\pi$  hypothesis agreed within the estimated errors with the quoted results based on the complete sample.

## HARD-PION CALCULATION OF $\pi$ - $\pi$ SCATTERING\*

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A mass-shell current-algebra calculation of  $\pi$ - $\pi$  scattering is given. The Weinberg scattering lengths and effective ranges are shown to be valid within a few percent. At the K meson mass, we find the essentially model-independent result  $\delta^0 - \delta^2 \simeq 35^\circ$ . The existing data at higher energies (up to 1 GeV) can be fitted by adjusting a model-dependent parameter.

During the past year, there has been renewed interest in the S-wave  $\pi$ - $\pi$  system. Weinberg<sup>1</sup> has commented that the success of the softpion current-algebra calculations of the  $K_{\rho 4}$ decay or single-pion production process<sup>2</sup> precludes a strong low-energy S-wave interaction. Using current-algebra techniques, he in fact obtains the remarkably small values for the I=0 and I=2 scattering lengths of  $a^0 \simeq 0.15 m_{\pi}^{-1}$ and  $a^2 \simeq -0.043 m_{\pi}^{-1}$ . On the other hand, recent efforts to extract the  $\pi$ - $\pi$  phase shifts by Walker et al.<sup>4</sup> and Schlein and Malamud<sup>5</sup> from pion production data have indicated the existence of a strong, I=0, S-wave interaction at higher energies leading to a resonance (a  $\sigma$  particle) somewhere between 700 MeV and 1 GeV. Fulco and Wong<sup>6</sup> have argued that in fact this implies the need for a considerably larger zero-energy scattering length than Weinberg's soft-pion prediction. In this note we will show that this is not necessarily the case, and if one generalizes the usual current-algebra analyses to one-shell, "hard-pion" calculations, agreement can be achieved with all the data from threshold to 1 GeV for the S- and P-wave phase shifts.

Recently<sup>7-11</sup> techniques have been developed for determining  $\pi$ ,  $\rho$ , and  $A_1$  vertex functions

without resorting to the usual soft-pion approximation of limiting the pion four-momenta to zero. The method depends on the assumptions that the axial and vector currents obey an SU(2) $\otimes$  SU(2) chiral algebra, the hypotheses of conserved vector current (CVC) and partial conservation of axial-vector current (PCAC), and that the intermediate states are saturated to a good approximation by single-particle  $\pi$ ,  $\rho$ , and  $A_1$  states.<sup>12</sup> The advantage of the technique is that it allows one to extend the usual currentalgebra analysis to pions significantly above threshold. Under the same assumptions, we have now generalized these results to n-point functions involving  $\pi$ ,  $\rho$ ,  $A_1$ , and  $\sigma$  mesons.<sup>13</sup> For scattering amplitudes, the condition of single-particle saturation implies that one should include pole diagrams using at the vertices the three-point functions previously determined<sup>7-9</sup> by the current algebra. However, in addition one must include a specific set of "seagull" diagrams to guarantee the satisfaction of CVC, PCAC, and the current algebra. The calculation was carried out using the "effective-Lagrangian" approach<sup>7</sup> and will be presented in detail elsewhere. We give here the results obtained for the  $\pi$ - $\pi$  scattering amplitudes. The  $\pi$ - $\pi$  S-matrix element has the form

$$\langle p_1^c, p_2^d | q_1^a, q_2^b \rangle = -i(2\pi)^4 \delta^4 (q_1^{+} q_2^{-} p_1^{-} p_2^{-}) NM_{cd, ab}(s, t, u),$$
(1)

where N is the conventional boson normalizing factor. The M function can be written as

$$M_{cd,ab}(s,t,u) = \delta_{ab} \delta_{cd}^{A}(s,t,u) + \delta_{ac} \delta_{bd}^{A}(t,u,s) + \delta_{ad} \delta_{bc}^{A}(u,s,t).$$
<sup>(2)</sup>

As discussed above, A will contain only a pole part and a sea-gull part. For  $\pi$ - $\pi$  scattering, only