$=740$, its $A/l = 1.36\times10^{-4}$ cm.

[7]The nominal 5.1×10^{-2}-cm-diam Pt wire, obtained from Matthey Bishop, Inc., was 99.999% pure. The resistivity ratio when mounted in the cryostat was $R_{300}/R_0 = 633$, its $A/l = 2.02\times10^{-4}$ cm.

[8]This does not include any deviation which may exist between the T_{62} (or T_{58}) temperature scale and the thermodynamic scale. A deviation that ranged as abruptly as that reported by D. T. Grimsrud and J. H. Werntz, Jr., Phys. Rev. 157, 181 (1967), would completely change the quantitative, though not the qualitative, conclusions of the present paper. In private conversations with Dr. H. Plumb and Dr. C. A. Swenson we learned that recent work above 2°K indicates a smoother deviation as a function of temperature. See H. Plumb and G. Cataland, Metrologia 1, 127 (1966), and J. S. Rogers, R. J. Tainsh, M. S. Anderson, and C. A. Swenson, to be published.

[9]This Ag sample was taken from the same spool as that of Ref. 6 and used in the "as received" condition. The electrical properties were not measured; the thermal conductivity was $5.4T$ W/cm °K^2. We are indebted to Professor J. S. Koehler for providing the Ag samples.

[10]This large uncertainty resulted from the small resistance of the Ag sample plus the small measuring current used in order to keep the resulting rise in temperature of the sample (situated in a vacuum) small. These measurements were made possible through the kind loan of a nanovolt potentiometer by Professor R. O. Simmons.

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INTRINSIC QUANTUM FLUCTUATIONS IN UNIFORM FILAMENTARY SUPERCONDUCTORS*

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Small depressions of supercurrent and critical temperature that indicate intrinsic thermodynamic fluctuations have been revealed in thin superconducting tin whisker crystals.

Quantum fluctuations are supposed to manifest their existence in thin superconductors by depression of the critical supercurrent densities and the critical temperature, according to several recent investigations.[1-6] However, disparities among the various accounts of this effect illustrate the present uncertainty about the fundamental nature and even the existence of fluctuations in systems characterized by long-range coherence. This Letter reports some new experiments on the critical supercurrent depression due to intrinsic fluctuations that may help to clarify this general problem.

Langer and Ambegaokar[1] have developed an elegant theory of the effect of intrinsic fluctuations on the resistive transition in thin superconductive filaments. They found that a fluctuation of order parameter in a volume $\sigma\xi(T)$, where σ is the cross-sectional area and $\xi(T)$ is the coherence length, has the minimum activation energy that permits phase slippage, and they calculated the steady-state temperature and current-density dependence of the consequent resistance near the critical temperature. However, all of the experiments—in thin microbridges formed by vapor deposition of thin films of tin—have shown a substantially larger effect. Typically, depressions of the effective critical temperature $\Delta T_c \gtrsim 10^{-2}$ °K have been observed instead of the predicted $\Delta T_c \sim 10^{-3}$ °K in comparable cross sections $\sim10^{-9}$ cm^2. Groff, Marčelja, Masker, and Parks[6] introduced a model based on phase fluctuations in a volume σL extending the full length L of their microbridges to explain their experiments. Hunt and Mercereau[5] assumed a fluctuation volume σl, where l is the electron mean free path, and included a noise temperature seven times the critical temperature to explain theirs.

Because the principal experimental difficulty in measurements of intrinsic fluctuations is elimination of overwhelming extraneous effects, we have made new measurements of the resistive transition in virtually perfect "whisker crystals" of tin. These measurements show a delicate supercurrent depression like that calculated by Langer and Ambegaokar instead of the larger effect previously reported for thin-film bridges.

We observed transitions of filamentary crystals of tin with cross-sectional areas $10^{-10} \lesssim \sigma \lesssim 10^{-8}$ cm^2 and lengths $0.02 < L < 0.1$ cm by recording the dc voltage appearing across the crystal as the temperature was gradually increased through the transition at various fixed currents $10^{-7} \lesssim I \lesssim 10^{-3}$ A.

The filamentary crystals were prepared as "whiskers" by the stress-accelerated recrystallization of electrolytic tin plate.[7] These crystals are usually virtually perfect single crystals and specimens were selected for uniformity of thickness and for the high elastic limit which identifies perfection.[8-11] They were manipulated by techniques that avoid severe plastic deformation and electrical contacts were effected by either soldering or mechanically clamping with Wood's metal, a Pb-Bi-Sn-etc. alloy that is superconducting at 3.7°K and expands on freezing at ~50°C.

Tracings of representative transitions at various currents are reproduced in Fig. 1. These transitions showed little hysteresis at low voltages and a sudden onset of the voltage with effective transition widths around 1 mdeg at low currents. The onset temperatures T were noted for about ten attempts at each of about 20 values of I. The principal source of uncertainty was the reproducibility of the onset temperature. The usual plots of $I^{2/3}$ vs T were constructed to determine the Ginzburg-Landau or "mean-field" critical current[12] $I_c \propto \Delta T^{3/2}$ from the data at temperatures well below T_c. This straight line was extrapolated through the fluctuation region to $I_c = 0$ to provide a reference line for analysis and to determine the critical temperature T_c. Plots of I vs $\Delta T = T_c - T$ such as the logarithmic plot in Fig. 2 show the fluctuation region at small ΔT, where the critical current I falls below the mean field line eventually dropping rapidly toward zero as $\Delta T \to \Delta T_c$.

The mean-field critical current[12] can be written

$$\frac{I_c(T)}{(\Delta T)^{3/2}} = \frac{\sigma J_c(T)}{(\Delta T)^{3/2}} = \frac{16}{3\sqrt{3}}\,\frac{e}{\hbar}\left[\frac{\Delta g(T)\xi(T)\sigma}{(\Delta T)^{3/2}}\right], \qquad (1)$$

where e is the electronic charge, $\hbar$ is Planck's constant, $\Delta g(T) = H_c^2(T)/8\pi$ is the condensation energy density, $\xi(T)$ is the coherence length, and σ is the cross-sectional area. The square bracket is temperature independent since $\Delta g(T)\xi(T) \propto (\Delta T)^{3/2}$. This equation arises physically from an appropriate balance of the condensation energy density Δg in a volume $\xi(T)\sigma$ against the kinetic energy $J_c\sigma\hbar/e$.

The theory of Langer and Ambegaokar describes the fluctuation-limited transition in the low-current-density limit ($J \to 0$) in terms of the ratio of a superconducting resistivity ρ_s to the normal resistivity ρ_n. Assuming parallel con-

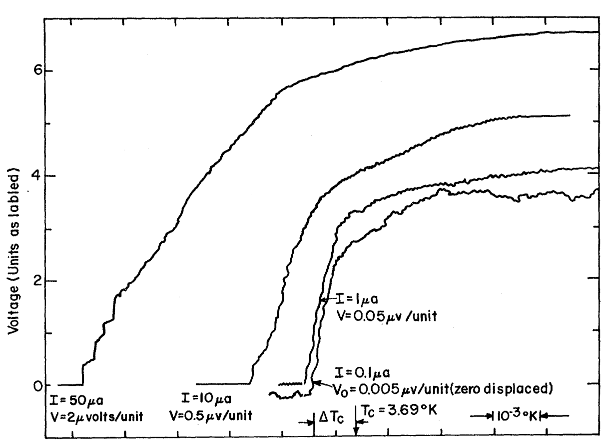

FIG. 1. Tracings of representative chart recordings of transitions placed on common temperature scale.

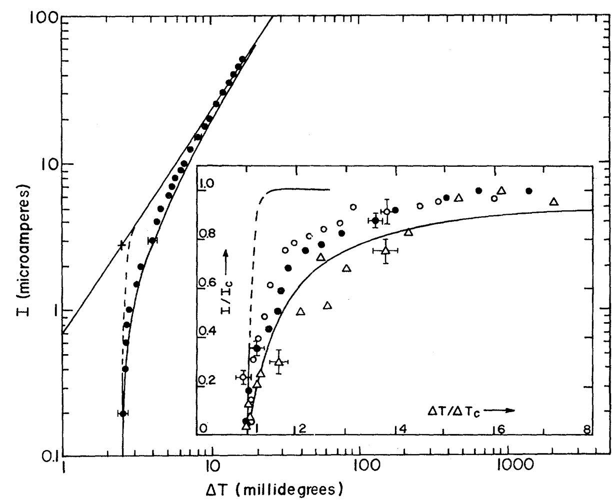

FIG. 2. Logarithmic plot of critical-current density I vs ΔT for crystal with area $\sigma = 1.1\times10^{-9}$ cm^2. Inset shows sensitive plot of normalized critical current $I/I_c = J/J_c$ vs $\Delta T/\Delta T_c$. •, $\sigma = 1.1\times10^{-9}$ cm^2; △, $\sigma = 4.1\times10^{-9}$ cm^2; ○, $\sigma = 0.8\times10^{-9}$. Solid curve from Langer and Ambegaokar; dashed curves based on Groff et al. Solid straight line is mean field I_c.

duction of supercurrents and normal currents, we identify the observed voltage $0 \leq V \equiv I\rho \leq I\rho_n \equiv V_n$, where $\rho^{-1} = \rho_s^{-1} + \rho_n^{-1}$, and use their Eq. (4.1):

$$\frac{\rho_s}{\rho_n}(J \to 0) = \frac{V}{V_n - V} = \exp\left\{\gamma - \frac{8\sqrt{2}}{3}\frac{\Delta g(T)\xi(T)\sigma}{k_B T_c}\right\}, \quad (2)$$

where $\gamma = \ln(\hbar^2 n^2 \sigma^2 / 4mk_B T_c) = \ln(3.2\times10^{35}\sigma^2)$, using for tin $T_c = 3.7$°K, $n = 1.2\times10^{23}$ cm^{-3}, with m the free electron mass and k_B the Boltzmann constant. Relevant values of the cross-sectional area $10^{-10} < \sigma < 10^{-8}$ make $\gamma \sim 35$ to 45. Just as in the mean-field critical-current equation, the quantity $\Delta g(T)\xi(T)\sigma$ appears. However, it arises here as the energy in a fluctuation volume $\xi(T)\sigma$ that is to be compared with $k_B T$, as in conventional fluctuation theory.

By identifying the fluctuation-limited critical current I with the midpoint of the transition, the effective critical temperature decrease at low current densities due to fluctuations can be defined by

$$(\Delta T_c)^{3/2} = \frac{3\gamma}{8\sqrt{2}} k_B T_c \left[\frac{\Delta g(T)\xi(T)\sigma}{(\Delta T)^{3/2}}\right]^{-1}. \quad (3)$$

Equations (2) and (3) describe rather sharp sigmoidal transition curves of ρ_s/ρ_n or V/V_n vs T with characteristic widths $W(\rho_s/\rho_n) = 4\Delta T_c/3\gamma$ and $W(V/V_n) = 8\Delta T_c/3\gamma$, respectively. However, the experiments showed a sharp onset of the transition which we compare with Eq. (2) after a shift by one half-width effected by a negligible adjustment in Eq. (3): $\gamma \to \gamma' = \gamma + 1 \approx 44$ in tin.

Eliminating $\Delta g(T)\xi(T)/(\Delta T)^{3/2}$ in Eqs. (1), (2), and (3) we obtain the useful result

$$(\Delta T_c)^{3/2} = \left(\frac{2}{3}\right)^{1/2} \frac{e}{\hbar} \frac{\gamma' k_B T_c}{[I_c(T)/(\Delta T)^{3/2}]} \quad (4)$$

which provides a unique connection between the measured quantities ΔT_c and the mean-field critical-current coefficient $I_c(\Delta T)/(\Delta T)^{3/2}$ with only the weakly varying γ' and T_c left as materials parameters. For tin Eq. (4) becomes $(\Delta T_c)^{3/2} \cong 2.8\times10^{-6}[I_c(T)/(\Delta T)^{3/2}]^{-1}$ with I_c in amperes. Figure 3 shows a plot of $\log(\Delta T_c)$ vs $\log[I_c(\Delta T)/(\Delta T)^{3/2}]$. The solid line is the theory and the points represent our data. Agreement is complete without adjustable parameters. This indicates that both the exponent of ΔT_c and the prefactor are correct. The experimental error is primarily due to uncertainty in determining the mean field line with sufficient precision. Notice that I_c extrapolates to a definite value at $\Delta T = \Delta T_c$; for our tin whiskers $I_c(\Delta T_c) \cong 2.8\times10^{-6}$ A, a quantity which we might imagine to be an equivalent fluctuation current.

Recasting the models of Groff et al. and of Hunt and Mercereau for a similar comparison yields, respectively, ΔT_c and $(\Delta T)^2 \propto [I_c(T)/(\Delta T)^{3/2}]^{-1}$. These results are inconsistent with our data. Groff et al. also compared ΔT_c with R^* which is the specimen resistance in the dirty limit and a calculable quantity proportional to L/σ in the clean limit. We tried this analysis on our data but recognized no systematic dependence on L. However, most of our crystals have $L \sim 0.03$ cm, where we find $\Delta T_c \sim \frac{2}{5}$ as large as Groff et al. find for comparable values of R^*.

A sensitive test of the fluctuation theories is the detailed shape of the drop of the fluctuation-limited critical current below the mean-field value. Figure 2 shows that our data nearly fit the shape predicted by Langer and Ambegaokar (solid lines) but not that given by Groff et al. (dashed lines). The inset shows the two theoretical curves and three sets of data to represent our range and present precision on a plot of $\Delta T/\Delta T_c$ against J/J_c. Again the Langer and Ambegaokar result agrees best although there may be some systematic deviation. The Groff et al. curve follows from their Eq. (10) since $f^2 = \Delta T_c/\Delta T$.

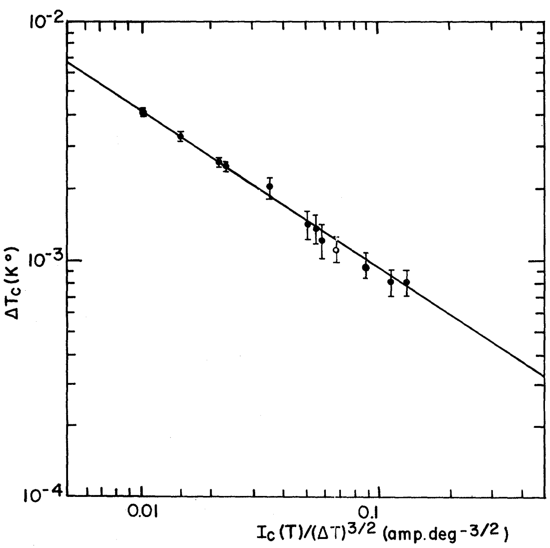

FIG. 3. Comparison of ΔT_c with mean-field critical-current coefficient $I_c(T)/(\Delta T)^{3/2}$. Solid line from theory following Eq. (3)

The experimental widths of the individual transitions (see Fig. 1) at low currents are $W \sim \Delta T_c \sim 10^{-3}$ °K rather than the somewhat smaller theoretical values $W \simeq 8\Delta T_c/3\gamma \sim 10^{-4}$ °K. Although they seem to be an order of magnitude narrower than those previously reported for microbridges[3-5] or whisker crystals,[13] these widths may still be limited experimentally. The transition widths increased slowly with current for $I > I(\Delta T_c) \sim 2.8\times10^{-6}$ A. For $I \gtrsim 10^{-4}$ A a step structure developed with highly reproducible jumps of voltage between distinct states that depend on both current and temperature. This effect, which is barely visible in Fig. 1, will be discussed elsewhere.

The cross-sectional areas of the microcrystals are elusive quantities. Our best values were obtained using Eq. (1) with $\Delta g(\Delta T) \cong (1.82)^2 H_0^2(\Delta T)^2/8\pi T_c \cong 893(\Delta T)$ (erg/cm^3) using $H_0 = 306$ Oe, and $\xi(T) = 0.85(\xi_0 l T_c/\Delta T)^{1/2}$ with $\xi_0 = \hbar V_F/\pi\Delta_0 = 2300$ Å using the experimental energy gap $\Delta_0 = 6.5k_B$ at $T=0$ and the Fermi velocity $V_F = (\pi^2 k_B^2/e^2)(\sigma_e/l)/\gamma_e$, where σ_e/l is the ratio of the electrical conductivity to the mean free path obtained from anomalous-skin-effect data and γ_e is the experimental electronic specific-heat coefficient. The result is $\xi(T) = 7.8\times10^{-3}(l/\Delta T)^{1/2}$ with an uncertainty of about 50% mostly due to σ_e/l.[14] Assuming that the electron mean free path is just the whisker diameter,[15] as suggested by the measured resistivity ratios, and neglecting the large anisotropy, the cross-sectional areas were calculated from

$$\sigma^{5/4} = [I_c(T)/\Delta T^{3/2}]/3.47\times10^9\ \mathrm{cm}^{5/2},$$

and found to agree with electron microscopic measurements of a diameter within the rather large experimental error.

Although our results agree closely with a theory of intrinsic fluctuations and show a much smaller fluctuation effect than previous experiments we have considered the possibility that the remaining effect is extraneous. Radio-frequency interference was excluded by double Mumetal shields and a double electrostatic screened room that had worked for a superconducting quantum magnetometer[16] with a sensitivity of $1/10^9$. Results here were indistinguishable from those obtained with standard shielding outside the room. Removal of shielding decreased reproducibility and broadened transitions. Thermal noise from room-temperature connections might produce a noise current $i \sim (k_B T/L')^{1/2}$, where $T \sim 300$°K and $L' \sim 10^{-7}$ H is the usual cold-circuit inductance. This yields $i \sim 10^{-7}$ A which is >10× too small to account for ΔT_c, but might have increased W. Increase of L' by 100× had no effect.

Magnetic field effects were small since the crystals are much smaller than a penetration depth at the relevant temperatures. Nevertheless, we measured the effect of applied fields in the range $0.01 \lesssim H \lesssim 13$ Oe. A negligible field effect was observed with $dT_c/dH \sim 10^{-3}$ deg/Oe in the mean-field region and several times smaller in the fluctuation-limited region. Heat-transfer problems should have been negligible although bath-temperature variations may have contributed to the widths. With the exception of the transition widths we believe that extrinsic perturbations were negligible, although they are nearly impossible to rule out definitively.

We conclude that the temperature dependence of the critical-current depression that we have observed near T_c in tin whisker crystals is in good agreement with the theory of Langer and Ambegaokar. This result indicates the existence of intrinsic thermodynamic quantum fluctuations in thin superconductors. We presume that the characteristic fluctuation volume is then $\sigma\xi(T)$ rather than σl or σL, at least if $l \gtrsim \xi(T)$. However, the theory seems to be incomplete in description of the resistive state since it gives only the observed onset temperatures and not the observed shape or width of the transition nor the observed voltage steps at higher currents. These effects may indicate that a partially ordered mixed state should be explicitly included in the theory.[17]

We gratefully acknowledge many helpful conversations with Professor V. Ambegaokar, Professor J. S. Langer, Professor M. E. Fisher, and Professor J. W. Wilkins, and the help of Mr. Walter Henkels on the early experiments and Mr. Thomas Smith on growth of the whiskers.

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†National Science Foundation Predoctoral Trainee.

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DECAY $\Xi^- \to \Lambda^0 e^- \bar{\nu}$ †*

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In the course of a search for leptonic decays of the Ξ hyperon, we have observed two unambiguous examples of

$$\Xi^- \to \Lambda^0 e^- \bar{\nu} \tag{1}$$

and obtain a branching fraction of 1.0×10^{-3} for this mode. The Ξ^- were produced in a 27-event/μb exposure of the Lawrence Radiation Laboratory 72-in. hydrogen bubble chamber to an incident K^- beam with momentum 1.7, 2.1, and 2.40 to 2.75 GeV/c. We have considered production events of the types

$$K^-p \to \Xi^- K^+, \tag{2a}$$
$$\to \Xi^- K^+\pi^0, \tag{2b}$$
$$\to \Xi^- K^0\pi^+, \tag{2c}$$
$$\to \Xi^- K^+\pi^+\pi^-, \tag{2d}$$
$$\to \Xi^- K^0\pi^+\pi^0, \tag{2e}$$

where the decay kink of the Ξ^- and the decay Λ^0 were observed. Some 2823 events fitted one of Reactions (2a)-(2e) as well as the normal decay sequence

$$\Xi^- \to \Lambda^0\pi^-, \tag{3a}$$
$$\Lambda^0 \to p\pi^-, \tag{3b}$$

with confidence level $\gtrsim 0.5\%$ for each of the three one-vertex fits.

Candidates for the beta-decay mode (1) satisfied the following criteria: (a) Lambda decay (3b) fits with confidence level $\gtrsim 0.5\%$. (b) Normal Ξ^- decay (3a) does not fit; confidence level $\lesssim 0.5\%$. (c) A two-vertex fit to one of Reactions (2a)-(2e) followed by the decay (1) is obtained with confidence level $\gtrsim 0.5\%$. Only nine events of the topologies giving rise to Reactions (2a)-(2e) satisfied the criteria (a), (b), and (c). Of these, three events have negative tracks from the Ξ^- decay which are nearly flat in the chamber, have measured laboratory-system momenta less than 200 MeV/c, but are clearly darker than minimum ionizing. A pion of 200 MeV/c or less has bubble density >1.5 times that for the minimum-ionizing beam tracks, a difference distinguishable in our pictures for any but steeply dipping tracks. Another four events have charged Ξ^- decay tracks with momenta greater than 200 MeV/c. We have imposed the additional requirement that this momentum be less than 200 MeV/c for an event to be a candidate for (1). The remaining two events are shown in Fig. 1.

Each of the events in Fig. 1 is an unambiguous example of the decay (1). Event A fits $K^-p \to \Xi^- K^+$ with subsequent $\Xi^- \to \Lambda e^- \bar{\nu}$ decay with a four-constraint $\chi^2 = 1.45$. Event B fits the same production-decay sequence with $\chi^2 = 0.34$. Neither event fits muonic decay $\Xi^- \to \Lambda\mu^-\bar{\nu}$, although four of the seven rejected events do fit this mode. None of these four events has been unambiguously identified as an example of Ξ muonic decay. In each of events A and B the electron tracks are nearly flat in the chamber, with measured momenta 100.3 ± 1.2 and 134.6 ± 1.8 MeV/c, respectively. If the negative decay tracks were pions, their relative ionizations would be 2.6 and 2.0, respectively (as muons, 2.0 and 1.6); therefore,