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COLLISIONAL DAMPING OF A PLASMA ECHO

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It is shown that the collisional damping on the free streaming motion of plasma can be very important in certain circumstances even though the collisional frequency is small compared with the plasma frequency. Both spatial and temporal echoes are treated in this note. In the latter case, for example, we found that the collisional damping goes as $\exp[-\beta\omega_p^2t^3]$ (where β is the collisional frequency).

Recently Gould, O'Neil, and Malmberg[1] demonstrated the possibility of generating a plasma echo by applying two pulses of longitudinal waves to a plasma. The theory is based on Vlasov's equation. It is found that the plasma echo was produced by the reconstruction of the phases of the two free-streaming motions of particles due to the applied pulses. In this paper we consider the collisional effect on the motion of particles. It is shown that for small collisional frequencies, even though the collisional effect on the collective motion of particles (plasma oscillations) can generally be neglected, the collisional damping of the free-streaming motion can be quite important in certain circumstances so as to make it impossible for the generation of plasma echoes to occur.

We shall use the simple Brownian motion[2] for the description of collisions between electrons. The main feature relevant to our problem is the diffusion in velocity space of this collision model. The approximation for small collision frequency using Fokker-Planck-type collision models is a singular perturbation problem. This is responsible for the somewhat unconventional result obtained below.

The governing equations are as follows:

$$\frac{\partial f}{\partial t}+v\frac{\partial f}{\partial x}-\frac{e}{m}E\frac{\partial f}{\partial v}=\beta\left[\frac{\partial}{\partial v}(vf)+u_0{}^2\frac{\partial^2 f}{\partial v^2}\right], \quad (1)$$

$$\frac{\partial E}{\partial x}=\frac{\partial E_e}{\partial x}-4\pi n_0 e\int_{-\infty}^{\infty}dv(f-F_0), \quad (2)$$

where $f(x,t,v)$ and $F_0(v)$ are the electron and ion distribution functions, respectively, and $E(x,t)$ and $E_e(x,t)$ are the total and externally applied electric field, respectively. The term on the right-hand side of (1) is the collision operator we are going to use in this paper, where β is the collision frequency and u_0 is the average velocity of electrons. In what follows we shall normalize time t by t_0, volocity v by u_0, length x by l, and the electric field E, E_e by φ_0/l. Letting $f=F_0+(e\varphi_0/mu_0{}^2)f_1$ and neglecting any quadratic term in f_1, we obtain Eqs. (1) and (2) in the dimensionless form as

$$\frac{\partial f_1}{\partial t}+v\frac{\partial f_1}{\partial x}-E\frac{\partial F_0}{\partial v}=\epsilon\left\{\frac{\partial}{\partial v}(vf_1)+\frac{\partial^2 f_1}{\partial v^2}\right\}, \quad (3)$$

$$\frac{\partial E}{\partial x}=\frac{\partial E_e}{\partial x}-\left(\frac{l}{\lambda_D}\right)^2\int_{-\infty}^{\infty}dvf_1, \quad (4)$$

where $\epsilon=\beta t_0$ and $\lambda_D=(mu_0{}^2/4\pi n_0 e^2)^{1/2}$. For the physical problem we are interested in, we have $\epsilon\ll 1$ and $l/\lambda_D=O(1)$. Assuming that all the external disturbances act through E_e, we shall Fourier analyze f_1, E as follows:

$$f(k,\omega,v)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}dtdx\,e^{-i(\omega t+kx)}f_1(x,t,v), \quad (5)$$

$$E(k,\omega)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}dtdx\,e^{-i(\omega t+kx)}E(x,t). \quad (6)$$

Then Eqs. (3) and (4) become

$$i(\omega+kv)f_1-\epsilon\left\{\frac{d^2 f}{dv^2}+\frac{d}{dv}(vf)\right\}=E\frac{dF_0}{dv}, \quad (7)$$

$$iK(E-E_e)=-\left(\frac{l}{\lambda_D}\right)^2\int_{-\infty}^{\infty}dvf(k,\omega,v). \quad (8)$$

We shall use Eq. (7) to solve for $f(k,\omega,v)$ in terms of E. First dropping the terms with ϵ (collision terms) in (7), we obtain the usual collisionless result, i.e.,

$$f_{\text{out}}=-iE\frac{dF_0/dv}{\omega+kv}. \quad (9)$$

This approximate solution is not valid near $\omega+kv=0$. Around there, we let

$$-i(\omega+kv)=(K^2)^{1/3}u, \quad (10)$$

$$F=(K^2\epsilon)^{1/3}f. \quad (11)$$

Substituting these in (7) and dropping terms of order $\epsilon^{1/3}$, we obtain another approximate equation which is valid near $\omega+kv=0$:

$$\left(\frac{d^2}{du^2}-u\right)F=\left(\frac{\partial F_0}{\partial v}\right)_{v=-\omega/k}. \quad (12)$$

The solution of (12) which goes into f_{out} given by (9) as $u\to\pm\infty$ is

$$F(u,k,\omega)=E\left(\frac{\partial F_0}{\partial v}\right)_{v=-\omega/k}\times\int_0^{\infty}\exp(-\tfrac{1}{3}t^3+ut)dt. \quad (13)$$

We can now compute the relationship between E and E_e by using (9) and (13) in the velocity integral of (8). Note that the range of v integration has to be split into two regions: one for which $|\omega+kv|\geqslant\delta$ (there $\epsilon^{1/3}\ll\delta\ll 1$), where we use $f=f_{\text{out}}$; and the other $|\omega+kv|>\delta$, where we use $f=(K^2\epsilon)^{-1/3}F$. The result is

$$E=E_e/\mathcal{E}(k,\omega), \quad (14)$$

$$\mathcal{E}(k,\omega)\equiv 1-\left(\frac{l}{\lambda_D}\right)^2\frac{1}{k^2}\times\left[\mathrm{P}\int dv\frac{\partial F_0/\partial v}{v+\omega/k}+i\pi\left(\frac{\partial F_0}{\partial v}\right)_{v=-\omega/k}\right], \quad (15)$$

where P indicates the principal value of the integral, and we note that the principal-value integral comes from f_{out} (nonresonant particles) while the second term in the square bracket comes from the resonant particles described by F. It is noted from (15) that the effect of the weak collision effect on the collective behavior of plasma is not very surprising. In fact this same result could be obtained by simply replacing the collision term with $-\epsilon f_1$ in (3). The collisional effect is more significant for the free-streaming motion. We shall illustrate this by the following two examples.

(a) Spatial echo problem.—The external electric field $E_e = \exp[i\omega_e t]\delta(x)$. We shall let the time dependence of f_1 be $\exp[i\omega_e t]$; then $E_e(k) = 1$. To get $f_1(x, v)$ we need the Fourier inversion of $f(k, v)$ given by (9) and (12), i.e.,

$$f_1(x, t, v) = \exp(i\omega_e t)\int_{-\infty}^{\infty}\frac{dk}{2\pi}e^{ikx}f(k, v, \omega_e). \quad (16)$$

Again, as before, the integral in (16) has to be split into two parts, i.e., for $|k+\omega/v| \geq \delta$ we use $f = f_{\rm out}$ and for $|k+\omega/v| < \delta$ we use $f = (K^2\epsilon)^{-1/3}F$. However, it is readily shown that for sufficiently large x, say $\epsilon^{1/3}x \sim O(1)$, the contribution of the integral from $f = f_{\rm out}$ becomes vanishingly small and we obtain

$$f_1(x, t, v) = \frac{H(x)}{\mathcal{E}(\omega_e, -\omega_e/v)}\frac{1}{v}\frac{\partial F_0}{\partial v}\exp\left[i\omega_e\left(t-\frac{x}{v}\right)-\frac{\epsilon\omega_e^2}{3}\left|\frac{x^3}{v^5}\right|\right], \quad v > 0,$$

$$= \frac{H(x)-1}{\mathcal{E}(\omega_e, -\omega_e/v)}\frac{1}{v}\frac{\partial F_0}{\partial v}\exp\left[i\omega_e\left(t-\frac{x}{v}\right)-\frac{\epsilon\omega_e^2}{3}\left|\frac{x^3}{v^5}\right|\right], \quad v < 0, \quad (17)$$

where $H(x)$ is the step function defined as

$$H(x) = 1, \quad x \geq 0,$$
$$= 0, \quad x < 0. \quad (18)$$

In (17) we did not include the motion of plasma collective oscillations which is heavily damped due to Landau damping at the distance $x \sim \mathcal{E}^{-1/3}$ from the disturbance.

(b) Temporal echo problem.—The external field is $E_e = \exp(ik_e x)\delta(t)$. We then let $f_1(x, t, v) = \exp(ik_e x)f_1(t, v)$. Now $f_1(t, v)$ is obtained by the Fourier inversion of $f_{\rm out}$ and F, i.e.,

$$f(t, v) = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{i\omega t}f(k_e, \omega, v). \quad (19)$$

The integral again is split into two parts and the result after some manipulation is

$$f_1(x, t, v) = \frac{H(t)}{\mathcal{E}(-k_e v, K_e)}\frac{\partial F_0}{\partial v} \times\exp[ik_e(x-vt)-\tfrac{1}{3}\epsilon K_e^2 t^3]. \quad (20)$$

Note that both in (17) and (20)[3] the damping factors are proportional to t^3 or x^3 instead of simply linear in t or x. Furthermore, in terms of dimensional variables (take $t_0 = 1/\omega_p$, $l = \lambda_D$ say), we have the damping factor in the spatial case as $\exp[-(\beta/\omega_p\lambda_D^3)x^3]$ (for thermal particles) and in the temporal case as $\exp[-\beta\omega_p^2 \times t^3]$. It is thus seen that the collisional damping of the plasma free-streaming motion will be very important for $x \sim O((\omega_p\lambda_D^3/\beta)^{1/3})$ and $t \sim O((\beta\omega_p^2)^{-1/3})$.

From the above analysis it is seen that the result (17), (20) depends only upon those particles which move with a velocity such that $\omega + kv \approx 0$. We can thus conclude that the results we obtained are quite independent of the simple collision model we used. It is seen that, as long as a collision operator is a diffusion type, locally at $v \approx -\omega/k$ this collision operator is in the form of the simple model we used in the present paper. In this connection we mention the turbulent diffusion due to weak instabilities in plasmas. The plasma echo measurements may provide a unique way of detecting the turbulent diffusions.

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