PRODUCTION OF THREE CHARGED PIONS IN $\bar{p}+n$ ANNIHILATION AT REST

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We have measured a sample of 2785 events of the reaction $\bar{p}+n\rightarrow 2\pi^-+\pi^+$ which are due to antiproton capture at rest in deuterium. An analysis of the ($I^GJ^P=1^-0^-$) three-pion final state has been attempted in terms of the π-π interaction.

The objective of this Letter is the communication of our present[1] data on the reaction

$$\bar{p}+n\rightarrow 2\pi^-+\pi^+ \qquad (1)$$

initiated by antiprotons brought to rest in a deuterium bubble chamber. This is possible since events with slow outgoing protons can be considered as being due to $\bar{p}$ annihilation on free neutrons. Reaction (1) is of particular interest because the three pions are in a pure quantum state ($I^GJ^P=1^-0^-$) which is identical to the final state of the η and K decays into three pions, and thus provides an opportunity of studying the π-π interaction up to a center-of-mass energy (1736 MeV) much higher than in η and K decays. The uniqueness of the quantum numbers is a consequence of G conservation if we assume S-wave absorption of antiprotons at rest. This assumption is supported by theory and experiment.[2]

Reaction (1) has also been studied, on the basis of 252 events, by the Padova-Pisa groups.[3] Our data, statistically more significant, are similar to theirs, but we do not agree with their interpretation. They concluded that Reaction (1) proceeds by 40% $\pi^-\rho^0$, 40% π^-f^0, and 20% constant background. We shall show that the 3π state has a complex structure, very sensitive to the detailed energy dependence of the π-π interaction, but we have not yet been able to find a satisfactory interpretation.

Data.—Our groups are continuing the analysis of $\bar{p}d$ film obtained with the BNL-Columbia 30-in. bubble chamber exposed to a separated beam of stopping $\bar{p}$ at the alternating-gradient synchrotron. About $\frac{2}{3}$ of our sample of Reaction (1) comes from part of a special scanning for three-prong stars with no obvious unbalance in momentum. The remaining third comes from a portion of the film where all (about 70 000) $\bar{p}n$ annihilations have been measured; from this subsample and imposing the selections stated below, we get the frequency for Reaction (1):

$$\frac{\bar{p}+n\rightarrow 2\pi^-+\pi^+}{\text{total } \bar{p}n}=2.4\pm 0.4\%. \qquad (2)$$

On the basis of kinematical fits,[4] we were able to identify the events belonging to Reaction (1). The analysis has been done in an independent way in the two laboratories, but the data are in good agreement. We therefore present the combined sample and we are confident that the possibility of systematic errors has been substantially reduced.

About 44% of all $\bar{p}n$ annihilations show a visible proton track; when the proton was too short to be seen, it has been treated either as an unmeasured track (1-constraint fit) or as a measured track with all three Cartesian components of the momentum equal to zero within suitable errors (4-constraint fit). The two procedures gave the same result. The proton momentum distribution is in good agreement with the Hamada-Johnson deuteron wave function up to 200 MeV/c.[5] We have selected 2785 events with proton momentum less than 150 MeV/c and, according to the impulse model, we consider them as being due to Reaction (1).

We have considered the following possible sources of contamination: (1) events from the reaction $\bar{p}+d\rightarrow 2\pi^-+\pi^++\pi^0+p$, with a very slow π^0; and (2) annihilations in flight. We have compared the π^0 spectrum of all 4π annihilations with the corresponding $\pi^\pm$ spectra. The events which fitted both the 3π and the 4π hypotheses (15% of the sample) contribute a clearly spurious peak at the very beginning of the π^0 spectrum. We have then estimated that only (3 ± 3)% of the events can be assigned to the 4π channel. An analysis based on the ratio of the probabilities for the 3π and 4π hypotheses gave 1.5%.

On the basis of the symmetry of the visible momentum relative to the antiproton direction, we have estimated a contamination of annihilations in flight of $3 \pm 2\%$.

The previous analysis of the π^0 spectrum and an analysis of the missing-mass distribution give us confidence that our selection criteria do not reject more than 2% of the 3π events.

The Dalitz plot for the 3π system is shown in Fig. 1(a). Each event has been plotted twice (because of the two π^-), and therefore the plot is completely symmetric with respect to the diagonal. The plot contains all the information about Reaction (1) as long as we consider the proton as a true spectator.[5] It shows considerable structure, mainly: (i) strong enhancement in the low $M^2(\pi_1^-, \pi_2^-)$ region where $M^2(\pi^+ \pi_1^-) \simeq M^2(\pi^+, \pi_2^-) \simeq 1.64$ GeV^2 (about f^0 mass); (ii) absence of events in the region $M^2(\pi^+, \pi_1^-) \simeq M^2(\pi^+, \pi_2^-) \simeq 1.08$ GeV^2 (hole near the center of the Dalitz plot); (iii) lack of events in the region where one $M^2(\pi^+, \pi^-)$ is small and the other one is large; and (iv) apparent abundant production of ρ^0 and f, as seen in the $M^2(\pi^+, \pi^-)$ distribution [Fig. 1(b)].

We have also selected a sample of 243 events with proton momentum higher than 200 MeV/c and another sample of 114 events enriched in annihilations in flight[6]; both samples give a rather isotropic Dalitz plot, indicating that the structures we observe in Fig. 1 are typical of Reaction (1) following atomic $\bar{p}$ capture. We note also that the presence of the "hole" near the center of the Dalitz plot per se supports the evidence that we are dealing with a pure state. Any incoherent admixture of other channels would likely fill the hole.

Interpretation. – We have attempted to fit the Dalitz-plot distribution by writing the amplitude for Reaction (1) as a superposition of amplitudes:

$$A_{t, t_3, l} = b_{t, t_3} F_{t, l}(|\vec{p}|) \cdot S_l(\vec{p}, \vec{q}), \qquad (2)$$

where t, t_3, and l are the isospin, its third component, and the angular momentum of the dipion formed by π_1 and π_2, $\vec{p}$ is their relative momentum in the di-pion c.m. system, $\vec{q}$ is the momentum of π_3 in the over-all c.m. system, and

$$S_l(\vec{p}, \vec{q}) = [l!/(2l-1)!!] P_l(\cos\theta) p^l q^l,$$

where $\cos\theta = \vec{p} \cdot \vec{q}/pq$, and P_l is a Legendre polynomial.[7] $F_{t,l}(p)$ are scalar "form factors"

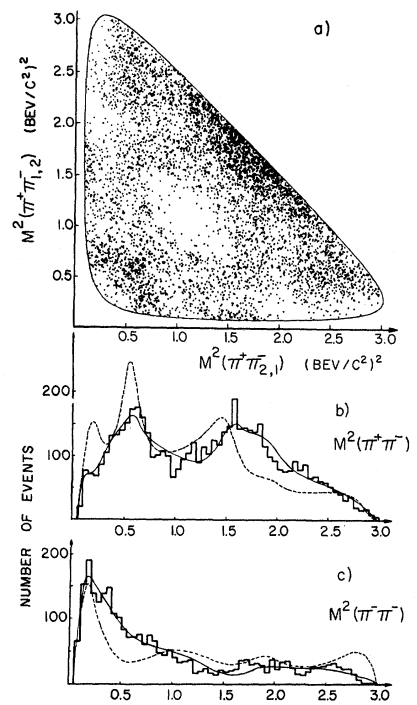

FIG. 1. (a) Symmetrized Dalitz plot of the $\pi^-\pi^-\pi^+$ final state, with its projections (b) on the $M^2(\pi^+, \pi^-)$ axis, and (c) on the $M^2(\pi^-, \pi^-)$ axis. The curves correspond to fit 1 (dashed line), fit 5 (solid line).

characterizing the interaction between π_1 and π_2. The ρ, f^0, and "σ" resonances have been introduced as Breit-Wigner amplitudes

$$F_{t,l}(p) = \frac{m_0 \Gamma_0}{m_0^2 - m^2 - i m_0 \Gamma(l, p)},$$

where

$$m = 2(m_\pi^2 + p^2)^{1/2},$$

$$\Gamma(l, p) = \Gamma_0 (m_0/m)(p/p_0)^{2l+1};$$

$$m_0 = 2(m_\pi^2 + p_0^2)^{1/2}$$

and Γ_0 are the resonance mass and width. The form of the effective range amplitude is

$$F_{t,0}(p) = 2/(\tfrac{1}{2} r a p^2 - 1 - i p a),$$

where a and r are the scattering length and effective range. The b's are products of Clebsch-Gordan coefficients.[8] All amplitudes are symmetrized with respect to the two negative pions. The total amplitude is then

$$A = \sum_{t, t_3, l} C_{t,l} \exp(i\varphi_{t,l}) A_{t, t_3, l}, \qquad (3)$$

where $C_{t,\,l}$ and $\varphi_{t,\,l}$ are constants to be optimized by a fitting program. The normalization is $\int |\sum_{t_3} A_{t,\,t_3,\,l}|^2 dm_{12}^2 dm_{13}^2 = 1$ and $\int |A|^2 dm_{12}^2 \times dm_{13}^2 = 1$. With this normalization the difference

$$\sum_{t,\,l} |C_{t,\,l}|^2 - 1$$

is the contribution of the interference terms.

We started introducing the amplitude corresponding to the ρ and f meson production. However, the di-pion decay angular distributions in the ρ and f mass bands do not agree with those predicted for the decay of a free resonance [Figs. 2(b) and 2(d)]. The addition of a constant coherent background does not improve the fit. We have also considered a $t=0$ S-wave resonance with mass and width optimized by the program. Moreover, we have introduced a $t=2$ S-wave low (π,π) mass enhancement in the form either of an effective range or of a Breit-Wigner amplitude.

The results of these fits are shown in Table I and some of them are compared with the experimental distributions in Figs. 1 and 2. None of these fits gave a good χ^2. We have also tried several other possibilities and refinements, for example, addition of the g meson, addition of S-, P-, and D-wave (π,π) interaction in the energy region above the known resonances; none of them improved appreciably the χ^2.

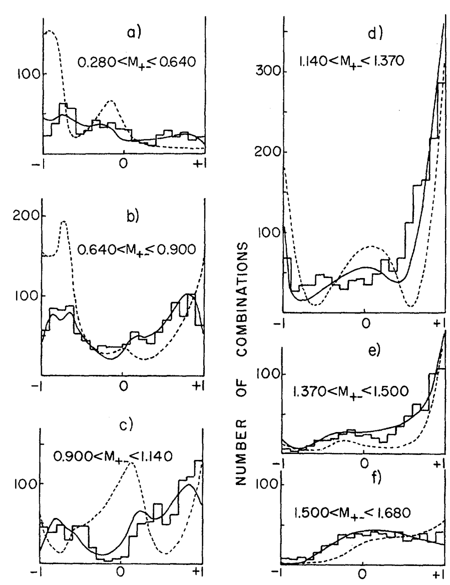

FIG. 2. Angular distributions of the π^+ relative to the (π^+,π^-) di-pion line of flight, in the di-pion c.m. system for various $M(\pi^+,\pi^-)$ regions. The curves are as in Figs. 1(b) and 1(c).

Conclusion. – The difficulty of treating a three-body system and the lack of a simple parametrization of the energy dependence of the pion-pion phase shifts are most likely the reasons for which we have not been able to find a satisfactory fit to the data. We feel, however, confident that the foregoing analysis allows the following conclusions.

(i) From the analysis of the reaction $\bar{p}+p \to \pi^+ + \pi^0 + \pi^-$ at rest,[9] it has been concluded that the singlet S state gives a uniform Dalitz plot, and accounts for $(3.5\pm0.6)\%$ of all $\bar{p}p$ annihilations. Our data are incompatible with a uniform distribution; furthermore, from the observed frequency of Reaction (1) we can conclude[10] that the singlet S-state contribution to the reaction $\bar{p}p \to \pi^+ + \pi^- + \pi^0$ accounts at most for $(1.4\pm0.3)\%$ of all $\bar{p}p$ annihilations.

(ii) ρ production seems to be very small and may even be consistent with zero. Comparing this result with those on the reaction $\bar{p}+p \to \pi^+ + \pi^0 + \pi^-$,[9,11,12] we confirm their conclusion that the (ρ,π) branching ratio, while large for the triplet state, is small for the singlet.

(iii) f^0 production seems to be rather large. The shape of the decay angular distribution in the f^0 mass region requires the presence of other, largely unknown, amplitudes: $t=0, 2$ S waves, and $t=1$ P or F waves; consequent-

Table I. Results of some of the fits tried. The following values were used for the ρ_2 and f^0 masses and widths: $\rho(0.77, 0.16)$; $f^0(1.25, 0.16)$ GeV/c. Typical error on $|C_{t,\,l}|^2$ is 0.02.

No. of Fit		1	2	3	4	5
(ρ^0,π^-)	$\|C_{1,1}\|^2$	0.39	0.04	0.06	0.02	0.02
	$\varphi_{1,1}$	23°	147°	146°	271°	165°
(f^0,π^-)	$\|C_{0,2}\|^2$	0.53	0.36	0.33	0.17	0.36
	$\varphi_{0,2}$	0°	0°	0°	0°	0°
a	$\|C_{0,0}\|^2$	...	0.65	...	...	...
	$\varphi_{0,0}$	...	270°	...	...	...
(σ^0,π^-)b	$\|C_{0,0}\|^2$	...	...	0.77	...	0.30
	$\varphi_{0,0}$	...	...	282°	...	152°
c	$\|C_{2,0}\|^2$	...	...	...	0.84	0.47
	$\varphi_{2,0}$	...	...	...	−91°	−45°
χ^2		1917	877	453	434	190
No. of intervals		71	71	73	71	75
No. of parameters		3	5	7	7	11

[a]Constant, coherent background.

[b]The fitted (m_0,Γ_0) in GeV/c^2 for the σ^0 were (1.2, 0.86) for fit 2, (0.48, 0.38) for fit 5.

[c]An effective range formula $(p\cot\delta = -1/a + \frac{1}{2}p^2 r)$ was used for the $t=2$ (π,π) interaction; the fitted values in units m_π^{-1} for the scattering length and the effective range were (0.36, 0.44) for fit 4, (0.40, 0.81) for fit 5.

ly, the (f^0, π^-) frequency is rather uncertain. However we note that the most recent data on $\bar{p}p$ annihilation[12] show an (f^0, π^0) frequency which is consistent with our best fits.

Although our analysis seems to require rather strong S-wave (π, π) interaction both in $t=0$ (very wide Breit-Wigner with $m_0 \simeq 500$ MeV/c^2 and $\Gamma_0 \simeq 400$ MeV/c^2) and in $t=2$ (low energy enhancement), we cannot exclude that a combination of other amplitudes could also give an equally good or better fit to the experimental data.

This result for the S-wave (π, π) phase shifts should be compared with that obtained from pion production,[13] pion-nucleon scattering,[14] and η and K decays.[15] We note, in particular, that the $t=2$ phase shift obtained from the analysis of pion production reactions[13,16] is small ($|\delta_0^2| < 20°$) below $M(\pi, \pi) = 1.0$ GeV, in contrast to the behavior obtained from our best fit. We note, however, that the (π^-, π^-) enhancement could be due to a rescattering effect of the type discussed by Aitchison[17] and Month.[18]

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[1]The data we present in this Letter constitute about $\frac{1}{5}$ of all events from Reaction (1) available in our film; the analysis is continuing.

[2]See L. Gray, P. Hagerty, T. Kalogeropoulos, G. Nicodemi, S. Zenone, R. Bizzarri, G. Ciapetti, M. Gaspero, I. Laakso, S. Lichtman, G. C. Moneti, C. Natoli, and G. Pertile, Phys. Rev. Letters 17, 501 (1966). In this paper, the theoretical justifications and the experimental verifications of the S-capture hypothesis and relevant references are given. In fact, if one takes the φ quantum numbers for granted (as determined from other experiments), then this work is an excellent verification of the S-wave absorption, at least as far as the $\varphi\pi^-$ reaction is concerned.

[3]A. Bettini, M. Cresti, S. Limentani, L. Peruzzo, R. Santangelo, S. Sartori, L. Bertanza, A. Bigi, R. Carrara, R. Casali, and P. Lariccia, Nuovo Cimento 47A, 642 (1967).

[4]In Syracuse the program chain FOG-CLOUDY-FAIR has been used, and in Rome the chain THRESH-GRIND-SLICE-SUMX.

[5]From the study of this reaction as well as others, we find that when the proton has large momentum, it cannot be treated as a spectator. For momenta less than ~150 MeV/c, it seems to play no role in the dynamics of the annihilations except for its (small) kinematical role. Furthermore, we have compared the Dalitz plot for events with proton momenta <45 MeV/c with that of events with proton momenta in the interval 45-150 MeV/c. The comparison gives $\chi^2 = 47$ with 50 degrees of freedom; this is indicative that there is no influence of the proton on the $\bar{p}n$ annihilation process. We notice that if the $\bar{p}n$ system annihilates in D wave, the proton should be in a D wave relative to it; consequently the subsample with higher proton momentum should have a stronger contribution from the D-wave annihilation. The equivalence of the two subsamples indicates that the contribution of D-wave annihilation is small.

[6]This last sample was selected on the basis of the (measured) missing mass ($-0.18 < \text{mm}^2 < -0.09$ GeV^2) and of the missing momentum; their average $\bar{p}$ momentum is approximately 200 MeV/c and it ranges between 100 and 300 MeV/c. Among our 2785 examples of Reaction (1), only 1% of the events have $\text{mm}^2 < -0.04$ GeV^2.

[7]Ch. Zemach, Phys. Rev. 140, B97 (1965). For $l=1$ and 2 we have used the relativistic extension suggested by Zemach [Phys. Rev. 133, B1201 (1964)] which, however, is in our case practically indistinguishable from the nonrelativistic formula given here.

[8]The relevant values are $b_{0,0} = b_{1,0} = 1$, $b_{2,0} = 60^{-1/2}$, $b_{2,2} = 6/60^{1/2}$.

[9]C. Baltay, P. Franzini, N. Gelfand, G. Lutjens, J. Severiens, J. Steinberger, D. Tycko, and D. Zanello, Phys. Rev. 140, B1039 (1965).

[10]Independently of the interpretation of the Dalitz plot, it can be shown that the ratio between the rates $\Gamma[(\bar{p}+p)_{I=1} \to \pi^+ + \pi^- + \pi^0]/\Gamma[\bar{p}+n \to \pi^- + \pi^- + \pi^+]$ must be between $\frac{1}{4}$ and 1. See Ref. 7 and A. Pais, Ann. Phys. (N.Y.) 9, 548 (1960); for a discussion of the comparison of the antiproton reaction frequencies in hydrogen and deuterium, see R. Bizzarri, to be published.

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[12]M. Foster, Ph. Gavillet, G. Labrosse, L. Montanet, R. A. Salmeron, P. Villemoes, J. Zoll, C. Chesquière, and E. Lillestol, in Proceedings of the Heidelberg Conference on Elementary Particles, Heidelberg, Germany, 1967 (to be published).

[13]See, for example, P. E. Schlein, Phys. Rev. Letters 19, 1052 (1967); E. Malamud and P. E. Schlein,

Phys. Rev. Letters 19, 1056 (1967); W. D. Walker, Rev. Mod. Phys. 39, 695 (1967); A. C. Clegg, in Proceedings of the Heidelberg Conference on Elementary Particles, Heidelberg, Germany, 1967 (to be published), where reference to previous analysis can be found.

[14]C. Lovelace, R. H. Heinz, and A. Donnachie, Phys. Letters 22, 332 (1966).

[15]L. Brown and P. Singer, Phys. Rev. 133, B812 (1964).

[16]J. P. Baton, G. Laurens, and J. Reignier, Phys. Letters 25, B419 (1967).

[17]See, for example, I. J. R. Aitchison, Nuovo Cimento 51A, 250, 272 (1967).

[18]M. Month, Phys. Letters 18, 357 (1965); note that the total energy of the 3π system (1836 MeV) is close to the energy (1780 MeV), where the triangular diagram due to the resonance f^0 should have a singularity, with the effect of an enhancement of the population of the Dalitz plot at the intersection of the two f bands.

PHASE CONTOURS OF COLLISION AMPLITUDES*

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Phase contours are curves along which the phase of a collision amplitude is a real constant. In their simplest form they describe the phase as a function of real energy and momentum transfer and provide a convenient summary of experimental results for a scattering process. More generally they can be used to provide consistency tests in the dynamics of strong interactions of elementary particles.

We will give a brief discussion of the following topics: (1) properties of phase contours, (2) phase contours for pion-nucleon scattering below 1.4 GeV, (3) phase contours in a Regge model for pion-nucleon scattering, and (4) the use of phase contours for studying consistency between resonance poles, zeros, and high-energy behavior in a crossing symmetric model.

(1) Properties of phase contours.—The phase $\varphi(s,t)$ of an invariant scattering amplitude $F(s,t)$ is defined by

$$\varphi(s,t)=\mathrm{Im}[\ln F(s,t)], \qquad (1)$$

where Im denotes the imaginary part, and s, t denote the invariant energy and momentum transfer variables. It is also necessary to define the phase at an initial point (s_0,t_0). When the amplitude has zeros or poles on the physical sheet, the phase depends on the route taken from (s_0,t_0) to (s,t), so we must always specify the route.

A phase contour is defined by

$$\varphi(s,t)=C, \qquad (2)$$

where C is a real constant. It is useful to study phase contours both for real s and t, and for complex s and fixed t, etc. Their properties include the following:

(a) Phase contours, for different values of C, do not meet each other except at zeros and poles of the amplitude and at certain other singularities.

(b) The phase change clockwise in the complex plane round a zero is -2π, and round a pole is 2π.

(c) For fixed t, and $s=s_1+is_2$, the phase is a harmonic function of s_1 and s_2, and the phase contours are orthogonal to the modulus contours in this complex s plane.

(d) From the optical theorem,

$$\mathrm{Im}F(s,0)>0, \quad \text{for } s>(m+M)^2. \qquad (3)$$

Assuming an asymptotically constant total cross section and the Pomeranchuk theorem,

$$F(s,t)\sim isB, \quad \text{as } s\to+\infty. \qquad (4)$$

We choose our initial point (s_0,t_0) as the limit along $(s+i0, t=0)$ as $s\to+\infty$, and take

$$\varphi(s\to+\infty,0)=\tfrac{1}{2}\pi. \qquad (5)$$

(e) The phase contours $\varphi(s,t)=0$, or π, cannot enter the region $0\le t<4m^2$, when $s>4m^2$.

(f) If the scattering amplitude has power behavior as $s\to\infty$, the phase is asymptotically constant. In the particular case of a symmetric amplitude, or a Regge term of even signature, we can write

$$F(s,t)\sim\beta(t)s^{\alpha(t)}\exp[i\pi\{1-\tfrac{1}{2}\alpha(t)\}]. \qquad (6)$$