INTERPRETATION OF 2- TO 5-GeV/c $\pi^- p \to \pi^0 n$ POLARIZATION DATA BY USE OF AN INTERFERENCE MODEL*

R. C. Lamb, R. A. Lundy, T. B. Novey, A. Yokosawa, and D. D. Yovanovitch
Argonne National Laboratory, Argonne, Illinois
(Received 27 November 1967)

Recent measurements of the polarization in the charge-exchange reaction $\pi^- p \to \pi^0 n$ at momenta between 2.0 and 5.0 GeV/c have been described by Drobnis et al.[1] It is advantageous to study the polarization at energies where the interference of one Regge-pole exchange in the t channel and resonances in the direct channel may be expected. Having tested the validity of such an interference model[2] near known πN resonances, one may proceed to fit the polarization data at other energies and thus determine resonance parameters which are less well known.

We assume an interference model where f and g (spin-nonflip and spin-flip amplitudes, respectively) are given as

$$f = f_{\text{Regge}} + f_{\text{res}} \tag{1}$$

and

$$g = g_{\text{Regge}} + g_{\text{res}}. \tag{2}$$

The differential cross section and polarization are, respectively, given as

$$d\sigma/dt = (\pi/q^2)[|f|^2 + |g|^2], \tag{3}$$

$$P = 2\,\mathrm{Im}(fg^*)/[|f|^2 + |g|^2]. \tag{4}$$

The amplitudes due to the exchange of the ρ Regge pole are given as[3]

$$f_{\text{Regge}}(s,t) = -(Mm/4\pi s^{1/2})F(s,t)b_1(t), \tag{5}$$

$$g_{\text{Regge}}(s,t) = (m/16\pi)F(s,t) \times [b_1(t) - \alpha(t)b_2(t)]\sin\theta, \tag{6}$$

where

$$F(s,t) = \left(\frac{1-e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}\right)\left(\frac{s}{2Mm}\right)^{\alpha(t)},$$

M is the nucleon mass; m, pion mass; $\alpha(t)$, the ρ trajectory; $b_1(t), b_2(t)$, the residues of the ρ Regge pole; and θ, center-of-mass scattering angle in the s channel. [Equations (5) and (6) are based on an asymptotic expression. We did not use an exact expression for lower energies since additional terms such as background integrals and other poles which may be important at lower energies are not well known. The simple expressions of Eqs. (5) and (6) are found to be adequate to explain the differential cross-section data at an incident momentum as low as 3.0 GeV/c.]

The resonant amplitudes are defined as

$$f_{\text{res}} = (\sqrt{2}/3q)\textstyle\sum_l (-1)^{I+\frac{1}{2}}(J+\tfrac{1}{2})A_l P_l \times (1 + t/2q^2), \tag{7}$$

$$g_{\text{res}} = (\sqrt{2}/3q)\textstyle\sum_l (-1)^{I+\frac{1}{2}}(-1)^{J-l-\frac{1}{2}} A_l P_l{}' \times (1 + t/2q^2)\sin\theta, \tag{8}$$

where A_l is given by the Breit-Wiegner formula

$$A_l = (i+\epsilon)(\Gamma_{\text{el}}/\Gamma)/(1+\epsilon^2)$$

with

$$\epsilon = 2(E_l - E)/\Gamma.$$

Here I is the isospin; J, total spin; q, πN center-of-mass momentum; l, orbital angular momentum; E, total energy; Γ, total width; and Γ_{el}, elastic width.

An analysis[4] of high-energy charge-exchange differential cross sections[5] where resonance effects are unimportant determines the trajectory $\alpha(t)$ and the residue functions $b_1(t)$ and $b_2(t)$. The sign of $b_2(t)$ near $t = 0$ was determined from the $\pi^\pm p$ elastic-scattering polarization data[6] at high energies. The following analysis includes only the case of $b_1(t)$ with no sign change.

Certain general qualitative features are apparent upon examination of this model. The polarization is proportional to $\mathrm{Im}(f \cdot g^*)$. If we limit the discussion to a single dominant resonance, the polarization at this resonant energy is

$$P \sim (\mathrm{Im} f_{\text{res}})(\mathrm{Re}\, g_{\text{Regge}}) - (\mathrm{Im}\, g_{\text{res}})(\mathrm{Re} f_{\text{Regge}}). \tag{9}$$

For small momentum transfers, we know that $\mathrm{Re} f_{\text{Regge}} < 0$ and that $\mathrm{Re}\, g_{\text{Regge}} > 0$.[4] A Regge recurrence scheme predicts that $J=l-\frac{1}{2}$ for the N_γ and $J=l+\frac{1}{2}$ for the Δ trajectory. The following can be seen from Eq. (9):

(a) If a resonance has $T=\frac{1}{2}$, and thus $\mathrm{Im}f_{\mathrm{res}}<0$, then $\mathrm{Im}g_{\mathrm{res}}<0$ provided the resonance is inconsistent with $J=l-\frac{1}{2}$; and thus the polarization is negative.

(b) If a resonance is in $T=\frac{3}{2}$, then $\mathrm{Im}f_{\mathrm{res}}>0$, and $\mathrm{Im}g_{\mathrm{res}}>0$ when the resonance is consistent with $J=l+\frac{1}{2}$; so the polarization is positive. It seems clear that the experimental data[1] giving predominately positive polarization bear out the Regge-trajectory assignments.

The polarization in the reaction $\pi^-p\to\pi^0n$ at momenta of interest to us was calculated considering the resonance listed below:

Momentum (GeV/c)	Resonances
2.07	$N(1683)$, $\Delta(1920)$, $N(2190)$, $\Delta(2390)$
2.50	$\Delta(1920)$, $N(2190)$, $\Delta(2390)$, $N(2650)$
2.72	$N(2190)$, $\Delta(2390)$, $N(2650)$
3.20	$\Delta(2390)$, $N(2650)$
3.47	$\Delta(2390)$, $N(2650)$

The resonance parameters used in the polarization calculations are given in Table I. Most of the parameters are taken from Barger and Cline.[7]

The polarization data at 2.07 GeV/c are in good agreement with the interference-model calculations when $N(2190)$ is assumed to have $J=l-\frac{1}{2}$ assignment, namely a $G_{7/2}$ state, as shown in Fig. 1. The calculation for 2.07 GeV/c was repeated with a $J=l+\frac{1}{2}$, ($G_{9/2}$) assignment for the $N(2190)$, leaving all other parameters unchanged. The result is also shown in Fig. 1, and is clearly inconsistent with the experimental data. In a similar manner we investigate the quantum numbers of the $\Delta(2390)$ and the $N(2650)$. Figure 1 shows the calculated polarization at 2.50 and 2.72 GeV/c for $\Delta(2390)$ assignments of $J=l+\frac{1}{2}$ and $l-\frac{1}{2}$. The experimental data at 2.72 GeV/c are consistent with a $J=l+\frac{1}{2}$ state. The results at 2.50 GeV/c, how-

Table I. Resonance parameters.

I	Mass	Γ	J	l	$\Gamma_{\mathrm{el}}/\Gamma$
$\frac{1}{2}$	1.683	0.105	$\frac{5}{2}$	3	0.80
$\frac{1}{2}$	2.190	0.240	$\frac{7}{2}$	4	0.15
$\frac{1}{2}$	2.650	0.425	11/2	6	0.07
$\frac{3}{2}$	1.929	0.170	$\frac{7}{2}$	3	0.35
$\frac{3}{2}$	2.390	0.275	11/2	5	0.11

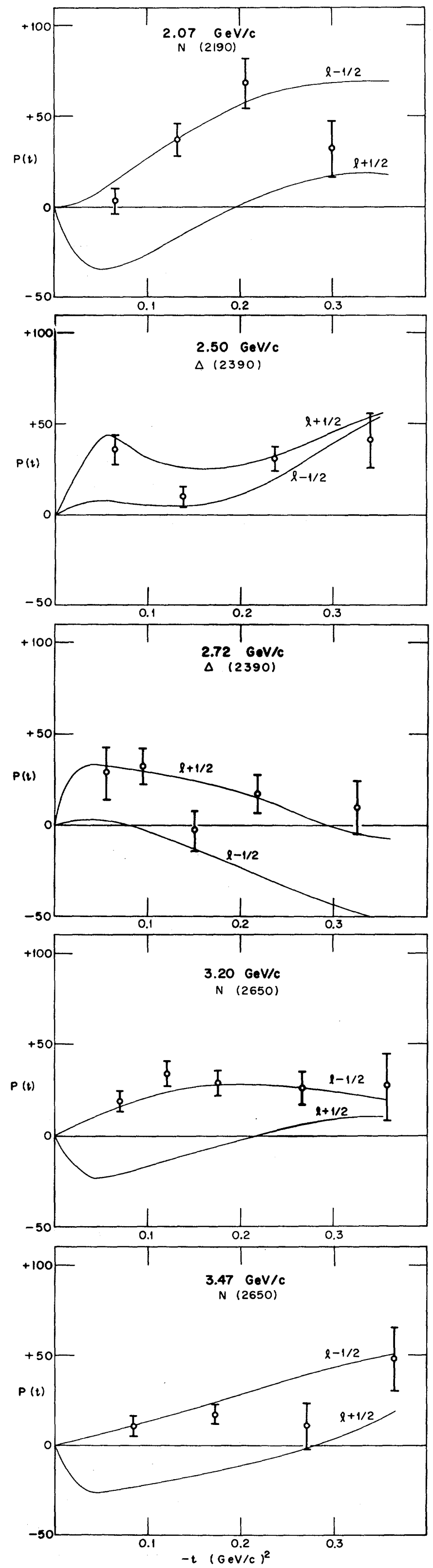

FIG. 1. Polarization in the charge-exchange reaction $\pi^-p\to\pi^0n$.

ever, are less conclusive. Also shown in Fig. 1 are calculations for 3.20 and 3.47 GeV/c with $N(2650)$ assignments of $J=l-\frac{1}{2}$ and $J=l+\frac{1}{2}$. From these data it is apparent that assignments of $J=l+\frac{1}{2}$ for the $\Delta(2390)$ and $J=l-\frac{1}{2}$ for the $N(2650)$ give the best fit to the experimental points.

We conclude, therefore, that the experimental data on the polarization in the reaction[1] $\pi^- p \rightarrow \pi^0 n$ can be successfully interpreted according to the above interference model.

We note here that if we assume the values of $b_1(t)$ with a sign change at around $t\sim 0.2$, the negative polarization is expected at higher momenta transfer.[8,9]

Some objection may be raised against the interference model due to the possibility of double counting.[10] This problem, while it may be significant in the calculation of differential cross sections, has been ignored by us.

The available data on high-energy charge-exchange polarizations[1,11] indicate that no substantial amount of polarization (probably less than 15%) is produced by mechanisms involving cuts, absorption, or other nonresonant effects. If such mechanisms were active, we would assume that their contribution at lower energies would not substantially effect our results.

We wish to acknowledge consultations with Dr. R. C. Arnold on the Regge-pole model, the assistance of R. George with computer programming, and the assistance of R. Miller in checking the numerical calculations.

---

*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]D. D. Drobnis, J. Lales, R. C. Lamb, R. A. Lundy, A. Moretti, R. C. Niemann, T. B. Novey, J. Simanton, A. Yokosawa, and D. D. Yovanovitch, to be published.

[2]R. J. N. Phillips, Nuovo Cimento 45A, 245 (1966); R. K. Logan and L. Sertorio, Phys. Rev. Letters 17, 834 (1966).

[3]S. C. Frautschi, M. Gell-Mann, and F. Zachariasen, Phys. Rev. 126, 2204 (1962).

[4]A. Yokosawa, Phys. Rev. 159, 1431 (1967).

[5]A. V. Stirling, P. Sonderegger, J. Kirz, P. Falk-Vairant, O. Guisan, C. Bruneton, and P. Borgeaud, Phys. Rev. Letters 14, 763 (1965); P. Sonderegger, J. Kirz, O. Guisan, P. Falk-Vairant, C. Bruneton, P. Borgeaud, A. V. Stirling, C. Caverzasio, J. P. Guilland, M. Yvert, and B. Amblard, Phys. Letters 20, 75 (1966).

[6]M. Borghini, G. Coignet, L. Dick, K. Kuroda, L. di-Lella, A. Michalowicz, P. C. Macq, and J. C. Oliver, Phys. Letters 24B, 77 (1967).

[7]V. Barger and D. Cline, Phys. Rev. Letters 16, 913 (1966).

[8]A. Yokosawa, in Proceedings of the International Conference on Polarized Targets and Ion Sources, Saclay, 1966 (La Documentation Francais, Paris, France, 1967), p. 127.

[9]D. Reeder and K. Sarma, private communication.

[10]R. Dolen, D. Horn, and C. Schmid, to be published; A. V. Stirling and C. B. Chiu, to be published; L. Durand, III, to be published.

[11]P. Bonamy et al., Phys. Letters 23, 501 (1966).