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[6]See Ref. 3; also W. Shockley, Science 158, 535 (1967), on $\vec{J}\times\vec{B}$ vs $\vec{J}\times\vec{H}$ in the Lorentz force.

[7]This experiment is in effect a proper formulation of, rather than an answer to, the "choice between the two magnetic dipole models," i.e., Ampérian versus magnetic charge, formulated by B. D. H. Tellegen [Am. J. Phys. 30, 650 (1962)]. Shockley and James (Ref. 2) and Penfield and Haus (Ref. 3) both predict that the magnetic-charge model will give the correct answer because changes in $\vec{G}_l$ [required by Shockley and James in (4) but neglected in Tellegen's formula for the Ampérian model] cause the Ampérian model to predict the same forces as the magnetic-charge model.

[8]A generalization of Eq. (18) for finite ρ_0 and its consideration from a viewpoint not using hidden momentum is Eq. (17) of E. I. Blount, Phys. Rev. 128, 2454 (1962).

[9]See W. Shockley and W. A. Gong, Mechanics (Charles E. Merrill Books, Inc., Columbus, Ohio, 1966). My attack on the unfamiliar Dirac electron problem analyzed in this correspondence was deliberately pursued by using the search-thinking tools in Mechanics, specifically the try-simplest-cases, conceptual-experiment, and idealized-limiting-cases tools. Detours through such complexities as the Hamilton-Jacobi equation, group velocity, and reduction of the separated z equation for Ψ of the form $U(\zeta)\Phi(\tau)$ to $u''-(u'/\zeta)+(f\zeta+2)f\zeta u=0$ were aspects of acquiring familiarity during the ten days of action with pencil and paper prior to the payoff hunch that a stationary electron was obviously the simplest and, because of relativistic invariance, an adequate case.

PION-PION SCATTERING, CURRENT ALGEBRA, UNITARITY, AND THE WIDTH OF THE RHO MESON

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The techniques of current algebra have been employed by Weinberg[1] to obtain an expression[2] for the pion-pion scattering amplitude in the low-energy limit. In this note we show that by making use of the condition of elastic unitarity[3] in a particularly simple fashion, we may extend this low-energy result through a useful range of physical energies. We predict S-wave phase shifts that vary roughly linearly with energy up to the region of the K-meson mass, at which point the isospin-0 phase shift is $+19°$ while the isospin-2 phase is $-12°$. The isospin-1, P-wave scattering length as determined by the low-energy current-algebra result, coupled with the knowledge of the mass of the ρ meson, gives a value for the width of the ρ decay into two pions that agrees well with experiment. If, in addition, we postulate that the ρ-meson dominates[4] the electromagnetic form factor of the pion, we obtain the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation[5] $2F_\pi^2 f_\rho^2 = m_\rho^2$ which cannot be proven[6] by current-algebra methods without additional postulates.

The current-algebra result for the pion-pion scattering amplitude reads

$$T_{a'b',ab}^{(CA)}(s,t,u) = F_\pi^{-2}[\delta_{a'b'}\delta_{ab}(s-m_\pi^2) + \delta_{a'a}\delta_{b'b}(t-m_\pi^2) + \delta_{a'b}\delta_{b'a}(u-m_\pi^2)], \quad (1)$$

where ab and $a'b'$ are the isospin indices of the initial and final pions, s is the usual square of the total center-of-mass energy with t and u the corresponding variables of the crossed channels, and F_π is the pion-decay constant with the experimental value of 94 MeV. The pion scattering amplitude may be decomposed into partial waves of definite angular momentum and isospin by writing

$$T_{a'b',ab}(s,t,u)=16\pi\sum_{lT}t_{lT}(s)(2l+1)\times P_l(\cos\theta)P_{a'b',ab}^{(T)},\qquad(2)$$

in which the $P^{(T)}$ are a set of isospin projection matrices. The current-algebra approximation gives, in this decomposition,

$$t_{00}^{(CA)}(s)=+(16\pi F_\pi^{\,2})^{-1}[7m_\pi^{\,2}+8k^2],$$
$$t_{02}^{(CA)}(s)=-(16\pi F_\pi^{\,2})^{-1}[2m_\pi^{\,2}+4k^2],$$
$$t_{11}^{(CA)}(s)=+(12\pi F_\pi^{\,2})^{-1}k^2,\qquad(3)$$

with

$$k^2=\tfrac{1}{4}(s-4m_\pi^{\,2}).\qquad(4)$$

The unitarity condition requires that in the elastic region, $4m_\pi^2<s<16m_\pi^2$,

$$\mathrm{Im}\,t_{lT}(s)=(k/\sqrt{s})\,|t_{lT}(s)|^2,\qquad(5)$$

which implies the familiar partial wave formula

$$t_{lT}=(\sqrt{s}/k)e^{i\delta_{lT}}\sin\delta_{lT},\qquad(6)$$

with the phases δ_{lT} real in this region; it also implies that

$$\mathrm{Im}[t_{lT}(s)]^{-1}=-k/\sqrt{s}.\qquad(7)$$

This leads us to define a real analytic function $h(s)$ with the imaginary part $-k/\sqrt{s}$ for all energies greater than the elastic threshold $s>4m_\pi^2$, but with no other singularities:

$$h(s)=-ik/\sqrt{s}+h_1(s),\qquad(8)$$

where $h_1(s)$ is chosen to cancel the spurious $\sqrt{s}$ singularity and introduce no others:

$$h_1(s)=\frac{2}{\pi}\frac{k}{\sqrt{s}}\ln\left[\frac{\sqrt{s}+2k}{2m_\pi}\right].\qquad(9)$$

We may now write

$$[t_{lT}(s)]^{-1}=h(s)+g_{lT}(s),\qquad(10)$$

with $g_{lT}(s)$ a meromorphic function save for an inelastic cut along the positive real axis, $s>16m_\pi^2$, and a cross channel cut in the negative real axis, $s<0$. The partial wave formula now reads

$$(k/\sqrt{s})\cot\delta_{lT}=h_1(s)+g_{lT}(s),\qquad(11)$$

which is a generalized effective-range expansion.[7]

One might be tempted to write $g_{0T}(s)$ as a finite polynomial in k^2 and use the resulting effective-range approximation to extrapolate the low-energy current-algebra results (3) into the physical region. This procedure, however, cannot be applied to S-wave amplitudes $t_{00}^{(CA)}(s)$ and $t_{02}^{(CA)}(s)$, for they both vanish in the gap $0<s<4m_\pi^2$ $(-m_\pi^2<k^2<0)$ below the elastic threshold. Thus $g_{0T}(s)$ has a pole in this gap, and it should not be expanded in a power series in k^2. Accordingly, we shall instead write the inverse function $[g_{0T}(s)]^{-1}$ as a first-order polynomial in k^2. We determine the coefficients of this expansion by fitting it to the current-algebra amplitude at the point where the latter vanishes, and thus obtain the simple form of the unitarily corrected amplitude,

$$t_{0T}(s)=\frac{t_{0T}^{(CA)}(s)}{1+h(s)t_{0T}^{(CA)}(s)}.\qquad(12)$$

Within the gap region, $0<s<4m_\pi^2$, the denominator in this expression differs from unity by only a few percent; so our result is consistent with the smoothness assumptions that are implicit in the idea of partial conservation of axial-vector currents (PCAC), and it is not sensitive to the position of the point where $[g_{0T}(s)]^{-1}$ is fitted to $t_{0T}^{(CA)}(s)$. The phase shifts that follow from Eq. (12) are shown in Fig. 1. They

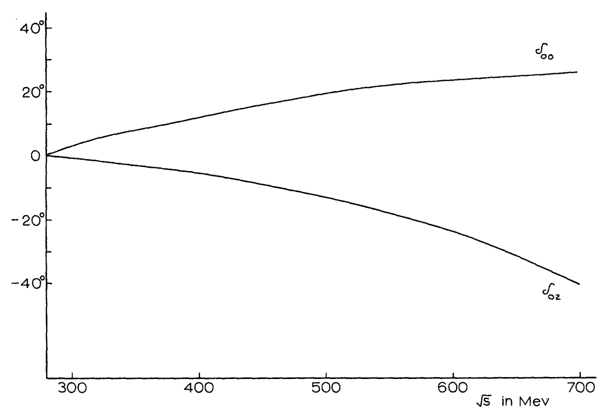

FIG. 1. S-wave pion-pion phase shifts for isospin 0, δ_{00}, and isospin 2, δ_{02}, as a function of the total center-of-mass energy that is obtained from Eq. (12).

agree in sign, but can differ by a factor of 2 or so from the phase shifts obtained in analyses[8] of pion-production data. In view of the difficulties involved in the experimental analysis, we feel that this is satisfactory. Our value for the phase difference that enters in the phenomenological description of the two-pion decay mode of the long-lived K meson, $\delta_{00}-\delta_{02}=+31°$, is entirely different from a value $-57°$ obtained recently.[9]

An effective-range approximation should be valid for the isospin-1, P-wave scattering amplitude, since in this channel the ρ-meson resonance appears, and $ks^{-1/2}\cot\delta_{11}$ vanishes at this resonance. The correct threshold behavior is obtained if we write this expansion as

$$g_{11}(s)=a_{11}/k^2+\tfrac{1}{2}r_{11}. \tag{13}$$

We may use the current-algebra scattering amplitude (3) at threshold to identify the scattering length as

$$a_{11}=12\pi F_\pi^{\,2}, \tag{14}$$

but it gives no information about the effective range r_{11}. This quantity can be determined, however, if we require that the amplitude has a resonance at the mass of the ρ meson. If we neglect the small variation of the function $h_1(s)$ in the resonance region, this procedure gives

$$e^{i\delta_{11}}\sin\delta_{11} =\frac{m_\rho\Gamma_\rho(k/k_\rho)^3(m_\rho/\sqrt{s})}{m_\rho^{\,2}-s-im_\rho\Gamma_\rho(k/k_\rho)^3(m_\rho/\sqrt{s})}, \tag{15}$$

in which

$$\Gamma_\rho=(3\pi F_\pi^{\,2}m_\rho^{\,2})^{-1}k_\rho^{\,5}, \tag{16}$$

and

$$k_\rho^{\,2}=k(m_\rho^{\,2})^2=\tfrac{1}{4}(m_\rho^{\,2}-4m_\pi^{\,2}). \tag{17}$$

If we take $m_\rho=750$ MeV, Eq. (16) yields $\Gamma_\rho=107$ MeV. The major effect of the variation of the function $h_1(s)$ is to increase this value by 8%, but it does not otherwise alter the functional form of Eq. (15). The momentum-dependent terms in Eq. (15) skew the resonance shape and shift the position of the peak to higher mass values. These corrections are included in the P-wave phase shift shown in Fig. 2, which gives[10] $\Gamma_\rho\simeq 130$ MeV for $m_\rho=775$ MeV. Our result was obtained by matching the current-algebra amplitude at threshold. If the effective-range approximation is constrained by the current-algebra amplitude at some other point in the gap $0<s<4m_\pi^{\,2}$, then the predicted ρ width will change less than 10%, which is again consistent with the PCAC smoothness assumption.

By comparing the width (16) with the decay-rate formula[11]

$$\Gamma_\rho=\frac{2}{3}\frac{g_{\rho\pi\pi}^{\,2}}{4\pi}\frac{k_\rho^{\,3}}{m_\rho^{\,2}}, \tag{18}$$

we find that the $\rho\pi\pi$ coupling constant is related to the pion-decay constant by

$$g_{\rho\pi\pi}^{\,2}=\tfrac{1}{2}(m_\rho^{\,2}/F_\pi^{\,2})(2k_\rho/m_\rho)^2. \tag{19}$$

If we assume ρ dominance,[4] $g_{\rho\pi\pi}=f_\rho$, where f_ρ^{-1} measures the strength of the ρ-photon transition, and neglect the mass of the pion relative to that of the ρ, we obtain the KSRF relation[5]

$$f_\rho^{\,2}=\tfrac{1}{2}(m_\rho^{\,2}/F_\pi^{\,2}). \tag{20}$$

We find it remarkable that the low-energy current-algebra amplitude can be simply extrapolated some 800 MeV, past the four-pion inelastic threshold, and give a good value of the ρ width. The lesson to be learned, evidently, is that the satisfaction of low-energy current-algebra constraints with a minimum momentum dependence consistent with general requirements such as unitarity and correct analytic structure can provide satisfactory results over a large energy interval.

We have enjoyed conversations with L. J. Clavelli, S. W. MacDowell, H. J. Schnitzer, R. H. Socolow, and C. M. Sommerfield.

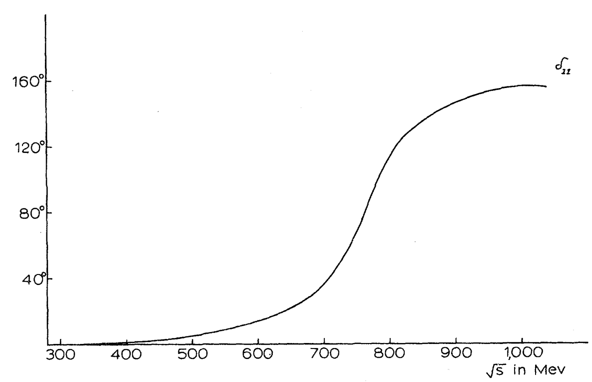

FIG. 2. P-wave pion-pion shift as a function of the total center-of-mass energy.

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SEARCH FOR A CHARGED MESON IN THE MASS REGION OF 960 MeV†

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Evidence has been reported in the past few years for a neutral and a charged meson state in the 960-MeV region. The existence of a neutral meson state (η') with $M=958\pm1$ MeV and $\Gamma<4$ MeV was established by several hydrogen-bubble-chamber experiments.[1] In the reaction $K^-p\rightarrow\Delta\eta'$, the η' was found to decay strongly into $\pi\pi\eta$ and electromagnetically into $\pi\pi\gamma$, from which decays its quantum numbers were determined to be $IJ^{PG}=00^{-+}$. Kienzle et al.[2] have reported evidence for a charged meson (δ) with a mass of 963 ± 5 MeV and $\Gamma<4$ MeV in the reaction $\pi^-p\rightarrow(MM)^-$, with δ^- decaying into one or three charged particles and possible neutrals.[3] Allison et al. report evidence for the reaction $K^-p\rightarrow\Sigma^\pm\pi^c\delta^\mp$ with $\delta^\mp\rightarrow\pi^\mp\pi^+\pi^-$ at 6.0-GeV/c incident momentum.[4] The coincidence of the mass and width of the δ and η' leads to two possible interpretations: (a) δ and η' are the same particle; in this case not only the isospin, but also the other quantum numbers of the η' are questioned. (b) δ and η' are two different particles; this interpretation has led to theoretical discussions on mass degeneracy,[5] to a possible assignment of the δ to a $^3P_0$ configuration in the quark-antiquark model for meson states,[6] and to the conjecture that δ might be the 0^{++} daughter trajectory of the ρ meson.[7] We report here an experiment which rules out possibility (a) and sets some upper limits on the cross section for production of the δ in $K^-n\rightarrow\Lambda\delta^-$ with subsequent decay of the δ into various states.

In this experiment the 72-in. bubble chamber, filled with deuterium, was exposed to a K^- beam at momenta 2.11 and 2.65 GeV/c. The reaction studied was

$$K^-d\rightarrow\Lambda X^-(p_s), \tag{1}$$

where X^- is a negative meson decaying into