ed-wave Born approximation for the inelastic scattering. To say any more would require the use of more sophisticated nuclear models then those used here. We want to thank H. Palevsky and J. Friedes for furnishing us with a preprint of their work before publication, as also did C. Wilkin and W. Czyż.

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CORRELATIONS IN PARTIAL WIDTHS OF NEUTRON-INDUCED REACTIONS*

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The nature of the spectrum of radiative transitions following slow-neutron capture has formed a subject of abiding interest in low-energy nuclear physics. The presence of anomalously strong high-energy γ rays in thermal capture has suggested to many workers the possibility of single-particle effects in the capture mechanism.[1,2] A powerful tool in the elucidation of this question is provided in the measurement of capture spectra in individual resonances in the epithermal neutron energy region.

As Lane and Lynn[2] have shown, anomalously strong $E1$ transitions to final states having a largely single-particle character are to be expected both in resonances ("internal and channel resonant capture") and between resonances ("direct or potential capture"). We have previously offered evidence for the presence of the latter, the direct-capture effect.[3,4] The purpose of this paper is to present evidence for the former effect.

Following the notation of Ref. 2, the resonance contribution to the collision matrix element for the (n,γ) reaction is as follows:

$$U(\mathrm{res}) = -ie^{-i\varphi c'}\sum_\lambda(\Gamma_{\lambda c'})^{\frac{1}{2}} \times(\Gamma_{\lambda\gamma f}{}^{\frac{1}{2}}+\delta\Gamma_{\lambda\gamma f}{}^{\frac{1}{2}})/(E_\lambda-E-\tfrac{1}{2}i\Gamma_\lambda). \quad (1)$$

The $\delta\Gamma_{\lambda\gamma f}{}^{1/2}$ in the above expression arises from the evaluation of the dipole integral in the external or "channel" region, $r>R$. The size of $\delta\Gamma_{\lambda\gamma f}{}^{1/2}$ is shown to be dependent on the reduced neutron widths of the initial and final states:

$$\delta\Gamma_{\lambda\gamma f}{}^{1/2}\approx\left(\frac{16\pi}{9}\right)^{1/2}\left(\frac{E_\gamma}{\hbar c}\right)^{3/2}\left(\frac{2}{R}\right)\theta_{fcf}\theta_{\lambda c'} \times\left(\frac{3}{4\pi}\right)^{1/2}\left(\frac{\bar{e}R}{k_f}\right), \quad (2)$$

where the θ's represent the dimensionless reduced-width amplitudes. (In Ref. 2 it is pointed out that the "internal" term $\Gamma_{\lambda\gamma f}{}^{1/2}$ may also contain a contribution dependent on the reduced neutron widths. It is not experimentally possible to distinguish these components.)

A consequence of the Lane and Lynn analysis is that for $\theta_{fcf}\neq 0$, a nonzero, positive correlation exists between partial radiative widths and reduced neutron widths. We assume that for those final states excited in the (d,p) reaction, $\theta_{fcf}\neq 0$ and the above correlation should be present.

The point of departure for our analysis is to assume that the initial-state (compound-nucleus) wave function is the sum of a single-particle-plus-core term and a complicated residual term corresponding to a hierarchy of more complex excitations. We then write the following expression for the radiative-width amplitude for an electric dipole transition from initial state i to final state j:

$$\Gamma_{\gamma ij}^{1/2} = A(\Gamma_{ni}^{0})^{1/2} + \Gamma_{cij}^{1/2}. \quad (3)$$

The first term results from the single-particle component of the initial state. The following correlation coefficients may be defined:

$$\rho_j \equiv \mathrm{corr}(\Gamma_{\gamma ij}^{1/2}, \Gamma_{ni}^{0\,1/2}), \quad (4)$$

$$\tau_{jj'} \equiv \mathrm{corr}(\Gamma_{cij}^{1/2}, \Gamma_{cij'}^{1/2}). \quad (5)$$

We make the plausible assumption that

$$\mathrm{corr}(\Gamma_{cij}^{1/2}, \Gamma_{ni}^{0\,1/2}) = 0, \quad (6)$$

and then the following relations will obtain:

$$\rho_j^2 = A_j^2 \langle \Gamma_{ni}^{0} \rangle_i / \langle \Gamma_{\gamma ij} \rangle_i, \quad (7)$$

$$\mathrm{corr}(\Gamma_{\gamma ij}^{1/2}, \Gamma_{\gamma ij'}^{1/2}) = \rho_j \rho_{j'} + \tau_{jj'}(1-\rho_j^2)^{1/2}(1-\rho_{j'}^2)^{1/2}. \quad (8)$$

It has been established that radiative-width and reduced-neutron-width amplitudes are Gaussian, and consequently the widths follow a Porter-Thomas or chi-squared distribution with one degree of freedom.[5] We must consider the problem of dealing with correlated chi-squared distributions. Experimentally, only the widths are measured and the sample correlation coefficients R and T are determined:

$$R \equiv \langle \mathrm{corr}(\Gamma_{\gamma ij}, \Gamma_{ni}^{0}) \rangle_j,$$

$$T \equiv \langle \mathrm{corr}(\Gamma_{\gamma ij}, \Gamma_{\gamma ij'}) \rangle_{j,j',j\neq j'}, \quad (9)$$

where R is averaged over the N final states j, and T is averaged over all $\frac{1}{2}N(N-1)$ pairs of final states.

The connection between these experimentally determined average sample coefficients R and T and the ρ and τ parameters is established by calculating the distribution of R and T from correlated distributions of width amplitudes with assumed values of ρ and τ.

The experiment was carried out with the fast-chopper, neutron time-of-flight facility[6] at the Brookhaven High Flux Beam Reactor with 192 g of Tm_2O_3. Thulium is monoisotopic ($A = 169$), and the low-lying states of ${}^{170}_{101}Tm_{69}$ are well known from comprehensive (d,p) and thermal (n,γ) experiments.[7] Capture events were observed as a function of neutron time-of-flight over a period of 80.9 h. A flight path of 21.7 m with a time resolution of 0.29 μsec/m was employed. The γ rays were detected in a 10 cm^3 Ge(Li) detector with an over-all resolution of 0.17% at 7 MeV. Events were recorded on magnetic tape, which was later scanned to sort out the γ rays corresponding to the various resonances of ${}^{169}Tm(n,\gamma){}^{170}Tm$. Relative transition strengths were obtained by the usual procedure of calculating the ratio of events falling in the detector two-escape peak to the total events recorded above 1.56 MeV. Absolute line intensities were obtained by comparing the intensities observed in the present experiment at thermal neutron energies with the absolute intensities determined in Ref. 7.

Table I presents the results of the thulium experiment including eight resonances with $J^\pi = 1^+$ and 15 final states is ${}^{170}Tm$ below 1.080-MeV excitation energy. These final states were also observed in the (d,p) experiment of Sheline et al.[7]

Several resonances below 83 eV have been omitted from the analysis for the following reasons: (a) The resonance spin and parity are known to be 0^+, (b) the spin is not known, or (c) the resonance is not well resolved in our experiment. The omitted resonances are at 14.4, 54.0, 59.2, 63.0, and 65.8 eV.[8] All other known resonances below 83.4 eV have been included. We have no reason to believe that exclusion of the above resonances introduces a statistical bias in the results. From the 120 entries of Table I we have calculated the R and T correlation coefficients. The results are the following:

$$R = +0.274, \quad T = +0.088.$$

A Monte Carlo calculation is employed to generate values of R and T for comparison to experiment in the following manner. For each resonance, $i = 1 \cdots M$, a set of numbers picked from a univariate normal distribution of zero mean and unit variance is chosen to form a vector $\vec{Z} = (z_{i0} \cdots z_{iN})$. A linear transformation $\vec{Y} = A\vec{Z}$ creates a vector $\vec{Y}$ which follows a multivariate normal distribution whose elements y_{i0} and y_{ij}, $j = 1 \cdots N$, represent reduced-

Table I. Photon intensities observed in resonances of $^{169}Tm(n,\gamma)^{170}Tm$. The neutron reduced widths are also shown at the bottom of the table.

$^{169}Tm(n,\gamma)^{170}Tm$	I_γ (photons per 10^3 neutrons captured)							
E_γ (MeV) \ E_n (eV)	3.9	17.5	29	34.8	38	44	50.7	83
6.594	1.6 ± 0.2	1.2 ± 0.3	2.8 ± 0.32	0.6 ± 0.28	1.3 ± 0.76	1.2 ± 0.48	1.2 ± 0.44	0.17 ± 0.39
6.556	7.2 ± 0.24	0.64 ± 0.22	2.1 ± 0.68	0.13 ± 0.234	1.3 ± 0.76	1.1 ± 0.48	-0.081 ± 0.32	1.3 ± 0.44
6.445	1.8 ± 0.16	0.46 ± 0.24	0.30 ± 0.59	0.72 ± 0.32	2.9 ± 0.96	0.80 ± 0.45	0.44 ± 0.44	1.38 ± 0.50
6.389	2.2 ± 0.2	0.29 ± 0.24	1.2 ± 0.68	1.8 ± 0.38	1.03 ± 0.82	1.7 ± 0.6	0.52 ± 0.40	0.39 ± 0.49
6.375	0.96 ± 0.09	0.22 ± 0.24	0.59 ± 0.59	1.3 ± 0.33	3.7 ± 1.0	0.50 ± 0.45	0.56 ± 0.44	0.17 ± 0.44
6.356	2.5 ± 0.17	0.68 ± 0.26	0.67 ± 0.59	0.92 ± 0.32	2.1 ± 0.92	3.8 ± 0.68	1.09 ± 0.44	2.0 ± 1.8
6.003	1.6 ± .17	1.6 ± 0.33	0.15 ± 0.74	0.39 ± 0.36	1.0 ± 0.9	1.0 ± 0.64	0.77 ± 0.48	1.6 ± 0.56
5.945	5.5 ± 0.22	0.92 ± 0.32	2.6 ± 0.88	0.86 ± 0.39	4.2 ± 1.1	-0.10 ± 0.50	1.5 ± 0.56	1.5 ± 0.64
5.911	2.0 ± 0.3	0.20 ± 0.31	1.5 ± 0.8	0.49 ± 0.42	0.62 ± 0.92	0.25 ± 0.55	0.44 ± 0.52	-0.22 ± 0.61
5.900	0.52 ± 0.17	0.31 ± 0.31	0.075 ± 0.74	2.9 ± 0.48	-0.10 ± 0.82	0.45 ± 0.55	-0.52 ± 0.48	0.0 ± 0.55
5.809	-0.16 ± 0.16	3.2 ± 0.4	-0.44 ± 0.74	0.078 ± 0.39	-0.41 ± 0.82	0.60 ± 0.55	0.68 ± 0.56	0.89 ± 0.66
5.736	2.7 ± .24	0.58 ± 0.33	0.9 ± 1.6	0.64 ± 0.44	0.8 ± 1.6	1.0 ± 0.68	0.32 ± 0.57	1.1 ± 0.64
5.730	1.3 ± 0.12	3.5 ± 0.46	+0.74 ± 0.82	0.29 ± 0.42	0.41 ± 0.92	0.94 ± 0.60	3.3 ± 0.64	0.22 ± 0.66
5.684	0.040 ± 0.18	-0.020 ± 0.31	-0.74 ± 0.74	1.6 ± 0.44	1.13 ± 1.03	3.0 ± 0.76	3.1 ± 0.68	0.22 ± 0.66
5.518	1.7 ± 0.2	0.92 ± 1.2	0.59 ± 0.81	0.72 ± 0.48	0.72 ± 1.13	0.70 ± 0.65	0.48 ± 0.64	0.96 ± 0.72
$\Gamma_n^{\,o}$ (mV)	3.5 ± 0.3	0.61 ± 0.10	0.036 ± 0.006	1.2 ± 0.05	.077 ± 0.0067	0.51 ± 0.03	0.79 ± 0.10	0.82 ± 0.04

neutron-width amplitudes and radiative-width amplitudes, respectively. The elements of A, governed by the relation between ρ and τ implicit in Eq. (8), are functions of ρ and τ. To include experimental errors σ_{ij}, the widths are formed as $y_{ij}^2 + (\sigma_{ij}/\langle\Gamma_{ij}\rangle_i)\chi_{ij}$, where χ_{ij} is a normally distributed random variable ($j = 0 \cdots N$).

R and T distributions are calculated by iteration of the above procedure until a frequency histogram of adequate accuracy is formed. Some of the results of such analyses are shown in Fig. 1. The percentile positions of the experimental values in the integral probability distributions of R and T, determined for 1000 iterations, are plotted as functions of the av-

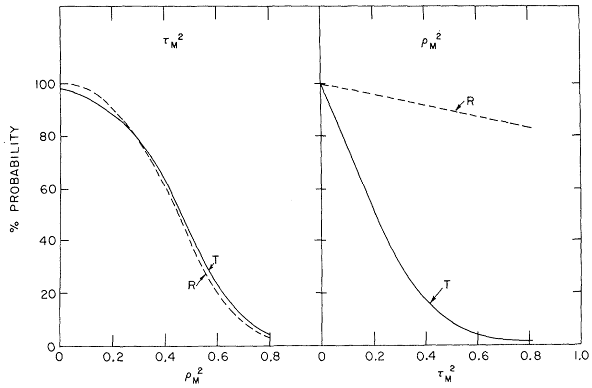

FIG. 1. The percentile positions of the average sample correlation coefficients $R = 0.274$ and $T = 0.088$ as a function of the mean theoretical parameter $\rho_M{}^2$ for $\tau_M{}^2 = 0$, and as a function of $\tau_M{}^2$ for $\rho_M{}^2 = 0$.

erage correlations $\rho_M{}^2$ and $\tau_M{}^2$ (averaged over final states). Figure 1 shows that the experimental values $R = +0.274$ and $T = +0.088$ fall at the 99.9 and 98.1 percentiles, respectively, for the values $\rho_j = \tau_{jj'} = 0$.[9] It is demonstrated, therefore, that a significant positive correlation exists.

Analysis of Fig. 1 and further calculations of the type outlined above enable us to draw several additional conclusions: (a) The value of $\rho_M{}^2$ lies in the range $0.2 \leq \rho_M{}^2 \leq 0.7$ for 10 to 90% confidence limits. (b) The value of τ^2 is less well determined by the available data, but it is probably near zero.

To summarize, we have domonstrated a significant positive correlation between reduced neutron and radiative widths in $^{169}\mathrm{Tm}(n, \gamma)^{170}\mathrm{Tm}$, verifying the suggestions of Lane and Lynn. It seems probable that the small but significant positive correlation between the radiative widths themselves is a result of the same process. The positive sign of the correlations is consistent with the multivariate normal distribution of amplitudes proposed by Krieger and Porter (Ref. 5).

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"HIDDEN LINEAR MOMENTUM" RELATED TO THE $\vec{\alpha}\cdot\vec{E}$ TERM FOR A DIRAC-ELECTRON WAVE PACKET IN AN ELECTRIC FIELD*

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The need for re-examination of linear momentum associated with electromagnetic fields in matter has been emphasized in several recent publications.[1-3] In particular, Costa de Beauregard[1] has pointed out a failure of the action-equals-reaction principle in the interaction between a current loop and an electric charge. He concludes that a force must act on the current loop when the moment of the loop changes in the presence of an electric field:

$$\vec{F} = \vec{E} \times \dot{\vec{M}}_1 / c, \tag{1}$$

where $\vec{M}_1 = I\vec{A}/c$ is the magnetic moment of current I flowing on the boundary of area $\vec{A}$. Costa de Beauregard does not explicitly calculate momentum in the electromagnetic fields. From considerations of electromagnetic momentum Shockley and James[2] independently concluded the necessity for the force $\vec{F}$ in the case in which the field $\vec{E}$ is undisturbed by I. (When the current loop is a conductor that does disturb $\vec{E}$ by acquiring an induced electric dipole, $\vec{F}$ is produced by the conventional $\vec{E}$ in $\nabla \times \vec{E} = -\dot{\vec{H}}/c$ acting on the surface charges on the conductor.)