ION-WAVE ECHOES*

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A method of directly observing the reversible nature of collisionless damping of electron plasma waves has recently been reported.[1] This method involves the occurrence of plasma-wave echoes. When an electron plasma wave is excited in a plasma, the phase mixing of the individual particles and particle-wave interactions cause the wave to damp. A second wave applied to the plasma at a different time or at a different location is similarly attenuated, but the individual particles retain the phase information imparted by the two waves. These particles will phase "unmix" causing the appearance of a third wave, "the echo." These electron plasma-wave echoes have been observed experimentally.[2] In this paper we wish to report the experimental observation[3] of ion-wave echoes which also arise from the reversible nature of collisionless damping. From a theoretical standpoint, ion-wave echoes are more complicated because the motion of both electrons and ions must be considered. However, from an experimental standpoint, there is an advantage in working with ion-acoustic waves whose damping and phase velocity are relatively independent of plasma density in the collisionless regime. At a given density ion-wave echoes can be observed over a large range of frequencies (over three octaves) and the perturbing effect on collisions on the ion-wave echo can be more easily interpreted. In contrast, the phase velocity and damping length of an electron plasma wave are sensitively dependent on the density.

A physical picture[4] of echo generation which parallels the simplified calculation of Gould, O'Neil, and Malmberg[1] can be made by considering noninteracting particles and idealized grids which act as gates that either let particles go through or absorb them completely. In Fig. 1(a) the first grid located at $x=0$ allows short bursts of particles to pass through at intervals of τ. The lowest harmonic of this modulation is $\omega_1=2\pi/\tau$. The velocity distribution of the particles then evolves as

$$f(v)=f_1(v)\cos(\omega_1 t-\omega_1 x/v).$$

The density perturbation is obtained by integrating $f(v)$ over velocity. The spread in velocity of the particles causes the density perturbation to phase mix to a uniform value for large x. At a position $x=l$ where sufficient phase mixing has taken place, a second grid modulates the stream of particles at a higher frequency ω_2. In Fig. 1(a) we have chosen for convenience $\omega_2=2\omega_1$. The resulting stream essen-

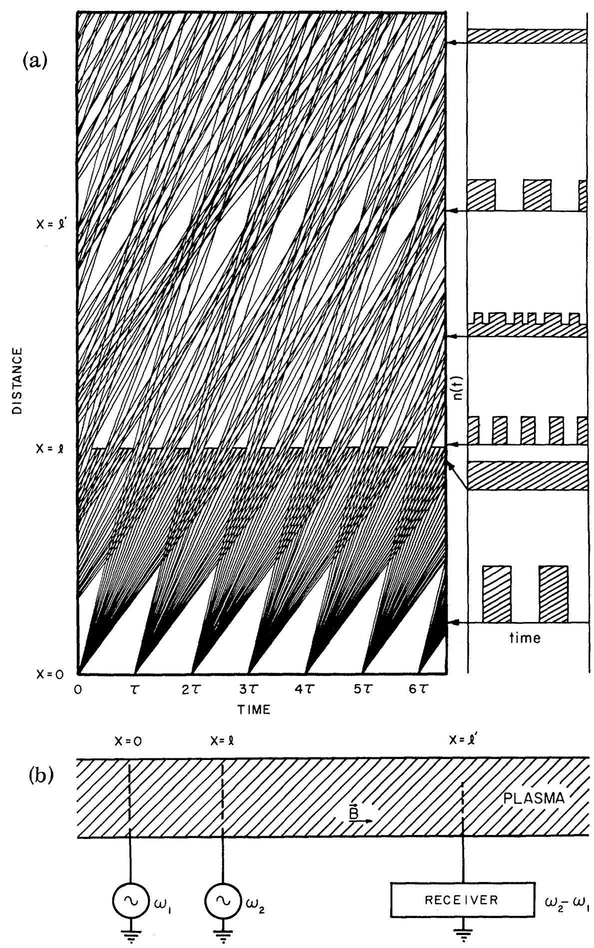

FIG. 1. (a) A simplified picture of echo generation by considering noninteracting particles and idealized grids. At $x=0$ particles are gated at intervals of τ or at a basic frequency of $\omega_1=2\pi/\tau$. At $x=l$ particles are modulated similarly at frequency $\omega_2=2\omega_1$. The echo appears at a position $x=l'=[2\omega_1/(2\omega_1-\omega_1)]l=2l$ with a frequency of $\omega_3=\omega_2-\omega_1$. On the right-hand side the density $n(x,t)$ is plotted as a function of time for various values of x. (b) Schematic of the experimental arrangement. The range of experimental parameters is $1.5<l<12$ cm; $35<\omega/2\pi<215$ kHz; $5\times10^9<N<2\times10^{11}$ cm^{-3}; $T_i=T_e=0.2$ eV; $B=4$ kG.

tially undergoes a product of these successive modulations, since in the absence of collisions or collective interactions all of the particles retain their phase information in the x-t diagram. The resulting velocity distribution evolves in time and space as

$$f(v)=f_{12}(v)\cos[\omega_1 t-\omega_1 x/v]\cos[\omega_2 t-\omega_2(x-l)/v].$$

This can be written

$$f(v)=\frac{f_{12}(v)}{2}\left\{\cos\left[(\omega_2+\omega_1)t-\frac{\varphi_2+\varphi_1}{v}\right]+\cos\left[(\omega_2-\omega_1)t-\frac{\varphi_2-\varphi_1}{v}\right]\right\},$$

where $\varphi_1=\omega_1 x$ is the phase imparted by the first grid and $\varphi_2=\omega_2(x-l)$ is the phase imparted by the second grid. In the first term, which oscillates at $\omega_2+\omega_1$ the phase $\varphi_2+\varphi_1$ does not vanish for $x>l$. This term will cause the integration over velocity space to phase mix to zero. However, in the second term, which oscillates at the difference frequency $\omega_2-\omega_1$, there is no velocity dependence in the argument if $\varphi_1=\varphi_2$. At this point there is no phase mixing and the echo will appear. Since φ_2 starts at a later position than φ_1 it must increase at a faster rate if it is to equal φ_1 for some x. From the definition of φ_1 and φ_2 this means that $\omega_2>\omega_1$ for the echo to appear. The point at which $\varphi_2=\varphi_1$, i.e., the echo position, is given by

$$x=l'=[\omega_2/(\omega_2-\omega_1)]l. \quad (1)$$

This oversimplified picture predicts the same echo position as obtained by more refined theories[5,6] which include self-consistent fields through a plasma dielectric function. However, it does not yield an echo asymmetric in shape about its maxima as calculated from these theories. This physical picture is helpful nonetheless in understanding the effect of collisions on the echo. When two particles interact, their velocities or slopes in the x-t diagram of Fig. 1(a) would change, resulting in a more smeared pattern and a weaker echo.

The experimental observation of ion-wave echoes was performed in a highly ionized cesium plasma of a Q device. Figure 1(b) shows the experimental arrangement. The device is operated single ended with a hot plate ionizing a neutral cesium beam at one end and a cold plate terminating the plasma at the other. The plasma is confined radially by an axial magnetic field of 4 kG. The density range of $5\times10^9<n<2\times10^{11}$ cm^{-3} and temperatures $T_e=T_i=0.2$ eV of the plasma were chosen such that $\lambda/l\geqslant1$, where λ is the mean free path for ions and l is the spacing between the two grids. The frequencies used were such that $\omega\tau_{ii}>1$.

An ion-acoustic wave of frequency ω_1 is continuously excited at a grid in the plasma and propagates along the plasma column. This wave is damped by spatial collisionless damping.[7] A second ion-acoustic wave at ω_2 is excited at a second grid separated from the first grid by a distance l which can be much greater than a damping length for the first wave. Then at a distance l' from the first grid, $l'>l$, the echo appears in the plasma at a frequency $\omega_3=\omega_2-\omega_1$. The echo was detected by three different methods: a small negatively biased grid which is carefully shielded, a 35-GHz microwave interferometer system, and an X-band microwave system. All three systems gave essentially the same results, but the simplicity and the higher signal-to-noise ratio of the biased grid made it by far the most satisfactory method of detection. The received signal is fed into a lock-in amplifier which is synchronized to a different frequency ω_3 generated by mixing the two input frequencies ω_1 and ω_2.

The amplitude of the echo as a function of distance from the first grid has been measured for various grid separations and excitation frequencies, as shown in Fig. 2. The same diagram also shows the received signal when the excitation frequencies of the two grids are interchanged. In this case $\omega_1>\omega_2$ and no echo occurs in agreement with the preceding argument. The position of the center of the echo with respect to the grid separation has been measured for various excitation frequencies. Figure 3 shows that the position of the echo l' bears a relationship to the grid spacing l as given by

$$l'=[\omega_2/(\omega_2-\omega_1)]l+l_0, \quad (2)$$

where l' is measured from the first grid. Comparing Eq. (2) with (1) we see that the correct linear dependence of the echo position on the separation between the excitation grids is observed although there is a constant displacement as indicated by the intercepts on Fig. 3. The displacement of the echo position could be explained if the ion-acoustic waves were excited in the plasma at a position slightly displaced from the grid itself. Preliminary results show that when the Q machine is operat-

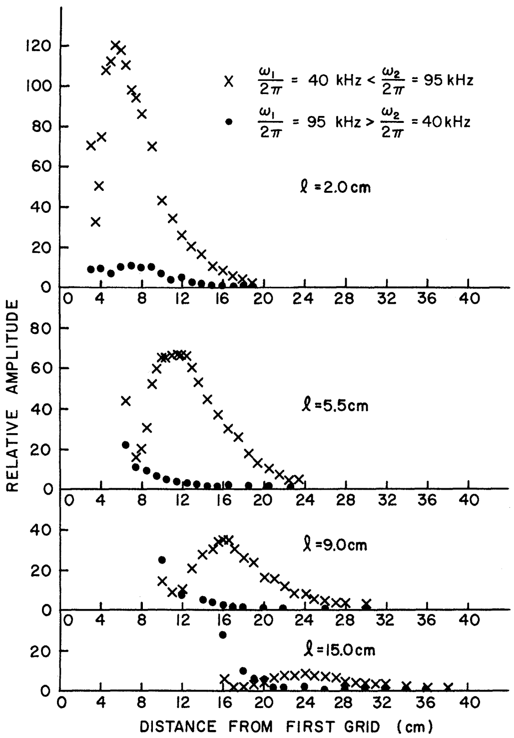

FIG. 2. Plots of received signal versus distance from the first grid for four values of l. The crosses are the signal received when $\omega_2 > \omega_1$. The solid points are the signal received when $\omega_1 > \omega_2$. For these points $T_i = T_e = 0.2$ eV, $N = 2\times10^{10}$ cm^{-3}. The estimated experimental error is ±5%. The density measured with a Langmuir probe in a magnetic field could be higher than the actual density by a factor of 2 [M. Hasmi, A. J. van der Houven van Oordt, and S. G. Wegrove, in Proceedings of the International Conference on the Physics of Quiescent Plasmas, Garching, Germany, 1967 (unpublished)].

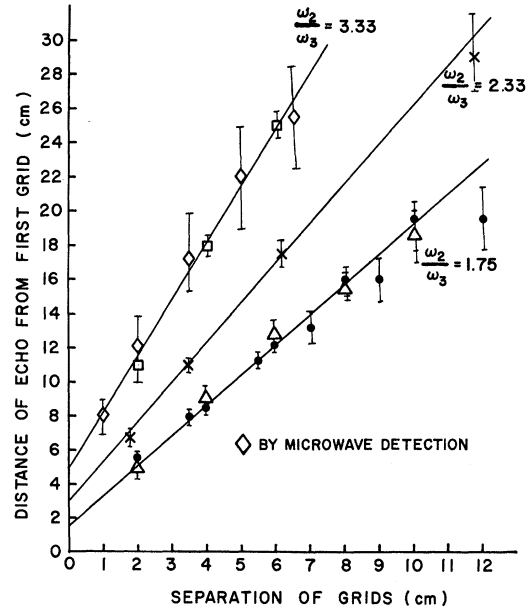

FIG. 3. A plot of the distance of the echo maximum from the first grid for different values of ω_2/ω_3. The slope of the solid line is given by ω_2/ω_3. The points indicated by diamonds were obtained from the X-band microwave system. All other points were obtained with the biased grid. $N = 2\times10^{10}$ cm^{-3}; $T_i = T_e = 0.2$ eV. Squares, $\omega_1/2\pi = 35$ kHz; $\omega_2/2\pi = 50$ kHz. Diamonds, $\omega_1/2\pi = 70$ kHz; $\omega_2/2\pi = 100$ kHz. Crosses, $\omega_1/2\pi = 40$ kHz; $\omega_2/2\pi$ 70 kHz. Circles, $\omega_1/2\pi = 40$ kHz; $\omega_2/2\pi = 95$ kHz. Triangles, $\omega_1/2\pi = 70$ kHz; $\omega_2/2\pi = 160$ kHz.

ed in a double ended configuration with no net plasma drift the echo is no longer displaced from the theoretically predicted position.

As shown by Gould, O'Neil, and Malmberg,[1] the echo is a second-order effect. A verification of this was made by measuring the change in the amplitude of the echo as a function of the amplitude of the excited ion waves. It was found that the amplitude of the echo[8] varied linearly with the product of the excited wave amplitudes over a range of excitation amplitudes $10^{-2} < \delta n/n_0 < 10^{-1}$.

Since the occurrence of the echo depends on the individual particles retaining the phase information imparted to them by the two excited waves, small angle collisions can have a strong effect in reducing the amplitude of the echo. The effect of ion-ion collisions on the amplitude of the echo was measured by increasing the plasma density. The effect of ion-neutral collisions was measured by bleeding an inert gas, argon, into the chamber. Both procedures resulted in a decrease in the relative echo amplitude. However, the effect of the ion-ion collisions was much stronger than the ion-neutral collisions as one would expect since small-angle long-range Coulomb collisions should be very effective in decreasing the echo amplitude with change in density. The change in the logarithm of the echo amplitude with change in density $d\ln A/dn$ is approximately 20 times larger for increase in ion density than for increase in neutral density.

Another method of observing the effect of collisions on the echo is to measure the amplitude of the echo as a function of the separation l between the two excitation grids. As the separation between the two grids is increased, the position of the echo moves further from the excitation grids and the particles which make up the echo have a greater probability of suffering a collision. As shown in Fig. 2 the amplitude of the echo decreases as l increases. The quantitative description of collisional effects will be reported in a future publication.

Our experimental observations together with preliminary interpretations can be summed up as follows:

(1) The position of the echo varies linearly with grid separation and has the proper dependence on the excitation frequencies. The displacement of the echo seems to be due to the fact that the drifting plasma alters the position where the waves are excited in the plasma.

(2) The echo does not appear unless $\omega_2 > \omega_1$.

(3) The amplitude of the echo depends linearly on the product of the amplitude of the two excited waves.

(4) Both ion-ion and ion-neutral collisions tend to decrease the amplitude of the echo.

(5) As shown in Fig. 2, the echo is asymmetrical. It rises to its maximum value faster than it dies out showing that the assumption of noninteracting particles and a simplified excitation scheme which predicts a symmetrical echo is not sufficient. Collective effects and the biased grid excitation must be considered.

In summary, our preliminary investigation of ion-wave echoes promises a useful means of studying collisional phenomena. The echo depends on the particles retaining the phase information imparted by the excited waves and is sensitive to relatively weak interactions. It may be possible to use the echo to study the effects of particle-wave or wave-wave interactions in a collisionless turbulent plasma.

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[2]J. H. Malmberg, C. B. Wharton, T. M. O'Neil, and R. W. Gould, to be published.

[3]N. Ahern, D. Baker, and A. Y. Wong, in Ninth Annual Meeting of the Division of Plasma Physics of the American Physical Society, Austin, Texas, 8-11 November 1967 (unpublished).

[4]A similar picture for temporal echoes was first given by K. R. MacKenzie, private communication.

[5]T. M. O'Neil and R. W. Gould, to be published.

[6]G. L. Johnston and B. D. Fried, to be published.

[7]A. Y. Wong, R. Motley, and N. D'Angelo, Phys. Rev. 133, A436 (1964).

[8]The echo amplitude $\delta n_E/n$ is down from the excited wave amplitude by 10^{-3}.

ANISOTROPY AND MASS ENCHANCEMENT OF THE CYCLOTRON EFFECTIVE MASS IN Pt †

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Detailed and accurate measurements of the cyclotron effective masses for both s- and d-like carriers have been made in Pt. Comparison with recent augmented-plane-wave band-structure calculations indicate that the many-body mass enhancement factor is roughly isotropic with a value of about 1.5.

Recent band-structure calculations in Pt[1,2] have attempted to expain its anomalously large heat capacity[3,4] and magnetic susceptibility.[4-6] These calculations have shown, however, that in order to achieve quantitative agreement with experiments many-body interactions must be considered. Two well-known contributions to the density of states in addition to that due to band structure are electron-electron and electron-phonon interaction. Recently a third mechanism has been suggested[7-10] that in special cases could greatly enhance the density of states. This is the so-called exchange enhancement due to short-lived spin fluctuations. Estimates of the magnitude of this exchange enhancement in Pt have varied from 2 to 8, but these estimates were based on simple models and appear, from the results of the present experiment, to be quantitatively in error.

Detailed and accurate de Haas–van Alphen (dHvA) measurements of both the extremal areas and cyclotron effective masses on all the sheets of the Fermi surface of Pt have been made. The extremal area measurements and a detailed description of the shape of the Pt Fermi surface deduced from them will be the subject of a separate Letter.[11] The Pt Fermi surface consists of three distinct sheets not