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ELECTROMAGNETIC MASS DIFFERENCE OF KAONS*

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Using the algebra of currents, modified Weinberg sum rules, and the tadpole model of Coleman, Glashow, and Schnitzer, we calculate the kaon electromagnetic mass difference in the soft-kaon limit to be -3.9 ± 0.6 MeV, in excellent agreement with experiment.

In the course of a recent calculation[1] of the electromagnetic mass difference of pions using chiral $SU(2)\otimes SU(2)$ current-algebra and soft-pion techniques, we introduced a modification of Weinberg's second sum rule[2] which rendered the result finite and in good agreement with experiment. In this note, we extend these considerations to the chiral $SU(3)\otimes SU(3)$ current algebra in order to calculate the second-order electromagnetic mass difference of kaons. We find that this method enables us to compute the "nontadpole" contribution[3,4] to the mass difference, and we find a considerably smaller value than that obtained by the authors of Ref. 4. When our result is combined with their phenomenological value for the tadpole contribution, the total mass difference thus calculated is in excellent agreement with experiment.

To order e^2, the kaon electromagnetic mass difference $\Delta(m_K^{\ 2}) = m^2(K^+) - m^2(K^0)$ is given by

$$\Delta(m_K^{\ 2}) = -[2(2\pi)^4]^{-1} \int d^4q (q^2)^{-1} (g_{\lambda\beta} + a q_\lambda q_\beta / q^2) T^{\lambda\beta}(p,q), \tag{1}$$

where

$$T_{\lambda\beta}(p,q) = (2\pi)^3 e^2 \int dx\, e^{-iqx} [\langle K^+(p)| T(V_\lambda^{\ \mathrm{em}}(x) V_\beta^{\ \mathrm{em}}(0)) |K^+(p)\rangle - \langle K^0(p)| T(V_\lambda^{\ \mathrm{em}}(x) V_\beta^{\ \mathrm{em}}(0)) |K^0(p)\rangle] + \text{contact term}. \tag{2}$$

We now take the soft-kaon limit ($p_\lambda \to 0$) and use the chiral $SU(3)\otimes SU(3)$ current algebra and partial conservation of axial-vector currents in the form

$$\partial^\lambda A_\lambda^{\ (i)} = m_K^{\ 2} F_K \varphi_K^{\ (i)}, \quad i = 4, 5, 6, 7, \tag{3}$$

to obtain

$$T_{\lambda\beta}(0,q) = e^2 F_K^{\ -2} [\Delta_{\lambda\beta}^{\ V(3)}(q) + \Delta_{\lambda\beta}^{\ V(8)}(q) - 2\Delta_{\lambda\beta}^{\ A(5)}(q) + g_{\lambda\beta} C(q) + M_{\lambda\beta}(q)], \tag{4}$$

where

$$\Delta_{\lambda\beta}^{V(i)} = \int dx\, e^{iqx} \langle T(V_\lambda^{(i)}(x) V_\beta^{(i)}(0))\rangle_0, \quad (5a)$$

$$\Delta_{\lambda\beta}^{A(i)} = \int dx\, e^{iqx} \langle T(A_\lambda^{(i)}(x) A_\beta^{(i)}(0))\rangle_0, \quad (5b)$$

$C(q)$ is the contribution of the contact term, and $M_{\lambda\beta}(q)$ is the tadpole contribution. The tadpole contribution arises from the presence of equal-time commutators of the axial current with its divergence which appear in the reduction and which, in contrast to the pion case, lead to a term dependent on $\langle T(S^{(3)} V_\lambda^{\mathrm{em}} V_\beta^{\mathrm{em}})\rangle_0$, with $S^{(3)}$ an isovector scalar density.[5]

Next, we write spectral representations for $\Delta_{\lambda\beta}^{V(i)}$ and $\Delta_{\lambda\beta}^{A(i)}$:

$$\Delta_{\lambda\beta}^{V,A(i)} = g_{\lambda\beta} F_{V,A}^{(i)}(q^2) - q_\lambda q_\beta G_{V,A}^{(i)}(q^2), \quad (6)$$

where

$$F_{V,A}^{(i)}(q^2) = -i\int_0^\infty dm^2 \frac{\rho_{V,A}^{(i)}(m^2)}{m^2-q^2-i\epsilon}, \quad (7a)$$

$$G_V^{(i)}(q^2) = -i\int_0^\infty dm^2 \frac{\rho_V^{(i)}(m^2)}{m^2(m^2-q^2-i\epsilon)}, \quad (7b)$$

and

$$G_A^{(5)}(q^2) = -i\left[\int_0^\infty dm^2 \frac{\rho_A'^{(5)}(m^2)}{m^2(m^2-q^2-i\epsilon)} + \frac{F_K^2}{m^2-q^2-i\epsilon}\right]. \quad (7c)$$

(We omit Schwinger terms since they cancel in the final result.) Since the tadpole term is gauge invariant by itself, gauge invariance ($q^\lambda T_{\lambda\beta} = q^\beta T_{\lambda\beta} = 0$) determines the contact term to be

$$C(q^2) = q^2(G_V^{(3)} + G_V^{(8)} - 2G_A^{(5)}) - (F_V^{(3)} + F_V^{(8)} - 2F_A^{(5)}). \quad (8)$$

The integrand of (1) then becomes

$$(g^{\lambda\beta} + aq^\lambda q^\beta/q^2) T_{\lambda\beta}(0,q) = e^2 F_K^{-2}[3q^2(G_V^{(3)} + G_V^{(8)} - 2G_A^{(5)}) + M_{\lambda\lambda}(q)]. \quad (9)$$

We now assume that the nontadpole contribution taken separately should yield a finite result (this point will be discussed further below). The finiteness condition is

$$\lim_{q^2\to\infty} (q^2)^2[G_V^{(3)} + G_V^{(8)} - 2G_A^{(5)}] = 0, \quad (10)$$

which implies

$$\int_0^\infty dm^2(\rho_V^{(3)} + \rho_V^{(8)} - 2\rho_A'^{(5)}) = 2M_K^2 F_K^2, \quad (11)$$

and

$$\int_0^\infty dm^2(m^2)^{-1}(\rho_V^{(3)} + \rho_V^{(8)} - 2\rho_A'^{(5)}) = 2F_K^2. \quad (12)$$

The sum rule (12) is a generalization to SU(3) ⊗ SU(3) of Weinberg's first sum rule, whereas (11) is the generalization of the modified sum rule introduced in Ref. 1. Assuming dominance of $\rho_V^{(3)}$ by the ρ meson, of $\rho_V^{(8)}$ by ω and φ, and of $\rho_A'^{(5)}$ by $K_A(1320)$, the sum rules become

$$g_\rho^2 + g_\omega^2 + g_\varphi^2 - 2g_{K_A}^2 = 2m_K^2 F_K^2, \quad (11')$$

$$\frac{g_\rho^2}{m_\rho^2} + \frac{g_\omega^2}{m_\omega^2} + \frac{g_\varphi^2}{m_\varphi^2} - \frac{2g_{K_A}^2}{m_{K_A}^2} = 2F_K^2, \quad (12')$$

and we get for the nontadpole part of the kaon electromagnetic mass difference

$$\Delta_{\mathrm{n.t.}}(m_K^2) = -3\alpha(8\pi F_K^2)^{-1}[g_\rho^2 \ln(m_\rho^2/m_K^2) + g_\omega^2 \ln(m_\omega^2/m_K^2) + g_\varphi^2 \ln(m_\varphi^2/m_K^2) - 2g_{K_A}^2 \ln(m_{K_A}^2/m_K^2)]. \quad (13)$$

To proceed further, we must appeal to experiment. The branching ratios[6] $\Gamma(\rho\to\mu^- + \mu^+)/\Gamma(\rho\to\mathrm{all}) = (5.1\pm1.2)\times10^{-5}$ and $\Gamma(\omega\to e^- + e^+)/\Gamma(\omega\to\mathrm{all}) = (1.2\pm0.3)\times10^{-4}$ yield $(g_\omega^2/g_\rho^2) = 0.70 \pm 0.35$. Combining this result with the empirical relation $F_K = 1.28F_\pi$,[7] we find from the sum rules (11′) and (12′) that

$$g_\varphi^2/g_\rho^2 = 1.40 \pm 0.95, \quad (14a)$$

$$g_{K_A}^2/g_\rho^2 = 1.20 \pm 0.65, \quad (14b)$$

and hence

$$[m(K^+) - m(K^0)]_{\mathrm{n.t.}} = 0.74 \pm 0.6\ \mathrm{MeV}. \quad (15)$$

We take the tadpole contribution to be that computed in Ref. 4 by a phenomenological fit to the baryon mass differences:

$$[m(K^+)-m(K^0)]_{\text{tadpole}} = -4.7 \text{ MeV}, \tag{16}$$

and thus our final result is[8]

$$m(K^+)-m(K^0) = -3.96 \pm 0.6 \text{ MeV}, \tag{17}$$

in excellent agreement with the experimental value of -3.9 ± 0.6 MeV.

It remains only to discuss our requirement that tadpole and nontadpole terms are separately finite. The tadpole term contributes to any $\Delta T = 1$ mass difference, its numerical contribution changing from one mass splitting to another only because of the coupling constant at at the lower vergex in Fig. 1. Cancellation between this t-channel pole term and the nontadpole part, which represents the contribution of s- and u-channel intermediate states in $T_{\lambda\beta}(p,q)$, thus seems unlikely to occur in all mass differences. In addition, the tadpole term is nearly pure octet, the nontadpole term nearly pure 27-plet. It therefore seems reasonable to assume that these terms must be handled separately. The value of the tadpole term can then be obtained, as was done by Coleman and Schnitzer,[4] by analysis of the baryon mass splittings.[9]

We differ from Coleman and Schnitzer[4] only in the treatment (and numerical value) of the nontadpole contribution. In their paper, they were successful in fitting all the mass differences in the baryon and pseudoscalar meson octets, with the sole exception of the K^+-K^0 case, in which they obtained too large a value for the nontadpole part. Our smaller value for this term brings the entire scheme into good agreement with experiment.

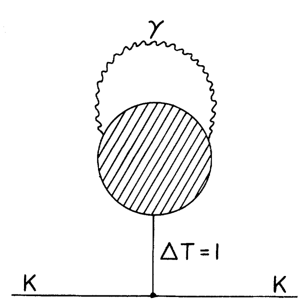

FIG. 1. Tadpole diagram describing the term proportional to $\langle T(S^{(3)}V_\lambda^{\text{em}}V_\beta^{\text{em}})\rangle_0$.

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so that the mass difference contribution is just

$$\delta(x^0)\{[A_0^{(4)}(x), \partial^\lambda A_\lambda^{(4)}(0)] - [A_0^{(6)}(x), \partial^\lambda A_\lambda^{(6)}(0)]\} = S^{(3)}(x)\delta^4(x).$$

More generally, we might expect in addition an isoscalar density which would not contribute to the mass difference. In the SU(3) limit, this term, which contributes to the kaon mass difference but not to the pion mass difference, gives a pure octet contribution to the mass operator. By contrast, the other terms in Eq. (4) give, in the SU(3) limit, a contribution proportional to $\Delta^V(q)-\Delta^A(q)$ which is of the same form as in the pion mass difference [see Eq. (4) in Ref. 1] and hence is a 27-plet contribution. Additional justification for identifying the $M_{\lambda\beta}$ term with the tadpole graphs of Ref. 3 comes from considering the t channel in $T_{\lambda\beta}(p, q)$: A t-channel pole, coming from a scalar-meson octet state, gives a contribution to $T_{\lambda\beta}$ of exactly the form of our $M_{\lambda\beta}$ term.

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