The results of a more thorough treatment of the data presented here will be published later along with data on other sources obtained during these flights and a later flight on 27 June 1967.

We are very grateful to the staff of the National Center for Atmospheric Research station at Palestine, Texas, for their hospitality and their successful conduct of the balloon flight operations. Philip Morrison and Allen Womack helped to improve the clarity of this paper.

†Work supported in part by the National Aeronautics and Space Administration under Grant No. NsG-386.

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REGGE CUTS IMPLY VANISHING TOTAL CROSS SECTIONS OR ESSENTIALLY CONSTANT DIFFRACTION PEAKS*

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(Received 30 November 1967)

It is shown that the infinite sequence of Regge cuts previously found in perturbation theory leads to high-energy behavior of scattering amplitudes with a power of the energy that is independent of the momentum transfer whenever the total cross sections are asymptotically constant.

It seems now well established that repeated exchange of Regge poles generates cuts in the complex angular-momentum plane.[1] If the intercept of the Pomeranchuk trajectory is strictly equal to 1, then exchange of several Pomeranchuk poles and a given trajectory $\alpha(t)$ leads to an infinite number of branch points that in general accumulate[2] for any $t<0$, at $\alpha(0)$. Up to now the current opinion seemed to be that nothing could be said about the corresponding contribution to the high-energy behavior. Nevertheless, we are going to show that, if one takes all the cuts into account, then, and for any $t<0$, the scattering amplitude $T(s,t)$ behaves asymptotically like $s^{\alpha(0)}(\ln s)^{\beta(t)}\cdots$, i.e., with a power that is independent of t, irrespective of whatever the value of the jumps over the cuts may be. For definiteness, we will present the explicit analysis for the case when $\alpha(t)$ is the Pomeranchuk trajectory itself, and later on comment on other cases.

(1) Pomeranchuk trajectory.—Here, the exchange of n Pomeranchuk poles gives a cut with a branch point located at

$$\alpha_c^{(n)}(t)=n\alpha_{\rm P}(t/n^2)-n+1. \tag{1}$$

Since we assume, as usual, that $\alpha_{\rm P}(0)=1$, it is then clear that for any $t<0$ one has[3]

$$\lim_{n\to\infty}\alpha_c^{(n)}(t)=\alpha_{\rm P}(0)=1. \tag{2}$$

Accordingly, the contribution of such cuts to the scattering amplitude in the s channel is, at high energy, of the form

$$\int_{-1/2}^{1}dl\, s^l g_t(l)\simeq T(s,t), \tag{3}$$

where $g_t(l)$ is essentially proportional to the product of the signature factor times the sum of the jumps across the cuts. We remark that by virtue of (2), $g_t(l)$ cannot vanish identically in any interval $1>l>1-\delta_0$, $\delta_0\neq 0$ fixed, for any $t<0$. The desired result will be proved rigorously from formula (3) by reducing the problem to the following theorem on Laplace transforms (Doetsch[4]):

Theorem.—If

$$\psi(x) = \int_0^\infty dy\, e^{-xy} \varphi(y),$$

$\varphi(y)$ bounded at $y=0$, and if

$$\limsup_{x\to\infty} \ln|\psi(x)|/x = -v, \quad v>0,$$

then $\varphi(y)$ vanishes identically if $y<v$. The theorem is proved by using the inverse Laplace transform, and applying suitable majorizations, to show that whenever $y<v$,

$$\int_0^y dy'\, \varphi(y') \equiv 0.$$

In our case, the integral in (3) may be reduced to a Laplace transform by defining $x=\ln s$ and making the change of variables $y=1-l$, so that[5]

$$T(s,t) = s\int_0^\infty dy\, e^{-yx} g_t(1-y). \quad (4)$$

Since the integral in (4) is clearly bounded, it follows that $\lim\sup[\ln|T|/\ln s]$ exists and is $\leqslant 1$. On the other hand, applying the theorem, and since $g(1-y)$ does not vanish identically in a neighborhood of $y=1$, we get that the lim sup has to be $\geqslant 1$. Therefore, we have proved that

$$\lim\sup \frac{\ln|T(s,t)|}{\ln s} = 1, \quad t<0. \quad (5)$$

Let us suppose that $T(s,t)$ behaves, for large s, as[6]

$$T(s,t) \simeq f(t) s^{\alpha_1(t)} (\ln s)^{\beta(t)} (\ln\ln s)^{\gamma(t)} \cdots; \quad (6)$$

then, from (5) we obtain at once that $\alpha_1(t)\equiv 1$, if $t<0$, QED.

The preceding analysis shows that the width of the diffraction peak, for elastic scattering, may at most change as $\ln\ln s$ for sufficiently large energy. Of course, this is much slower than the logarithmic behavior one expects from a naive Regge-pole analysis. In fact, what happens is that the "effective" trajectory is flat at negative values of t.

However, for $t>0$ the picture is quite different since the pole always lies above the branch points [cf. Eq. (1)]; accordingly, the essentially constant diffraction peak we obtain does not lead to a Gribov paradox.[7]

As far as the experimental situation is concerned, a constant diffraction peak at high energy seems to be consistent with elastic-scattering data. In particular, πN is nonshrinking; on the other hand, NN is shrinking while $N\bar{N}$ is antishrinking, so that since the asymptotic differential cross sections should be equal for these two processes,[8] one may believe that they tend to the same constant limit.

(2) General case.—The analysis is quite similar to the previous one, and one gets the same result, i.e., that the corresponding scattering amplitude behaves like

$$T(s,t) \simeq f(t) s^{\alpha(0)} (\ln s)^{\beta(t)} \cdots, \quad t<0, \quad (7)$$

where α is the dominant Regge trajectory.

It is interesting to remark that this gives a new type of connection between high energy in one channel and low energy in the crossed channel, namely that the behavior is given by the intercept of the exchanged trajectory.

In general, (7) gives also essentially nonshrinking diffraction peaks. Experimentally, however, shrinking has been observed in $N\pi$ charge exchange.[9] In fact, because of the small variation of $\ln s$ in the available energy range, one can still fit the data with formula (7); for instance, the usual fit to determine the ρ trajectory,

$$\frac{\ln|d\sigma/dt|}{\ln(s/s_0)} = \alpha_\rho(t), \quad (8)$$

is replaced by

$$\frac{\ln|d\sigma/dt|}{\ln(s/s_0)} = \alpha_p(0) + \beta(t)\frac{\ln\ln(s/s_0)}{\ln(s/s_0)}. \quad (9)$$

Taking as a typical number $s_0 = 1$ GeV, one gets that the coefficient of $\beta(t)$ in (9) only varies from 0.4 to 0.5 in the energy range 5-25 GeV, so that $\beta(t)$ simply gives the slope of the "effective" trajectory. Of course, what happens in this case is that one is only at intermediate energy.

(3) Discussion.—A way out of the situation just described would be to assume that the Pomeranchuk intercept is not exactly 1, but rather that[2,10] $\alpha_P(0)=1-\epsilon$, $\epsilon\neq 0$; then the branch points no longer accumulate at $\alpha(0)$, but rather give an effective twisted trajectory, $\alpha_{\rm eff}(t)$, that decreases as t separates from 0 along negative values. In this case, one obtains again a behavior similar to the Regge-pole type for the diffraction peak, i.e., in $\ln s$. However, the total cross section will tend to zero at infinity like a power $1/s^\epsilon$ of the energy. We refer to the work of Srivastava[2] for a detailed

analysis of this situation.

A last remark is that there is always the probability that the cuts discussed in Ref. 1 do not really appear in the scattering amplitudes, since they are obtained by considering particular terms in a perturbation expansion. However, according to Mandelstam,[1] this is very unlikely.

We are indebted to Professor N. N. Khuri and Professor A. Martin for illuminating discussions, and to Professor B. Zumino for his kind hospitality at the Physics Department of New York University, where this work was performed.

Note added in proof.—The result proved in this article was stated by Gribov.[11] We thank Professor A. Martin for bringing this fact to our attention.

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*Research supported in part by the National Science Foundation.

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$$\lim_{s\to\infty}\frac{\mathrm{Im}T(us,t)}{\mathrm{Im}T(s,t)}$$

exists if u is near 1. This assumption amounts to requiring that the relative importance of possible oscillations decreases as $s\to\infty$, and suffices to insure that

$$\lim_{s\to\infty}\ln|T(s,t)|/\ln s$$

exists. To our knowledge it is the most general condition that allows us to obtain asymptotic results.

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SUBTRACTIONS IN A DISPERSIVE APPROACH TO INTERNAL SYMMETRY BREAKING

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It is shown how the subtractions, in a dispersive approach to SU(3) and SU(2) breaking, reproduce the results of the tadpole model, under the assumption of certain widely used forms for the medium-strong interaction.

We wish to re-evaluate, within the light of recent theoretical developments, the results obtained by field-theoretical models of SU(3) breaking such as the ω-φ mixing model,[1] the tadpole model,[2] and the "fifth interaction."[3] We will show their basic similarities and rederive many of their results in what we believe to be a physically sounder fashion; in particular, the intramultiplet relations and the connection between SU(2) and SU(3) breaking of the tadpole model will be shown to arise in a natural manner.