BREAKING CHIRAL SYMMETRY*

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This note will explore what can be learned about chiral-symmetry breaking under two assumptions: (1) The Lagrangian is invariant under SU(3)⊗SU(3) except for a term which transforms under the representation $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$, and (2) certain vertex functions are as smooth as possible. No assumption is made concerning the magnitude of the symmetry-breaking term.[1] We find the natural appearance of a pseudoscalar nonet π, K, η, and η^*, and of a scalar kaon[2] κ lying below or just above the $K\pi$ threshold. In addition, we obtain a refinement of the Ademollo-Gatto theorem[3] for the K_{e3} form-factor $f^+(0)$.

We will begin by proving theorems about a general broken symmetry group G.[4] The Lagrangian is assumed to take the form

$$\mathcal{L}=\mathcal{L}_0+\epsilon_i\varphi_i, \tag{1}$$

where $\mathcal{L}_0$ is invariant under G, and the φ_i are local fields forming a basis for some definite real representation R of G. It follows that the currents $J_a^{\ \mu}$ constructed according to Noether's theorem satisfy the partial-conservation and commutation laws

$$\partial_\mu J_a^{\ \mu}=\epsilon_i T_{ij}^{\ a}\varphi_j, \tag{2}$$

$$[\int J_a^{\ 0}(x,t)d^3x,\ \varphi_i(y,t)]=T_{ij}^{\ a}\varphi_j(y,t), \tag{3}$$

where $T_{ij}^{\ a}$ is the real antisymmetric matrix representing the generator T^a in the representation R. We define 1-, 2-, and 3-point functions

$$\lambda_i\equiv\langle\varphi_i\rangle_0, \tag{4}$$

$$\Delta_{ij}(p^2)\equiv i\int d^4x\,e^{-ip\cdot x}\langle T\{\varphi_i(x),\varphi_j(0)\}\rangle_0, \tag{5}$$

$$(p+p')^\mu f_{ija}^{\ +}(p^2,p'^2,q^2)+q^\mu f_{ija}^{\ -}(p^2,p'^2,q^2)$$
$$\equiv\Delta_{ik}^{\ -1}(p^2)\Delta_{jl}^{\ -1}(p'^2)\int d^4x d^4y\,e^{ip\cdot x-ip'\cdot y}\langle T\{\varphi_k(x),\varphi_l(y),J_a^{\ \mu}(0)\}\rangle_0, \tag{6}$$

$$g_{ijk}(p^2,p'^2,q^2)\equiv\Delta_{il}^{\ -1}(p^2)\Delta_{jm}^{\ -1}(p'^2)\Delta_{kn}^{\ -1}(q^2)\int d^4xd^4y\,e^{ip\cdot x-ip'\cdot y}\langle T\{\varphi_l(x),\varphi_m(y),\varphi_n(0)\}\rangle_0, \tag{7}$$

where $q\equiv p-p'$. Equations (2) and (3) impose on these the conditions[5]

$$\epsilon_i T_{ij}^{\ a}\lambda_j=0, \tag{8}$$

$$\Delta_{ij}^{\ -1}(0)\lambda_j^{\ a}=\epsilon_j^{\ a}, \tag{9}$$

$$(p^2-p'^2)f_{ija}^{\ +}(p^2,p'^2,0)=\lambda_k^{\ a}g_{ijk}(p^2,p'^2,0)+\Delta_{ik}^{\ -1}(p^2)T_{kj}^{\ a}-T_{ik}^{\ a}\Delta_{kj}^{\ -1}(p'^2), \tag{10}$$

where

$$\lambda_j^{\,a} \equiv T_{jk}^{\ \ a}\lambda_k, \quad \epsilon_j^{\,a} \equiv T_{jk}^{\ \ a}\epsilon_k. \tag{11}$$

Equation (8) follows upon taking the vacuum expectation value of Eq. (2). Equation (9) follows upon multiplying Eq. (5) by $\epsilon_i^{\,a}$, using Eq. (2), integrating by parts using Eqs. (3) and (4), and setting $p_\mu = 0$. Equation (10) follows upon multiplying Eq. (6) by q_μ, setting $q^2=0$, and using Eqs. (2), (3), and (7).

We will not attempt to extract from Eq. (10) all the information it contains. Instead, we differentiate with respect to p^2, set $p^2 = p'^2 = 0$, multiply with $\lambda_i^{\,c}, \lambda_j^{\,b}$, add the same equations with cba replaced with bca and acb, and subtract the same equations with cba replaced with cab, bac, and abc. Using the symmetry of $g_{ijk}(p^2,0,0)$ in j and k, and the antisymmetry of $f_{ija}^{\ +}(0,0,0)$ in i and j, we find that

$$2\lambda_i^{\,a}\lambda_j^{\,c} f_{ijb}^{\ \ +}(0,0,0)$$

$$= \Delta_{ij}^{\ -1}(0)'\lambda_j^{\,d}[C_{abd}\lambda_i^{\,c} + C_{acd}\lambda_i^{\,b} + C_{bcd}\lambda_i^{\,a}], \tag{12}$$

where C_{abc} are the real, totally antisymmetric structure constants

$$[T_a, T_b] = C_{abc}T_c \tag{13}$$

and

$$\Delta_{ij}^{\ -1}(0)' = [d\Delta^{-1}(p^2)/dp^2]_{p^2=0}. \tag{14}$$

In order to make contact with physics, we will further assume that the functions f^+ and g appearing in Eq. (10) are as smooth as reasonably possible, over some range $0 \leq -p^2$, $-p'^2 \leq M^2$, i.e., in this range, $f_{ija}^{\ +}(p^2,p'^2,0)$ is nearly equal to the constant $f_{ija}^{\ +}(0,0,0)$ and $g_{ijk}(p^2,p'^2,0)$ is at most linear in p^2 and p'^2.[6] It follows that $\Delta_{ij}^{\ -1}(p^2)$ is linear in p^2; so

$$\Delta(p^2) \cong Z_T^{\ 1/2}(p^2+\mu^2)^{-1}Z^{1/2} \quad (0 \leq -p^2 \leq M^2), \tag{15}$$

where μ^2 is the physical mass matrix and $Z^{1/2}$ is a positive wave-function renormalization matrix.[7]

Now we return to the case of interest, $G = \mathrm{SU}(3) \otimes \mathrm{SU}(3)$ broken by $R = (\underline{3}, \underline{3}^*) \oplus (\underline{3}^*, \underline{3})$. It is apparent from Eqs. (9)-(12) that our theorems are almost exclusively concerned with the "would-be Goldstone bosons" which have the same isospin, hypercharge, and parity as the $\lambda_{ij}^{\ a}$, i.e., the same as π, K, η, and κ. (Since isospin and hyperchange are not broken, their generators annihilate λ.) The representation $(\underline{3}, \underline{3}^*) \oplus (\underline{3}^*, \underline{3})$ contains two states with the quantum numbers of the η, and one each with the quantum numbers of the π, K, and κ. We defer a discussion of the mixed effects in the η channel to a future work, and will concern ourselves here with only the π, K, and κ channels.[8] In these channels, $Z^{1/2}$ and μ^2 are diagonal and Eq. (15) therefore tells us that there is just one particle in each of these channels. Letting "a" denote the particle whose quantum numbers are the same as λ^a or T^a, we find from (2) and (9) that

$$\epsilon_a = \mu_a^{\ 2}F_aZ_a^{\ -1/2}, \tag{16}$$

$$\lambda_a = \mu_a^{\ -2}\epsilon_aZ_a = F_aZ_a^{\ 1/2}, \tag{17}$$

where λ_a and ϵ_a are the single nonvanishing components of $\lambda_i^{\,a}$ and $\epsilon_i^{\,a}$, and F_a is defined by the one-particle matrix element

$$\langle 0|J_a^{\ \mu}(0)|a,p\rangle \equiv F_a p^\mu (2\pi)^{-\frac{3}{2}}(2E_a)^{-\frac{1}{2}}. \tag{18}$$

There are only two nonvanishing ϵ_i and two nonvanishing λ_i, corresponding to the two vectors of $(\underline{3}, \underline{3}^*) \oplus (\underline{3}^*, \underline{3})$ with even parity and zero isospin and hypercharge. Thus there is one relation among the three ϵ_a and among the three λ_a, which turns out to say that $\epsilon_\pi = \epsilon_K + \epsilon_\kappa$ and $\lambda_\pi = \lambda_K + \lambda_\kappa$. We then conclude from (16) and (17) that

$$\mu_\pi^{\ 2}F_\pi Z_\pi^{\ -1/2} = \mu_K^{\ 2}F_KZ_K^{\ -1/2} + \mu_\kappa^{\ 2}F_\kappa Z_\kappa^{\ -1/2} \tag{19}$$

$$F_\pi Z_\pi^{\ 1/2} = F_KZ_K^{\ 1/2} + F_\kappa Z_\kappa^{\ 1/2}. \tag{20}$$

It is not possible to eliminate all of the unknown Z_a, but we can deduce that the $\mu_a|F_a|$ are <u>not</u> subject to a triangle inequality, i.e., either

$$\mu_\kappa \leq \big|\mu_\pi|F_\pi| - \mu_K|F_K|\big|/|F_\kappa| \tag{21a}$$

or

$$\mu_\kappa \geq \big|\mu_\pi|F_\pi| + \mu_K|F_K|\big|/|F_\kappa|, \tag{21b}$$

where (21a) applies only when F_π and F_K have the same sign, and (21b) applies when they are of opposite sign. Also, by letting a, b, c in Eq. (12) correspond to κ, K, and π, and dividing by $2\lambda_K\lambda_\pi Z_K^{\ -1/2}Z_\pi^{\ -1/2}$, we find that the renormalized K_{e3} form factor is

$$f^+(0) = (F_\pi^{\ 2} + F_K^{\ 2} - F_\kappa^{\ 2})/2F_KF_\pi. \tag{22}$$

Note that in the limit of exact SU(3), $F_\pi = F_K$, and $F_\kappa = 0$; so $f^+(0) = 1$. Further, $f^+(0) - 1$ is of second order in the SU(3)-breaking parameters $F_K - F_\pi$ and F_κ, in agreement with the Ademollo-Gatto theorem. The Cabibbo[9] theory relates $f^+(0)$ to the measured amplitudes for K_{e3}, π_{e3}, $K_{\mu 2}$, and $\pi_{\mu 2}$ decay:

$$|F_K[F_\pi f^+(0)]^{-1}| = (\tan\theta_A/\tan\theta_V)_{\rm eff} = 1.28. \quad (23)$$

To proceed any further, we will need independent information about the ratios F_K/F_π and F_κ/F_π. The spectral-function sum rules[10] yield values which give fair agreement with (22) and (23), but there are reasons for being skeptical about some of these sum rules, namely those which equate the integrals of all the spin-one spectral functions for the vector current. These sum rules cannot be derived in a straightforward way from the algebra of fields,[11] and also lead to incorrect predictions[12] for the ω, ρ, and φ meson masses. To be conservative we will use only the sum rules whose validity is assured by the algebra of fields, i.e., those which state the equality of all Schwinger terms, and those which equate the integrals of vector and axial-vector spectral functions with the same isospin and hypercharge. (These are all we need to derive the A_1-ρ mass ratio[13] and the π^+-π^0 mass difference.[14]) Saturating the sum rules for the strangeness-changing spectral functions with the $K^*(890)$, $K_A(1250)$, κ, and K, we find that

$$\frac{g_S^{\,2}}{m^2(K^*)} + F_\kappa^{\,2} = \frac{g_S^{\,2}}{m^2(K_A)} + F_K^{\,2} = 2F_\pi^{\,2},$$

where $g_S^{\,2} = g^2(K^*) = g^2(K_A)$. We eliminate $g_S^{\,2}$ to obtain

$$\frac{2F_\pi^{\,2} - F_\kappa^{\,2}}{2F_\pi^{\,2} - F_K^{\,2}} = \frac{m^2(K_A)}{m^2(K^*)} = 1.98. \quad (24)$$

If we also require that (23) be satisfied and use (22), we further find that

$$2F_K^{\,2}/(F_K^{\,2} + F_\pi^{\,2} - F_\kappa^{\,2}) = 1.28. \quad (25)$$

From (24) and (25) we obtain

$$F_K^{\,2}/F_\pi^{\,2} = 1.17, \quad F_\kappa^{\,2}/F_\pi^{\,2} = 0.34. \quad (26)$$

The inequality (21) thus becomes

$$\mu_\kappa \leqslant 670 \text{ MeV or } \mu_\kappa \geqslant 1150 \text{ MeV}.$$

We favor the first inequality because it is the one that holds when F_π is of the same sign as F_K, as would be expected for weak SU(3) breaking.[15] The $K\pi$ threshold is at 630 MeV, and if the κ lies below this mass (as we suspect[16]), then it would only decay electromagnetically by $\kappa \to K + 2\gamma$. The existence of such a particle is not precluded by current experimental evidence.

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†On leave from the University of California, Berkeley, California.

[1]M. Gell-Mann [Physics (N.Y.) 1, 63 (1964)] has considered chiral SU(3) ⊗ SU(3) as a weakly broken symmetry group.

[2]The association of a scalar kaon with the divergence of the strange vector current originates with Y. Nambu, Phys. Rev. Letters 4, 380 (1960).

[3]M. Ademollo and R. Gatto, Phys. Rev. Letters 13, 264 (1964).

[4]Only the continuous connected component of G is relevant to our analysis.

[5]Equation (9) reduces to a proof of the Goldstone theorem for the case $\epsilon = 0$ [cf. Eq. (18) of J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. 127, 965 (1962)]. We originally deduced Eq. (9) in Lagrangian field theory without making use of current algebra.

[6]The same smoothness assumption is used by H. Schnitzer and S. Weinberg, Phys. Rev. 164, 1828 (1967).

[7]Originally, we took g to be constant for small momenta. This gave the result $Z = 1$, with the consequences $F_K \approx 1.4F_\pi$, $F_\kappa \approx 0.4F_\pi$, and $\mu_\kappa \approx 890$ MeV. However, these values of F are in poor agreement with the SU(3)⊗SU(3) spectral-function sum rules [S. Glashow, H. Schnitzer, and S. Weinberg, Phys. Rev. Letters 19, 139 (1967); T. Das, V. Mathur, and S. Okubo, Phys. Rev. Letters 18, 761 (1967)], and furthermore, we cannot have $Z = 1$ if there are gauge-invariant couplings of vector and axial-vector mesons. It is for these reasons that we allow g to have linear dependence on p^2 and p'^2. Recent work of J. Sucher and C. Woo [Phys. Rev. Letters 18, 723 (1967)] on the σ model does use the hypothesis $Z = 1$.

[8]When our method is applied to the mixed channel, the principal new result is the relation

$$F_K^{\,2}\mu_K^{\,2} + F_\kappa^{\,2}\mu_\kappa^{\,2} = \tfrac{3}{4}(F_\eta^{\,2}\mu_\eta^{\,2} + F_{\eta'}^{\,2}\mu_{\eta'}^{\,2}) + \tfrac{1}{4}F_\pi^{\,2}\mu_\pi^{\,2}.$$

We wish to thank S. Shei and C. Sommerfield for their assistance in this connection. A detailed presentation of this work may be found in the contribution of S. Glashow to the Proceedings of the 1967 Erice Summer School (to be published).

[9]N. Cabibbo, Phys. Rev. Letters 13, 264 (1964).

[10]See Glashow, Schnitzer, and Weinberg, Ref. 7, and Das, Mathur, and Okubo, Ref. 7.

[11]T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev.

Letters 18, 1029 (1967).

[12]J. J. Sakurai, Phys. Rev. Letters 19, 803 (1967); A. Dar, to be published.

[13]S. Weinberg, Phys. Rev. Letters 18, 507 (1967).

[14]T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low, and J. E. Young, Phys. Rev. Letters 18, 759 (1967).

[15]Saturating the $T=Y=0$ spectral-function sum rules with ω, φ, D, and E, we obtain the result

$$m^{-2}(D)\cos^2\theta+m^{-2}(E)\sin^2\theta = [1-(F_\eta^{\ 2}+F_{\eta'}^{\ 2})/2F_\pi^{\ 2}] \times[m^{-2}(\omega)\cos^2\theta'+m^{-2}(\varphi)\sin^2\theta'],$$

where θ and θ' are unknown angles. This gives the inequality

$$0.74F_\pi^{\ 2}\le(F_\eta^{\ 2}+F_{\eta'}^{\ 2})\le1.38F_\pi^{\ 2}.$$

Combined with Eq. (26) and the mixed-channel formula of Ref. 8, this inequality provides the upper bound $\mu_K \le 1410$ MeV. But it is again clear that the heavier κ mass may only be realized with very large SU(3) breaking (e.g., $F_{\eta'}^{\ 2}>2F_\eta^{\ 2}$).

[16]The equality $\mu_K^{\ 2}-\mu_\pi^{\ 2}=m^2(K^*)-m^2(\rho)$ is both mysterious and well satisfied. Its natural generalization to even parity states is $\mu_K^{\ 2}=m^2(K_A)-m^2(A_1)$ and yields $\mu_K=620$ MeV.

EXPERIMENTAL TESTS OF THE VECTOR-DOMINANCE MODEL*†

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In this Letter we report the results of an experiment designed to perform some independent tests of the validity of the vector-dominance model[1] of electromagnetic interactions of hadrons. The vector-dominance model relates the electromagnetic current $J_\mu(x)$ of hadrons with the phenomenological fields of vector mesons $\rho_\mu(x)$, $\varphi_\mu(x)$, and $\omega_\mu(x)$ via

$$J_\mu(x)=-\frac{m_\rho^{\ 2}}{2\gamma_\rho}\rho_\mu(x)-\frac{m_\omega^{\ 2}}{2\gamma_\omega}\omega_\mu(x)-\frac{m_\varphi^{\ 2}}{2\gamma_\varphi}\varphi_\mu(x). \quad (1)$$

It follows from (1) that the electromagnetic form factors of nucleons and pseudoscalar mesons as well as electromagnetic decays of mesons can all be expressed in terms of measurable quantities γ_ρ, γ_ω, and γ_φ, which couple the vector meson to the photon. In particular, the photoproduction of ρ^0 mesons on complex nuclei can be thought of as via the diagram[2] (Fig. 1) where the photon materializes itself

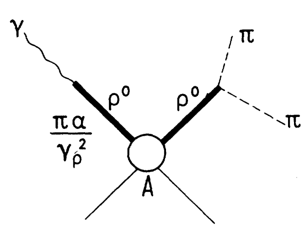

FIG. 1. Feynman diagram for photoproduction of ρ^0 on complex nuclei.

into ρ^0 with a coupling strength $\alpha\pi/\gamma_\rho^{\ 2}$ and the ρ^0 meson subsequently scatters diffractively off the whole nucleus. This diagram for photoproduction of ρ^0 mesons then carries the following two important implications.

(I) A factor $-m^{-2}$ enters from the ρ^0 propagator and the ρ^0 decay spectrum can be shown[2] to be of the form

$$R(m)=(m_\rho/m)^4 f_{\rm BW}(m)\cdots, \quad (2)$$

where $m^2=p_{\pi^+}+p_{\pi^-}$, and where $f_{\rm BW}(m)$ is the relativistic Breit-Wigner mass formula for the decay[3] $\rho^0\to\pi^+\pi^-$:

$$f_{\rm BW}(m)=\frac{1}{\pi}\frac{m_\rho\Gamma(m)}{(m_\rho^{\ 2}-m^2)^2+m_\rho^{\ 2}\Gamma^2(m)}, \quad (3)$$

with

$$\Gamma(m)=\frac{m_\rho}{m}\left[\frac{(\frac{1}{2}m)^2-m_\pi^{\ 2}}{(\frac{1}{2}m_\rho)^2-m_\pi^{\ 2}}\right]^{3/2}\Gamma_0.$$

Equation (2) provides a mass shift of ≈ 20 MeV/c^2 and has been used as an explanation for the difference between the mass $m_\rho=765$ MeV/c^2 (ρ^0 mesons produced from πN interactions[4]) and that of $m_\rho'=740$ MeV/c^2 (ρ^0 mesons produced in photoproduction experiments[5]).

The first purpose of the present experiment is to study the spectrum of $\rho\to\pi^+\pi^-$ in the region of high $\pi^+\pi^-$ invariant mass, $930<m<1130$