SINGLE-PARTICLE STRENGTH IN NUCLEI*

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Sum rules are derived which give the distribution of states consisting of an A-particle nucleus times a single-particle form factor among all physical states of the $(A+1)$-particle system. A similar derivation of Butler's new stripping theory clarifies some of its physical content.

Direct deuteron-stripping reactions examine the single-particle character of states of an $(A+1)$-particle nucleus relative to the ground state of an A-particle nucleus.[1] The projection of the latter state onto an $(A+1)$-particle bound state is normally expressed as a product of a spectroscopic factor times a normalized single-particle wave function, often called the form factor. The sum rules given the distribution of the product of the form factor times the A-particle ground state among all states of the $(A+1)$-particle system. They relate spectroscopic factors to overlaps of the form factor with the elastic-scattering wave functions of the extra nucleon on the A-particle core.

The sum rules are derived by inserting a complete set of states in a matrix element. The new stripping theory of Butler et al.[1] uses a similar technique and we show here that, as far as the spectroscopic factors are concerned, the same complete set must be used. The distribution of "single-particle stripping strength" thus obtained is very analogous to the distribution of single-particle strength given by the sum rule, and we can draw conclusions as to which terms may be neglected.

For simplicity we restrict the discussion to the case of a spin-zero A-particle nucleus and we neglect antisymmetrization. A detailed treatment will be given elsewhere but the important consequences of antisymmetrization are stated finally.

Bound states of the $(A+1)$-particle system are denoted by $\Psi_n(A+1;J_nM_n)$. The projection of the A-particle ground state $\chi_0(A;J_0=M_0=0)$ is defined by

$$\langle\chi_0|\Psi_n\rangle = S_n^{1/2}(j=J_n,l)\varphi_n(\vec{r},\vec{\sigma};j,l,m=M_n), \quad (1)$$

where S_n is the spectroscopic factor and the form factor φ_n is a normalized function of $\vec{r}$, the relative distance of particle $A+1$ from the center of mass of the first A particles. The orbital angular momentum l is restricted to one value by parity, and we shall drop explicit reference to the quantum numbers jlm.

We consider the overlap function

$$\langle\chi_0\varphi_n|\varphi_n\chi_0\rangle = 1. \quad (2)$$

The sum rules are obtained by inserting the unit operator of the $(A+1)$-particle Hamiltonian into the matrix element:

$$1 = \sum_q |\Psi_q\rangle\langle\Psi_q| + \sum_\lambda \int d^3k\, |\Psi_\lambda^{(+)}(\vec{k})\rangle\langle\Psi_\lambda^{(+)}(\vec{k})| + \sum_\alpha |\Psi_\alpha^{(+)}\rangle\langle\Psi_\alpha^{(+)}|. \quad (3)$$

The $\Psi_\lambda^{(+)}(\vec{k})$ are scattering states in which particle $A+1$ is incident with relative momentum $\vec{k}$, on the bound state χ_λ of the A-particle system. Rearrangement channels and channels with three or more incident particles needed to specify a complete set are denoted by $\Psi_\alpha^{(+)}$.

For the scattering state with $\lambda=0$, we define

$$\langle\chi_0|\Psi_0^{(+)}(\vec{k})\rangle = \varphi^{(+)}(\vec{k},\vec{r},\vec{\sigma}). \quad (4)$$

This wave function gives the exact elastic-scattering amplitude for the nucleon on the A-particle nucleus. As long as the elastic channel is open, it can be shown from unitarity and time-reversal invariance that the contributions of all other channels at a given energy are related to that of the elastic channel by functions of the S-matrix elements.[2]

Thus, after using the definitions (1) and (4), we obtain for the sum rules

$$1 = S_n + \sum_{q\neq n} S_q |\langle\varphi_n|\varphi_q\rangle|^2 + \int d^3\vec{k}\, f[S(k)]\, |\langle\varphi_n|\varphi^{(+)}(\vec{k})\rangle|^2, \quad (5)$$

where f is a function of the jl-partial wave S-matrix at the energy given by k.

These sum rules are model independent and give a definition of the energy distribution of single-particle strengths among all states of the $(A+1)$-particle system. They also allow the possibility of a direct calculation of spectroscopic factors from a knowledge of form factors and elastic-scattering wave functions, quantities needed for the usual distorted-wave Born-approximation analyses of stripping experiments. Results of such calculations will be reported elsewhere.

The basic idea of the new stripping theory of Butler et al.[1] is to relate stripping matrix elements going to final bound and continuum neutron states, using their common single-particle strength as defined here. To show this we assume final uncoupled motion of the proton, as in Ref. 1, and define the direct d-p stripping matrix element to a final state, n, as

$$M_n = \langle\langle\Psi_n|\chi_0\rangle\langle\chi_0|\Psi_d^{(+)}\rangle\rangle = S_n^{1/2}\langle\varphi_n|\varphi_{dn}\rangle, \qquad (6)$$

where

$$\Psi_d^{(+)}(1,\cdots,A+1) = \int d^3r_{A+2}\Psi_p^{(-)*}(\vec{k}_p,\vec{r}_{A+2})V_{np}(\vec{r}_{A+2}-\vec{r}_{A+1})\Psi_i^{(+)}(1,\cdots,A+2).$$

The complete wave function for the system with incident deuteron plane wave is $\Psi_i^{(+)}$, and $\Psi_p^{(-)}(\vec{k}_p)$ is some single-particle wave function for the proton. The insertion of a complete set of proton wave functions $\Psi^{(+)}(\vec{k}_p', \vec{r}_{A+2})$ is not essential to the development of the new theory and will not be considered further here. Projections onto states other than the ground state χ_0 of the target in Eq. (6) are taken to give small compound-nucleus contributions to the total matrix element.[1]

In analogy to Eq. (2), we can write Eq. (6) as

$$M_n/S_n^{1/2} = \langle\chi_0\varphi_n|\varphi_{dn}\chi_0\rangle. \qquad (7)$$

As before we insert the unit operator (3) and identify the various terms. Bound-state contributions are given in terms of matrix elements

$$M_q = S_q^{1/2}\langle\varphi_q|\varphi_{dn}\rangle.$$

It is important to notice that, for $q \neq n$, M_q is not the physical direct-reaction matrix element to the final state q because the fixed proton momentum, $\vec{k}_p$, in $\Psi_d^{(+)}$ and φ_{dn} takes physical values for the state n only.

The continuum contribution from $\lambda = 0$ is determined from Eq. (4):

$$M_c = \int d^3k\langle\varphi_n|\varphi^{(+)}(\vec{k})\rangle\langle\varphi^{(+)}(\vec{k})|\varphi_{dn}\rangle.$$

In addition there are continuum contributions from $\lambda \neq 0$ and α which we collectively denote by M_c'. Then the relation between M_c and M_n becomes

$$\frac{M_n}{S_n^{1/2}}[1-S_n] = \sum_{q\neq n} S_q\langle\varphi_n|\varphi_q\rangle\frac{M_q}{S_q^{1/2}} + M_c + M_c'. \qquad (8)$$

In the original derivation of the theory,[1] the bound-state terms $q \neq n$ and the residual continuum terms M_c' were implicitly neglected. Terms corresponding to M_c' can certainly not be neglected in the sum rule (5). On the other hand it could be argued that M_c' is small because of random phases, but this point needs further examination.

However, we can definitely assert that the bound-state terms cannot be neglected. To obtain them, relations similar to Eq. (8) can be derived starting from M_q instead of M_n. The set of linear equations for M_n and M_q can then be formally solved. Without doing this explicitly, we consider the approximation $\varphi_n \approx \varphi_q$. This should not be far out in many physical situations. After the neglect of M_c', the solution of the set of equations is

$$M_n = \frac{S_n^{1/2}}{1-\sum S_q}M_c. \qquad (9)$$

We have thus restored the rough proportionality to S_n of the direct cross sections to a set of final bound states n characterized by the same jl values. Also Eq. (9) has the satisfactory feature that both M_c and $1-\sum S_q$ vanish simultaneously when all the single-particle strength is in the bound states.

For reasons of simplicity, antisymmetrization has not been included here. However, it is clear on both physical and mathematical grounds that its inclusion is essential whenever complete sets of states are being considered. Mathematically the reason is that the unit operator (3) must be given in terms of the antisymmetrical eigenfunctions of an identical-particle

Hamiltonian. Then Eqs. (2) and (7) must also be antisymmetrized. Physically we have neglected the possibility that there is single-particle strength in χ_0 or, in other words, that the single-particle state, φ_n, is partly occupied. The consequences to the sum rule (5) and the relation (9) are what might be expected. The sum rule must give $1-\Lambda$ instead of unity, where Λ is an exchange integral giving the single-particle strength already contained in χ_0. With a reasonable approximation, similar to that used for the other bound states, the denominator in Eq. (9) should be replaced by $1-\sum S_q-\Lambda$.

To conclude, we can say that the insertion of a complete set of states has led to the useful sum rules (5). The new stripping theory can be understood as relating the "single-particle stripping strengths" of a bound state and the continuum.

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[1]S. T. Butler, R. G. L. Hewitt, B. H. J. McKellar, and R. M. May, Ann. Phys. (N.Y.) 43, 282 (1967).

[2]The functions can be obtained by writing the sum rule in terms of the $\Psi^{(-)}$, using the standard relationships between the $\Psi^{(-)}$ and $\Psi^{(+)}$, and finally inverting a matrix.

TWOFOLD INCREASE OF THE HIGH-ENERGY X-RAY FLUX FROM CYGNUS XR-1†

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In a series of measurements using a balloon-borne scintillation counter we have found a significant change in the high-energy (>23 keV) x-ray flux from Cygnus XR-1. On 16 and 25 May 1967 we flew the same detector with which we had measured the spectrum of Cygnus XR-1 from 23 to 97 keV on 19 September 1966. These two May observations were in agreement with each other and were about twice the magnitude of the result reported earlier by us.[1]

There have been earlier reports of a decrease by a factor 3-6 in the 1- to 10-keV x-ray flux from Cygnus XR-1[2-4] relative to the early (self-consistent but unconfirmed) June 1964 measurement by Bowyer et al.[5]

In Ref. 1 we compared our September 1966 data with McCracken's[6] April 1965 data and found the flux he measured to be 1.5 times what we measured and to be greater than ours with at least 97% confidence. Among the reasons why such differences are not usually considered to be significant are the following: (a) Calculations, rather than measurements, of detectors' absolute sensitivity are usually used; (b) pressure altitudes are often not measured precisely and atmospheric absorption corrections are therefore subject to error; (c) malfunctions occur during flight, often with uncertain effects on the results; (d) experimenters do not agree on the correct way to analyze data.

The present comparison of our May 1967 and September 1966 results is not affected by the first problem or the last. In addition, while our September 1966 results contained a possible 15% uncertainty due to discrepancies in pressure measurements, the 16 May experiment included an in-flight comparison of a carefully calibrated pressure sensor with a Winzen barocoder like that relied on in September and consistently showed a negligible difference of 0.05 mbars.

During our May 1967 flights several malfunctions did occur; however, we believe that they produced no unknown effects on our results. These were the following: (a) complete loss of data for one energy band due to failures in the data recording system but not in the detector or amplifiers, (b) corona discharge during a well-defined part (excluded here) of the 16 May flight (stable background levels with normal statistical fluctuations before and af-