be

$$v_{Fe}(0.83) = 9.5\times10^5 \text{ m/sec},$$
$$v_{Fe}(1.67) = 6.7\times10^5 \text{ m/sec},$$
$$v_{Fh}(2.5) = 2.6\times10^5 \text{ m/sec}, \qquad (7)$$

where the number in parentheses is the number of electrons ($\times10^{17}/\text{cm}^3$) in ellipsoids characterized by the listed velocity in the specified direction. It is seen that the velocity determined experimentally [Eq. (6)] is intermediate between the extremes given in Eqs. (7). In fact, the result of Eq. (6) is within 15% of the weighted average of the roots of the products ($v_{Fe}v_{Fh}$) of Eqs. (7). A more quantitative comparison awaits a detailed calculation of the effect for bismuth, similar perhaps to that of Walpole and McWhorter[11] in their studies of nonlocal helicon propagation.

The assumption $\omega_c \gg \omega$ warrants a brief discussion, since the nonlocal magnetosonic waves are observed at low fields where this condition may no longer be valid. First, it is noted that the effect of finite frequency, if present, is to cause the curve of n vs $1/B$ (Fig. 1) to bend in the opposite direction to that observed in the low-field region. Second, from the known cyclotron resonance masses[10] the finite frequency is expected to cause errors of less than ten percent in the determination of Δn over the region covered by Fig. 2.

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ISOMER SHIFTS AND THE SELF-CONSISTENT CRANKING MODEL*

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It has recently become possible to estimate the difference in the rms charge radius between the ground and first excited state of a rotational nucleus. The necessary data can be obtained from Mössbauer measurements of the chemical shift[1] and, independently, from the shift in muonic atoms,[2] of the nuclear gamma-ray energy. It is naturally of interest to compare such results with predictions of nuclear models.

The purpose of this note is to present calculations of charge radii of rotating nuclei based on the self-consistent cranking model.[3,4] (Hereafter, Ref. 4 is referred to as II.) This model depicts collective rotation, in the context of the Hartree-Fock-Bogoliubov theory, as the rotation of a deformed average field additionally stretched by the Coriolis force, which, at the same time, diminishes the Cooper pair correlations. The Coriolis-perturbed variational wave function or the corresponding single-particle density matrix determines the expectation value of any observable as a function of the angular velocity or angular momentum of the ground-state band. The parameters of the calculation are those of Case II given in II. The notation in the ensuing equations is also defined in II.

The radial proton operator R^2 is defined by

$$R^2 = \sum_{\substack{kl \\ \text{(protons)}}} \langle k|r^2|l\rangle a_k^\dagger a_l, \qquad (1)$$

where $r^2=x^2+y^2+z^2$ and the sum runs over a complete set of deformed single-particle states taken, in practice, from the Nilsson model.[5] The change in the charge radius for a rotating nucleus is given by

$$\delta\langle R^2\rangle=\langle R^2\rangle_\Omega-\langle R^2\rangle_{\Omega=0}=\sum_{\substack{kl\\ \text{(protons)}}}\langle k|r^2|l\rangle\delta\rho_{kl}, \quad (2)$$

where the change in the density matrix $\delta\rho$ is given to order Ω^2 in the cranking velocity by Eq. (59a) of Ref. 3, which is sufficient to describe low angular-momentum states. The result (2) is conveniently decomposed as follows:

$$\delta\langle R^2\rangle=\delta\langle R^2\rangle_{\text{crank}}+\delta\langle R^2\rangle_{\text{C ap}}+\delta\langle R^2\rangle_{\text{stretch}}. \quad (3)$$

The first contribution is essentially a pure cranking model result which neglects the changes in the average field and the attenuation of pair correlations. Explicitly, the contribution can be written as

$$\delta\langle R^2\rangle_{\text{crank}}=-\Omega^2\frac{\partial\mathscr{I}_p}{\partial(m\omega_0^2)}-\frac{1}{4}\Omega^2\frac{\partial\mathscr{I}_p}{\partial\lambda_p}\frac{\mu_p}{b_p}, \quad (4a)$$

where $\mathscr{I}_p$ is the proton contribution to the moment of inertia. The special form of the first term of (4a) follows from the use of the Nilsson potential, if one understands that the parameter $m\omega_0^2$ in the spherical part of the Hamiltonian $V=\frac{1}{2}m\omega_0^2R^2$ is to be treated as the independent variable, holding all other parameters fixed. The derivative is explicitly given by

$$\frac{\partial\mathscr{I}_p}{\partial(m\omega_0^2)}=-\sum_{\substack{klm\\ \text{(protons)}}}\frac{\langle k|j_x|l\rangle\langle l|r^2|m\rangle\langle m|j_x|k\rangle}{E_k+E_l}\times\left[\frac{2(U_lV_m+U_mV_l)(U_kU_m+V_kV_m)}{E_l+E_m}+\frac{(U_lU_m-V_lV_m)(U_kV_m-U_mV_k)}{E_k+E_m}\right], \quad (5a)$$

where j_x is the angular momentum operator, U_k and V_k are the occupation amplitudes, and E_k the quasiparticle energies. The first term in (4a) arises from the effect of the Coriolis force on independent quasiparticles, while the second term arises from an adjustment of the Lagrange multiplier λ_p to maintain the correct average number of protons in the presence of the Coriolis perturbation. The derivative $\partial\mathscr{I}_p/\partial\lambda_p$ is defined in II, the remaining quantities are defined by

$$\mu_p\equiv 2\sum_{\substack{k\\ \text{(protons)}}}\frac{\langle k|r^2|k\rangle U_k^2V_k^2}{E_k},$$

$$b_p\equiv\sum_{\substack{k\\ \text{(protons)}}}\frac{U_k^2V_k^2}{E_k}. \quad (5b)$$

Allowing the proton gap Δ_p to diminish with rotation ("Coriolis antipairing") gives the additional contribution

$$\delta\langle R^2\rangle_{\text{C ap}}=\delta\Delta_p[\nu_p-(a_p/b_p)\mu_p], \quad (4b)$$

where

$$\nu_p\equiv\sum_{\substack{k\\ \text{(protons)}}}\frac{\langle k|r^2|k\rangle U_kV_k(U_k^2-V_k^2)}{E_k},$$

$$a_p\equiv\frac{1}{2}\sum_{\substack{k\\ \text{(protons)}}}\frac{U_kV_k(U_k^2-V_k^2)}{E_k}, \quad (5c)$$

and $\delta\Delta_p$ is given to order Ω^2 in II.

The final contribution arises from the change in deformation ("centrifugal stretching") and is given by

$$\delta\langle R^2\rangle_{\text{stretch}}=\delta\beta\left[m\omega_0^2\sum_{\substack{k\\ \text{(protons)}}}\frac{\langle k|r^2Y_{20}|l\rangle\langle l|r^2|k\rangle(U_kV_l+U_lV_k)^2}{E_k+E_l}+\frac{\partial\Delta_p}{\partial\beta}\nu_p+\frac{\partial\lambda_p}{\partial\beta}\mu_p\right], \quad (4c)$$

Table I. Values of $[I(I+1)]^{-1}\delta\langle R^2\rangle/\langle R^2\rangle$ (units of 10^{-4}).

		Theory			
Nucleus	Crank	CAP	Stretch	Total	Experiment
Sm^{152}	0.058	0.020	3.21	3.29	2.3 ± .7[a]
					1.2 ± .3[a]
					0.83 ± 0.28[b]
					0.97 ± 0.12[c]
Sm^{154}	0.026	0.0004	0.331	0.358	
Gd^{154}	0.013	0.003	1.45	1.47	
Gd^{156}	0.002	-0.014	0.281	0.268	0.16 ± 0.06[d]
					0.10 ± 0.03[e]
Gd^{158}	0.008	-0.021	0.201	0.188	0.078 ± 0.033[d]
Gd^{160}	0.012	-0.020	0.136	0.128	
Dy^{160}	-0.006	-0.021	0.163	0.135	
Dy^{162}	0.007	-0.022	0.132	0.116	
Dy^{164}	0.015	-0.022	-0.023	-0.030	
Er^{164}	0.028	-0.018	0.197	0.208	
Er^{166}	0.038	-0.019	0.066	0.085	
Er^{168}	0.038	-0.019	0.071	0.090	
Er^{170}	0.029	-0.017	0.114	0.125	
Yb^{170}	0.056	-0.004	0.160	0.212	0.20 ± 0.06[f]
Yb^{172}	0.056	-0.003	0.203	0.256	
Yb^{174}	0.054	-0.003	0.143	0.193	
Yb^{176}	0.056	-0.004	0.172	0.224	
Hf^{176}	0.085	0.005	0.225	0.315	
Hf^{178}	0.082	0.003	0.137	0.222	
Hf^{180}	0.076	0.002	0.145	0.223	
W^{182}	0.119	-0.009	0.298	0.408	~0.22[g]
W^{184}	0.114	-0.013	0.378	0.479	
W^{186}	0.133	-0.017	0.797	0.914	
Th^{230}	0.019	-0.013	0.327	0.333	
Th^{232}	0.018	-0.010	0.122	0.130	
Th^{234}	0.015	-0.010	0.061	0.066	
U^{230}	0.015	-0.013	0.683	0.685	
U^{232}	0.013	-0.011	0.175	0.178	
U^{234}	0.012	-0.009	0.050	0.052	
U^{236}	0.013	-0.008	0.023	0.027	
U^{238}	0.012	-0.008	0.010	0.015	
Pu^{238}	0.010	-0.011	0.012	0.010	
Pu^{240}	0.009	-0.011	0.0004	-0.0012	
Cm^{244}	0.008	-0.014	-0.015	-0.021	

where $\partial\Delta_p/\partial\beta$, $\partial\lambda_p/\partial\beta$, and $\delta\beta$ are defined (to order Ω^2) in II. The last two terms in Eq. (4c) arise, respectively, from a change in the gap parameter due to the change in deformation and from another consequent adjustment of the proton number Lagrange multiplier.

Using the relation $\Omega^2=(\hbar/\mathscr{I})^2I(I+1)$, valid for low angular-momentum states, the calculated values of $[I(I+1)]^{-1}\delta\langle R^2\rangle/\langle R^2\rangle$ are presented in Table I and compared with the few available measurements for the $I=2^+$ state.

The theory certainly gives the right order of magnitude of the change in radius but the theoretical values seem to be too large, in general, by factors 2 to 3. On the other hand, the experimental data are, with the exception of Yb^{170}, given for cases which are not the best examples of rotational nuclei. Moreover, much of the data is still somewhat crude and possibly subject to some reinterpretation. The calculations may be improved in the future when it becomes possible to perform Hartree-Fock-Bogoliubov calculations for heavy nuclei using realistic forces in place of the schematic quadrupole and pairing force.

The chief qualitative conclusions of the present work is that the main contribution to the change in radius, in most cases, indeed comes from the centrifugal stretching of the nucleus, as is widely assumed. However, the effect of the Coriolis force on independent quasiparticle motion [Eq. (4a)] is by no means entirely negligible, being typically of the order of 25% of the total, although sometimes almost 50% for cases in which the stretching is small.[6] It should be noted that the particle number correction in Eq. (4a) is of the same magnitude as the main term given by Eq. (5a) and usually of opposite sign, accounting for the smallness of the overall contribution. The C ap contribution is also quite small in most cases,

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for, as can be seen from Eq. (4b), $\delta\Delta_p$ is multiplied by the difference of two roughly equal terms which are themselves individually small, being sums over terms which largely cancel from both sides of the Fermi sea. One of the two terms, as was noted, comes from the particle-number correction. It should also be emphasized that the few cases of shrinkage in radius and deformation need not be taken seriously, since one could probably obtain a small expansion with a not unreasonable change of parameters.[4]

It should be emphasized that although the present calculation generally supports the assumption that "centrifugal stretching" is the main effect in the isomer shift, it does not support quantitatively the relation between the change in radius and the change in deformation usually assumed in phenomenological analyses. For example, it is customary to calculate the fractional change in deformation $\Delta\beta/\beta$ or the change in the electric quadrupole moment by assuming that the change in radius comes entirely from a change in ellipticity of a uniform charge distribution. Since the dynamic effects of rotation are neglected, such an estimate must be considered as rather crude. Thus, the most meaningful direct comparison of theory and isomer-shift data should be made with regard to the change in radius, rather than the change in deformation. In this sense, the present microscopic computations are an improvement over earlier ones of $\Delta\beta/\beta$.[4,7]

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SEARCH FOR PARTICLES WITH FRACTIONAL CHARGE $\geq \frac{4}{3}e$ IN COSMIC RAYS*

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The flux of relativistic particles with a charge $\geq \frac{4}{3}e$ was measured in cosmic rays at sea level using a scintillation counter hodoscope. An upper limit equal to 1.3×10^{-10} $\mathrm{cm^{-2}\ sr^{-1}\ sec^{-1}}$ was established at 90% confidence level.

Despite the negative results of a large number of searches for quarks which seek to identify the quark through the magnitude of the fractional charge,[1-3] it is still possible that quarks with charges of $\frac{1}{3}e$ and $\frac{2}{3}e$ exist but have not been observed in these experiments because of an instability with respect to decay into compounds of two quarks, four quarks, or two quarks and an antiquark, through processes such as

$$q \to \bar{q}\bar{q} + (qqq), \quad q \to qqqq + (\bar{q}\bar{q}\bar{q}), \quad \text{or } q \to qq\bar{q} + (q\bar{q}),$$

where (qqq) is a baryon and $(q\bar{q})$ is a meson; the overline signifies the antiparticle.

If the mass of the quark is very large, the three-quark forces must be very large to account for the relatively small mass of the baryons, and the quark-antiquark forces must be very large since the meson is light. Certainly it is not then possible to exclude the possibility that the masses of the $qqqq$, qq, or the $qq\bar{q}$ might be smaller than the mass of a single quark, though much larger than the baryon