MAGNETOSONIC WAVES IN BISMUTH

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We report the observation of magnetosonic waves in bismuth at microwave frequencies. The waves are characterized at low magnetic fields by a phase velocity near the Fermi velocities.

Several writers[1-3] have predicted the existence of magnetosonic (sometimes called magnetoacoustic) waves in semimetals. These waves arise from the fast Alfvén wave which is a compressional wave when propagating perpendicular to the external magnetic field. If the magnetic field is reduced so that the magnetic "pressure" becomes less than the thermal pressure of the carriers, the result is a nonlocal acoustic-like wave propagating at a velocity near the thermal speeds of the carriers. These waves have not previously been observed in solids although Alfvén waves in bismuth have been reported by a number of workers.[4-7] In this paper we report the first observation of magnetosonic waves as manifested by the alteration of the Alfvén-wave dispersion relation at low magnetic fields where the wave velocity becomes comparable to the Fermi velocities of the electrons and holes.

The experiments were done using a conventional X-band microwave spectrometer in which the klystron was locked onto the cavity absorption maximum. The magnetic field of a superconducting solenoid was amplitude modulated by means of a small copper solenoid wound around the microwave cavity. The amplitude-modulated reflected microwave signal was crystal detected at one arm of a "magic tee" bridge and fed directly into a phase-sensitive, lock-in detector.

The sample (approx $1\times8\times8$ mm^3) was placed on the narrow wall of the TE_{103} rectangular microwave cavity in the center of the cavity's long dimension so that the rf magnetic field was maximum along the surface of the sample. Propagation in this configuration is perpendicular to the magnetic field. This fact is important since for $k\perp B$ the fast magnetosonic wave velocity differs from the Alfvén velocity because of nonlocal effects. For $k \parallel B$ the fast magnetosonic velocity is the Alfvén velocity, and the slow magnetosonic wave (essentially the acoustic plasma oscillation) is strongly Landau damped.[3,8] Analysis of the field dependence of the wave vector yields the dispersion relation of the waves.

The dispersion relations for the two Alfvén modes (ordinary and extraordinary) propagating at an angle θ to the magnetic field are

$$\frac{c^2k^2}{\omega^2}=\frac{c^2}{v_A^2}=\sum\frac{\omega_p^2}{\omega_c^2}, \tag{1}$$

$$\frac{c^2k^2}{\omega^2}=\frac{c^2}{v_A\cos^2\theta}=\sum\frac{\omega_p^2}{\omega_c^2}\frac{1}{\cos^2\theta}, \tag{2}$$

where the sum is taken over all types of particles, and ω_p and ω_c are the plasma and cyclotron frequencies, respectively. In writing the simple dispersion relation of Eqs. (1) and (2), it is assumed that $v_A \gg v_{Fe}, v_{Fh}$, $\omega_c\gg\omega$, and $\sum\omega_p^2/\omega_c^2\gg\epsilon_l$ $\omega\tau\gg1$, where v_{Fe} and v_{Fh} are the electron and hole Fermi speeds, ϵ_l is the lattice dielectric constant, and τ is the collision time. [Since the present configuration is one in which the propagation vector k is perpendicular to B, we are concerned only with the ordinary mode above, namely Eq. (1).]

However, if the magnetic field is lowered until the Alfvén-wave velocity becomes comparable with the Fermi speeds of the carriers, the compressional nature of the wave becomes important in determining the dispersion. In this case the wave velocity no longer follows the Alfvén dispersion law, but approaches a constant speed related to the particle thermal speeds, which in gas plasma is the ion acoustic speed.

Yokota[3] has solved the problem for Alfvén waves propagating in the nonlocal limit in a degenerate gas. The result he obtained is

$$v^2=v_A^2+\tfrac{2}{5}v_{Fe}v_{Fh}, \tag{3}$$

assuming spherical Fermi surfaces for both sets of carriers. However, the form of the dispersion relation is expected to be the same even for the anisotropic Fermi surfaces of bismuth.

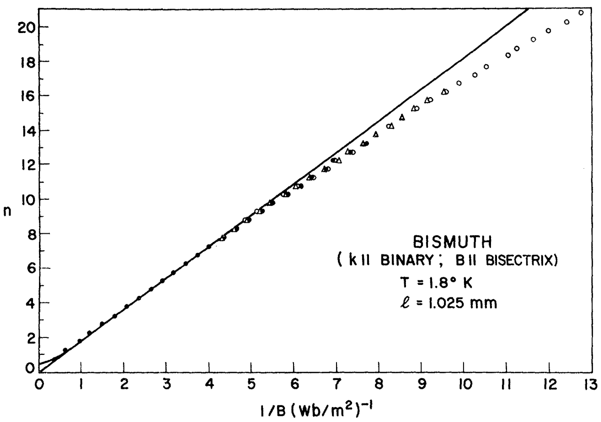

FIG. 1. A plot of the number of wavelengths n in the sample versus reciprocal magnetic induction, $1/B$. The sample is 1.025 mm thick and the microwave frequency is 10.80 GHz. The presence of several different points arises from the different curves taken on different days.

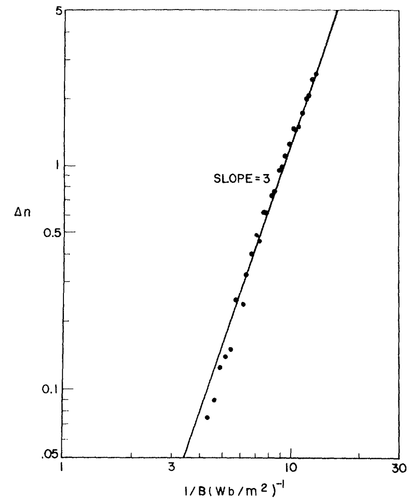

FIG. 2. The deviation Δn from a straight-line fit in Fig. 1 as a function of reciprocal magnetic induction in the low-field region. The slope of the curve has been arbitrarily taken as 3.

Figure 1 shows a plot of an experimentally determined dispersion relation obtained for a sample of bismuth ($k \parallel$binary, $B \parallel$bisectrix) at 10.80 GHz. The vertical axis shows the number of wave lengths n in the sample as a function of the reciprocal of the magnetic induction, $1/B$. The reciprocal-field dependence is linear as expected, in the intermediate-field range (from approximately 0.25 to 1.0 Wb/m^2), where $v_A \gg v_F$ and local conditions prevail. In the high-field region, the points tend to scatter somewhat (because of quantum effects as reported previously[9]) and rise above the linear portion. This rise is due to the large lattice dielectric constant of bismuth, which is indicated by the curved section at very high fields. The linear portion gives the Alfvén speed in the intermediate-field range as

$$v_A = 6.24 \times 10^6 B \quad \text{(m/sec)},$$

where B is in Wb/m^2. This is in good agreement with previously reported values.

It is quite clear that in the low-field region ($B \lesssim 0.2$ Wb/m^2) the experimental points deviate progressively from the straight line fit. This deviation is due to the onset of nonlocal behavior of the magnetosonic waves. Using Eq. (3), it is possible to compare the deviation in wave length from that expected for a pure Alfvén wave.

If we define Δn as $n_A - n_{obs}$, where n_{obs} is the observed number of wave lengths at a magnetic field B and n_A is the number expected on the basis of a pure Alfvén-wave dispersion, i.e., $n_A = fl/v_A$, where l is the sample thickness and f the frequency of the microwave, it is then easily shown by use of Eq. (3) that, for $v_A^2 \gg \frac{2}{5} v_{Fe} v_{Fh}$,

$$\Delta n \cong K B^{-3}, \tag{4}$$

where

$$K = f l v_{Fe} v_{Fh} / 5\gamma^3 \tag{5}$$

and $\gamma = v_A / B$.

Figure 2 shows a log-log plot of Δn, determined from Fig. 1, as a function of $1/B$. A straight line has been drawn through the points with a slope of 3 as predicted by Eq. (4). The fit is quite good except where Δn is small and experimental error is appreciable.

The value of K in Eq. (5) may be determined from Fig. 2, and we find

$$(v_{Fe} v_{Fh})^{1/2} = 3.8 \times 10^5 \text{ m/sec}. \tag{6}$$

Since the exact theory of magnetosonic waves has not been worked out for the Fermi surfaces of bismuth, we are consequently limited to a semiquantitative comparison of velocities. In the binary direction the Fermi velocities of the carriers in bismuth are calculated[10] to be

$$v_{Fe}(0.83) = 9.5\times10^5 \text{ m/sec},$$

$$v_{Fe}(1.67) = 6.7\times10^5 \text{ m/sec},$$

$$v_{Fh}(2.5) = 2.6\times10^5 \text{ m/sec}, \quad (7)$$

where the number in parentheses is the number of electrons ($\times10^{17}/\text{cm}^3$) in ellipsoids characterized by the listed velocity in the specified direction. It is seen that the velocity determined experimentally [Eq. (6)] is intermediate between the extremes given in Eqs. (7). In fact, the result of Eq. (6) is within 15% of the weighted average of the roots of the products ($v_{Fe}v_{Fh}$) of Eqs. (7). A more quantitative comparison awaits a detailed calculation of the effect for bismuth, similar perhaps to that of Walpole and McWhorter[11] in their studies of nonlocal helicon propagation.

The assumption $\omega_c \gg \omega$ warrants a brief discussion, since the nonlocal magnetosonic waves are observed at low fields where this condition may no longer be valid. First, it is noted that the effect of finite frequency, if present, is to cause the curve of n vs $1/B$ (Fig. 1) to bend in the opposite direction to that observed in the low-field region. Second, from the known cyclotron resonance masses[10] the finite frequency is expected to cause errors of less than ten percent in the determination of Δn over the region covered by Fig. 2.

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ISOMER SHIFTS AND THE SELF-CONSISTENT CRANKING MODEL*

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It has recently become possible to estimate the difference in the rms charge radius between the ground and first excited state of a rotational nucleus. The necessary data can be obtained from Mössbauer measurements of the chemical shift[1] and, independently, from the shift in muonic atoms,[2] of the nuclear gamma-ray energy. It is naturally of interest to compare such results with predictions of nuclear models.

The purpose of this note is to present calculations of charge radii of rotating nuclei based on the self-consistent cranking model.[3,4] (Hereafter, Ref. 4 is referred to as II.) This model depicts collective rotation, in the context of the Hartree-Fock-Bogoliubov theory, as the rotation of a deformed average field additionally stretched by the Coriolis force, which, at the same time, diminishes the Cooper pair correlations. The Coriolis-perturbed variational wave function or the corresponding single-particle density matrix determines the expectation value of any observable as a function of the angular velocity or angular momentum of the ground-state band. The parameters of the calculation are those of Case II given in II. The notation in the ensuing equations is also defined in II.

The radial proton operator R^2 is defined by

$$R^2 = \sum_{\substack{kl \\ \text{(protons)}}} \langle k|r^2|l\rangle a_k^\dagger a_l, \quad (1)$$