only even G parity (for example f^0 and ρ). Moreover, the $I=0$ isospin exchange seems to be dominant in reaction (1) since the $I=\frac{1}{2}$ nucleon isobars are produced copiously, but the $I=\frac{3}{2}$ isobar state $N_{3/2}^*(1240)$ is suppressed. In reaction (4) only the $I=1$ isospin-exchange amplitude is allowed, and both $I=\frac{1}{2}$ and $\frac{3}{2}$ isobars are produced. It is interesting to note in this respect that the cross section at 6 GeV/c for

$$\pi^+ p \to N_{1/2}^*(1400)\pi^+, \quad N_{1/2}^* \to \pi N$$

is approximately 34 μb, whereas for

$$\pi^- p \to N_{1/2}^*(1400)\pi^0, \quad N_{1/2}^* \to \pi N$$

it is approximately 8 μb. We have not obtained a value of the inelasticity of the $N_{1/2}^*(1400)$ at the present time since the many-prong (>2) events are not analyzed.

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[1]See the compilation by A. H. Rosenfeld et al., Rev. Mod. Phys. 39, 1 (1967).

[2]See, for example, Marc Ross and Y. Y. Yam, Phys. Rev. Letters 19, 546 (1967). Earlier references can also be found there.

[3]L. D. Roper, Phys. Rev. Letters 12, 340 (1964); P. Bareyre et al., Phys. Letters 18, 342 (1965).

[4]E. Gellert et al., Phys. Rev. Letters 17, 884 (1966).

[5]The criteria used to obtain the sample of events for reactions (1) and (3) were (a) 790 MeV < missing mass < 1090 MeV and (b) consistency of observed or measured bubble density with that required by the kinematic fit. For reactions (2) and (4), the criteria were (a) −300 MeV < missing mass < 450, (b) consistency of observed or measured bubble density with that required by the kinematic fit, (c) error in the missing mass < 500 MeV, and (d) χ^2 probability 5%. In all the $\pi^- p$ film and in 20% of the $\pi^+ p$ film, no preselection was used to reduce the number of elastic events that were measured. However, in the remaining 80% of $\pi^+ p$ film, two-pronged events found in scanning which had an identifiable proton were not measured. Since this preselection to remove elastic events also removed events of reaction (2), the event sample and equivalent cross section for reaction (2) presented here are based on the 20% of $\pi^+ p$ film in which no preselection was used.

[6]No $I=2$ $\pi^+\pi^+$ resonance is observed in reaction (1). This gives an upper limit of 15 μb for production of any $\pi^+\pi^+$ resonance, assuming a width of approximately 100 MeV and a mass less than 2.2 GeV, with 99% confidence level.

[7]The $N_{1/2}^*(1525)$ is not clearly resolved from the broad $N_{1/2}^*(1688)$ in our data.

[8]The cluster of events in the higher $\pi^- p$ mass region of Fig. 1(c) is due to reflections of $\pi^-\pi^0$ resonant states (ρ^- and g_1^-).

[9]A number of fits have been made to each distribution of which one is shown in the figures. In each fit the assumed background shape is fixed, the various fits differing in the amount of peaking in the low-mass region of the background. The mass values obtained are insensitive to the background changes, while their widths and intensities vary widely.

[10]See, for instance, J. D. Jackson, Nuovo Cimento 34, 1644 (1964).

[11]K. J. Foley et al., Phys. Rev. Letters 19, 397 (1967). Our results of γ for events in the $N_{1/2}^*(1400)$ region with $t \leq 0.2$ GeV^2 are 6.0 ± 1.7 and 6.3 ± 1.5 GeV^{-2} for reactions (1) and (4), respectively.

MAGNETIC MOMENTS, FORM FACTORS, AND MASS SPECTRUM OF BARYONS*

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We have been able to derive and correlate the following baryon properties: (a) absolute values of the magnetic moments, (b) form factors $G_M(t)$ and $G_E(t)$, (c) mass spectrum, and (d) decay rates, in a relativistic theory based on the unitary representations of the dynamical group O(4, 2) ~ SU(2, 2). We are then able to make a number of new predictions.

The starting point of the theory is a conserved four-vector current operator j_μ constructed from the generators of the dynamical group and from the momentum operators $P_\mu = (p'+p)_\mu$ and $q_\mu = (p'-p)_\mu$, where p_μ' and p_μ are the baryon momenta in a vertex. In a recent paper[1] where the general theory is described we have considered a simple current operator that gives positive magnetic moments and "physical" mass spectra.[2] No attempt was made there to fit the experimental properties of the hadrons with the theory. In this paper we shall consider, as a continuation of the previous work, the most general linear conserved current and fit the theory to the experiment.

General current and requirements on its matrix elements. – The most general linear conserved current is of the form

$$j_\mu = (NN')^{-1} \times(\alpha_1\Gamma_\mu + \alpha_2 P_\mu + \alpha_3 P_\mu S + i\alpha_4 L\mu\nu q^\nu), \quad (1)$$

where N, N' are normalization factors to be determined; Γ^μ, the algebraic current operator constructed on the representation of O(4, 2); $S = L_{46}$, the Lorentz scalar generator in O(4, 2); and P_μ and q_ν have been defined above.

We require the following physical conditions on the matrix elements of the current operator:

(1) Constancy of charge. All levels of the O(4, 2) tower should have the same charge q, i.e., between the physical states $|\bar{n}\rangle$ we have

$$\langle\bar{n}|j_0|\bar{n}\rangle = q = \text{const, for all } |\bar{n}\rangle. \quad (2)$$

Note that the physical states are the "tilted" O(4, 2) states[1]

$$|\bar{n}\rangle = \mathfrak{N}^{-1}\exp(i\theta_n L_{45})|n\rangle. \quad (3)$$

(2) Current conservation. This means that for a vertex function, $(p'-p)_\mu j^\mu = 0$ and can be expressed in terms of the boosted O(4, 2) states $|\bar{n};p\rangle = e^{i\vec{\xi}\cdot\vec{M}}|\bar{n}\rangle$ as

$$M_{n'}\langle\bar{n}'|j_0|\bar{n};p\rangle = M_n\langle\bar{n}';-p|j_0|\bar{n}\rangle. \quad (4)$$

It follows that in the limit $\xi\to 0$ the "tilted states" are orthogonal with metric j_0: $\langle\bar{n}'|j_0|\bar{n}\rangle = \delta_{n'n}$.

Consequences. – From (1) and (2) we obtain, as in I,

$$[N(n)]^{-2} \times\{\alpha_1 n\cosh\theta_n + 2M_n\alpha_2 + 2M_n\alpha_3\sinh\theta_n\} = q, \quad (5)$$

and from (4) we obtain, after some manipulations described in I, the mass spectrum

$$M_n^2 = \left[2\left(\alpha_3^2 + \frac{\alpha_2^2}{n^2}\right)\right]^{-1}\left\{\alpha_1^2 + 2\beta\alpha_3 + \frac{2\gamma\alpha_2}{n^2} + \left[\left(\alpha_1^2 + 2\beta\alpha_3 + \frac{2\gamma\alpha_2}{n^2}\right)^2 - 4\left(\beta^2 + \frac{\gamma^2}{n^2}\right)\left(\alpha_3^2 + \frac{\alpha_2^2}{n^2}\right)\right]^{1/2}\right\} \quad (6)$$

and the "tilting angle"

$$\sinh^{-1}\theta_n = \frac{1}{n}\frac{\gamma - \alpha_2 m^2}{\beta - \alpha_3 m^2}. \quad (7)$$

Note that so long as $\alpha_3 \neq 0$, the mass spectrum has a saturation value for $n\to\infty$ at

$$m_{\text{sat}} = (2\alpha_3^2)^{-1}[\alpha_1^2 + 2\beta\alpha_3 + \alpha_1(\alpha_1^2 + 4\beta\alpha_3)^{1/2}]. \quad (8)$$

If we insert (7) into (5) we can determine the normalization factor $N(n)$ if $q\neq 0$. Otherwise, (5) is a consistency equation.

Next we evaluate for the current (1) the magnetic and electric form factors of the ground state $j^P = \frac{1}{2}^+, n = \frac{3}{2}$. There is no loss of generality in assuming $N(\frac{3}{2}) = 1$. For this state, if we denote

$$F_\mu(t) = \langle\bar{n}|j_\mu e^{i\xi\cdot M}|\bar{n}\rangle = \langle\bar{n}=\tfrac{3}{2}, j=\tfrac{1}{2}^+|j_\mu e^{i\xi L_{35}}|\bar{n}=\tfrac{3}{2}, j=\tfrac{1}{2}^+\rangle, \quad (9)$$

the form factors are given by

$$G_M(t) = F_1(t)\sinh\tfrac{1}{2}\xi;$$
$$G_E(t) = F_0(t)\cosh^{-1}\tfrac{1}{2}\xi = F_3(t)\sinh^{-1}\tfrac{1}{2}\xi. \quad (10)$$

The computation of the matrix elements (9) follows the usual procedure[1,3] and one obtains the following results:

$$G_M(t) = \mu\left(1 - \cosh^2\theta\frac{t}{4m^2}\right)^{-2},$$
$$\mu = -\tfrac{1}{2}\alpha_1\cosh\theta - M\alpha_4, \quad (11)$$

and

$$G_E(t) = q\left(1-\cosh^2\theta\frac{t}{4m^2}\right)^{-2} + \frac{t}{4m^2}\left(1-\cosh^2\theta\frac{t}{4m^2}\right)^{-3} \times\left(B_1 + B_2\cosh^2\theta\frac{t}{4m^2}\right), \quad (12)$$

with

$$q=\tfrac{3}{2}\alpha_1\cosh\theta+2M\alpha_2+3M\alpha_3\sinh\theta,$$

$$B_1=-q-\mu(4\sinh^2\theta+3)+4\alpha_3M\sinh\theta\cosh^2\theta,$$

$$B_2=q+3\mu-4M\alpha_3\sinh\theta.$$

Determination of the parameters. – We assume that the theory outlined above applied to both the proton and the neutron towers. It contains six parameters for the neutron tower and six parameters for the proton tower. It is possible, by making certain assumptions about the SU(2) transformation properties of these coefficients, to reduce their number. However, we do not attempt to do this here. On the contrary, we want to infer these properties *a posteriori* from the results. The 12 input parameters are then the following:

(a) The mass of the ground state of the tower: $n=\frac{3}{2}$, $m_{3/2}=0.94$ BeV (for both neutron and proton tower).

(b) The charges and the magnetic moments of the ground state ($q=0$, $\mu=-1.91$ for n, and $q=1$, $\mu=2.79$ for p).

(c) The tilting angle θ, the same for n and p and fixed to satisfy $\cosh^2\theta=5.0$. This gives the experimental "singularity" in the magnetic form-factor expression.

(d) One point on the curve $G_E(t)$ for n and p.

(e) One point on the mass spectrum curve for n and p.

These requirements give the following values (for the solid mass curve, Fig. 2): For the proton tower,

$$\alpha_1=-6.29,\ \alpha_2=7.46,\ \alpha_3=1.43,$$

$$\alpha_4=4.48,\ \beta=-4.02,\ \text{and}\ \gamma=2.68;$$

for the neutron tower,

$$\alpha_1=4.42,\ \alpha_2=-4.83,\ \alpha_3=-1.02,$$

$$\alpha_4=-3.20,\ \beta=2.81,\ \text{and}\ \gamma=-1.47.$$

If we use a slightly different mass curve (see dashed line in Fig. 2), the parameters α_1, α_2, β, and γ change slightly:

$$\alpha_1=-5.79,\ \alpha_2=6.57,\ \beta=-3.60,\ \text{and}\ \gamma=2.15.$$

Predictions. – (1) Double-pole form of the magnetic form factors, in excellent agreement with the experiment [Eq. (11)].[4]

(2) The exact equality

$$G_M^{\ p}(t)/\mu_p=G_M^{\ n}(t)/\mu_n,\ \text{for all}\ t. \qquad (13)$$

(3) A definite deviation of $G_E^{\ p}(t)$ from $G_M^{\ p}(t)/\mu$, in agreement with experiment (see Fig. 1); $G_E^{\ p}(t)$ has a minimum value of about -0.0108 at $t=-7.89$ and approaches 0 as $t\to\infty$.

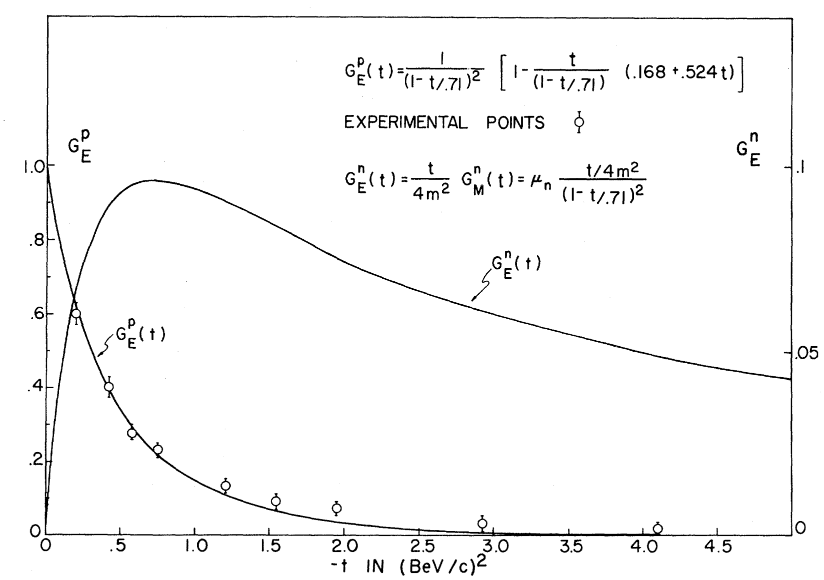

FIG. 1. Electric form factors of proton and neutron.

(4) The equality

$$G_E^{\,n}(t)=\frac{t}{4m^2}G_M^{\,n}(t)$$
$$=\mu_n\frac{t}{4m^2}\left(1-\cosh^2\theta\frac{t}{4m^2}\right)^{-2}, \qquad (14)$$

which has been speculated by experimentalists,[5] and which seems to be in agreement with experiment. [This follows immediately from Eq. (13) for the adopted choice $B_1=-B_2$.] The neutron electric form factor has a maximum at $t=-0.71$ $(\mathrm{BeV}/c)^2$,

$$G_{E,\max}^{\,n}(t=-0.71)=0.096,$$

which agrees with experiment.[6]

(5) All form factors for arbitrary transitions $N_A^*\to N_B^*+\gamma$ for the $I=\frac{1}{2}$, $Y=1$ states, completely determined. The measurement of form factors of such processes as $e+p\to N_{1/2}^*(1400)+e\to N+\pi+e$ could provide a crucial test for the theory.

(6) A lot of new states with definite spin and parity doubling as determined by the unitary (most degenerate) fermion representation of the group O(4, 2). (See the weight diagram by Barut and Kleinert.[7])

(7) The partial decay widths of the resonances, determined with the tilting angle of the form of Eq. (7). This has been reported separately.[8]

(8) The mass spectrum of the $I=\frac{1}{2}$, $Y=1$ baryon resonances. Figure 2 shows two possible fits. It should be remarked that we have not yet incorporated any splitting of levels in j with the same n. The mass formula depends only on n and is the mass of the degenerate O(4) multiplet before splitting.

(9) A definite saturation value, Eq. (8), for the mass spectrum. For the above choice of parameters $M_{\rm sat}\cong 3.7$ GeV. But again this value will depend, of course, on the way the spin-dependent terms enter into the mass spectrum.

(10) Parameters of the isoscalar and isovector towers, evaluated from proton and neutron towers, as follows: for the isoscalar tower,

$$\alpha_1^{\,s}=-0.94,\quad \alpha_2^{\,s}=1.12,\quad \alpha_3^{\,s}=0.21,\quad \alpha_4^{\,s}=0.64;$$

for the isovector tower,

$$\alpha_1^{\,v}=-5.36,\quad \alpha_2^{\,v}=6.15,\quad \alpha_3^{\,v}=1.23,\quad \alpha_4^{\,v}=3.84.$$

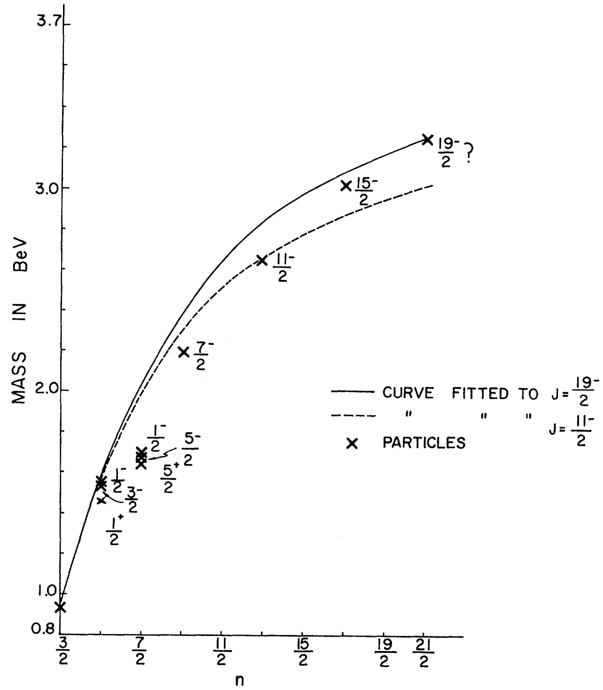

FIG. 2. Mass spectrum of the $I=\frac{1}{2}$, $Y=1$ baryon tower.

The ratio of the current coefficients for isovector and isoscalar case is close (within 15%) to the values obtained from SU(4) plus vector-meson dominance models,[8,9] namely

$$(5/3)m_\rho^{\,-1}/\tfrac{1}{3}m_\omega^{\,-1}\cong 5.1.$$

But these latter theories give magnetic moments which are also 15% too small.

There is another current component that couples the nucleon tower to the $I=\frac{3}{2}^-$ Δ tower. If this is also taken into account the values of the constants obtained above change only slightly. The electric form factor $G_E^{\,p}(t)$ can be fitted even better and other predictions remain essentially the same. This new current describes the transitions $N\to N^*(1236)+\gamma$.

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[1]For the general theory and earlier references see A. O. Barut, D. Corrigan, and H. Kleinert, to be published. Hereafter referred to as I.

[2]By physical mass spectrum we mean one which is increasing with j as to be applicable to baryons. The simple Majorana theory predicts in this context an unphysical spectrum of the form $1/(j+\frac{1}{2})$.

[3]See A. O. Barut and H. Kleinert, Phys. Rev. 160, 1149 (1967); H. Kleinert, Phys. Rev. 163, 1807 (1967), and thesis, University of Colorado, 1967 (unpublished).

[4]See the latest published curve up to $|t| \cong 10$ GeV/c^2 [W. Albrecht et al., Phys. Rev. Letters 18, 1014 (1967)], and the more recent measurements from Stanford up to $|t| \approx 25$ (GeV/c)^2 (R. Taylor et al., to be published).

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[6]J. R. Dunning et al., Phys. Rev. 141, 1286 (1966).

[7]A. O. Barut and H. Kleinert, Phys. Rev. 161, 1464 (1967).

[8]A. O. Barut and K. C. Tripathy, Phys. Rev. Letters 19, 108, 918 (1967).

[9]J. Schwinger, Phys. Rev. Letters 18, 923 (1967).

GENERALIZED DECK EFFECT AND $\bar{K}$**(1300) PRODUCTION IN $K^- + p \to K^- + \pi^+ + \pi^- + p$ AT 5.5 BeV/c †

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We report on a comparison of the generalized Deck effect (discussed recently by Ross and Yam[1]) with our data from a K^-p experiment in which the 30-in. Midwestern Universities Research Association hydrogen bubble chamber was exposed to a 5.5-BeV/c separated K^- beam at the zero-gradient synchrotron of the Argonne National Laboratory. In a sample of four-prong events (exposure equivalent to 1 event/0.3 μb) we identified 3368 examples of the reaction

$$K^- + p \to K^- + \pi^+ + \pi^- + p. \quad (1)$$

1304 of these events with an invariant mass, $M(K^-\pi^+)$, in the interval 0.84-0.94 BeV are due in large part to the reaction

$$K^- + p \to \bar{K}^{*0} + \pi^- + p, \quad \bar{K}^{*0} \to K^- + \pi^+. \quad (2)$$

The background to $\bar{K}^*$ events is estimated to be less than 15%; it is mostly associated with $N^{*++}(1236)$ production. There is little if any ρ^0 [<~7% of reaction (1)] or $Y^{*0}(1520)$, $Y^{*0}(1770)$, and/or $Y^{*0}(1815)$ (all Y^{*0} <~6%).

The $M(\bar{K}^{*0}\pi^-)$ distribution shows a broad enhancement in the mass region 1.2-1.5 BeV.[2] Part of this enhancement may be shown (by a detailed study of decay angular distributions[3]) to be due to $\bar{K}$**(1430) production. The remainder of the enhancement is presumably due to the "Deck" background as well as to possible other $\bar{K}^*\pi$ resonances. The purpose of the present study was to compare the data with the background predicted by the Ross-Yam model in order to see whether the data could or could not be understood without invoking the existence of genuine resonances.

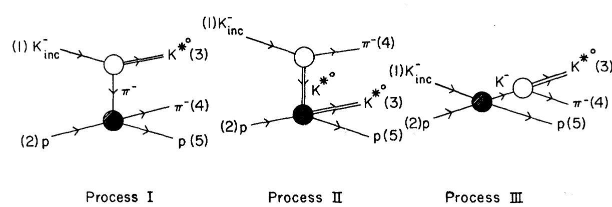

FIG. 1. Three processes associated with the dissociation, $K \to K^* + \pi$, considered in the model.

The Ross and Yam model we want to consider involves three processes corresponding to the dissociation of K^- into $\bar{K}^{*0}$ and π^- with (virtual) elastic scattering of each of the three particles with the target proton, as shown in Fig. 1. In addition to the usual Deck model (process I in Fig. 1) the model includes two other processes and mutual interferences. The relative phases of the amplitudes are determined by means of approximating each (virtual) elastic-scattering amplitude by the corresponding asymptotic form associated with the vacuon exchange. For example, the invariant amplitude for the process II in Fig. 1 is

$$\mathfrak{M} = gF(m_1^2, m_4^2, y^2)\frac{1}{y^2 - m_{K^*}^2}[-2iM_{35}q_{35}\sigma_T(\bar{K}^{*0}p)e^{\frac{1}{2}A(\bar{K}^{*0}p)\tau}]\left[p_{14} - \frac{p_{14}\cdot y}{M_{K^*}^2}\right]\cdot\epsilon(\lambda),$$