SOME CONSIDERATIONS ON THE INTERMEDIATE-BOSON HYPOTHESIS AND *CP* NONCONSERVATION

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The consequences of the assumptions that (1) the intermediate boson W has pair strong interactions and (2) $W^{\pm}=a^{\pm}$ particles of Lee (thus leading to CP nonconservation) are studied and found to lead to a consistent picture.

It is the purpose of the following note to point out that the propositions that (1) the intermediate vector boson W has pair strong interactions[1] with known hadrons and (2) $W^{\pm}=a^{\pm}$ particles of Lee[2] lead to a consistent theoretical and experimental picture. The (a^+, a^-) particles, postulated in connection with electromagnetic CP nonconservation, can be pair produced in strong interactions with the known hadrons, have charge $Q_K=(+e, -e)$, and satisfy[3]

$$C_{\mathrm{st}}|a^+\rangle=|a^+\rangle,\ \ C_{\mathrm{st}}|a^-\rangle=|a^-\rangle,$$

$$|a^-\rangle\equiv C_{\gamma}T|a^+\rangle;$$

where C_{st} is the particle-antiparticle conjugation operator, C_γ the charge conjugation operator, and T $(=T_\gamma)$ the time reversal operator. For simplicity, we shall assume that $a^{\pm}$ and the associated K_μ current are both unitary singlets $(I=N=S=0)$. As pointed out by Lee,[2] unitary (and isotopic) singlet current K_μ can be readily contructed from the fields associated with the a particles.

(1) The W-pair theory.—A particular model of W with quadratic strong interactions, the triplet version with nonzero triality, has been considered some time back by Ryan et al.[1] and Pepper et al.[1] and found consistent with experiment. In the present context, postulate (2) supplies a physically meaningful additional quantum number Q_K in place of triality conservation of the triplet version.[1] Of course both Q_K conservation and triality conservation will forbid such unwanted strong reactions as $W^++p\rightarrow W^-+p+2\pi^+$ which can lead to unobserved fast double β decay. The existence of W pairs with strong interactions will also circumvent the question raised by Dyson[4] concerning the nonexistence of a weak charged boson, since the W is now on a similar footing with π, K, and deuteron in having strong interactions. Such $(W\overline{W})$ pairs can presumably generate excited W states as well via, say, a bootstrap-type mechanism.

(2) Mismatch of discrete symmetries in $W^{\pm}=a^{\pm}$ hypothesis.—The usual mismatch theory[5] takes the following form:

$$\begin{array}{ccc} H_{\mathrm{st}} & H_\gamma & H_{\mathrm{wk}} \\ \hline C_{\mathrm{st}} \neq & C_\gamma \neq & C_{\mathrm{wk}} \\ P_{\mathrm{st}} \neq & P_\gamma \neq & P_{\mathrm{wk}} \\ T_{\mathrm{st}} \neq & T_\gamma \neq & T_{\mathrm{wk}}=T_{\mathrm{st}} \end{array} \tag{1}$$

Here each interaction Hamiltonian H_i ($i=$ strong, γ, and weak) is separately invariant under its own P_i, T_i, and C_i, while the CPT theorem takes the form $C_{\mathrm{st}}P_{\mathrm{st}}T_{\mathrm{st}}=C_\gamma P_\gamma T_\gamma=C_{\mathrm{wk}}P_{\mathrm{wk}}T_{\mathrm{wk}}$. A specific mismatch model of $H_{\mathrm{st}}(\mathrm{SU}(3))$ [the SU(3)-invariant part of H_{st}] and H_γ in terms of quark fields and spin-1, unitary-singlet, a-particle field has been presented by Lee.[2]

There is, however, an important new feature introduced when we add on the hypothesis $W^{\pm}=a^{\pm}$. This is most aptly illustrated by examining the implications of the assumption given in Eq. (1) that $T_{\mathrm{st}}=T_{\mathrm{wk}}$ (or equivalently, $C_{\mathrm{st}}P_{\mathrm{st}}=C_{\mathrm{wk}}\times P_{\mathrm{wk}}$) for semiweak vertices like $(\bar{p}nW)$, $(\Lambda\bar{p}W)$, $(\bar{l}\nu_l W)$, etc. For instance, a straightforward interpretation of operation $C_{\mathrm{st}}P_{\mathrm{st}}$ $(=C_{\mathrm{wk}}P_{\mathrm{wk}})$ to $\bar{p}nW^+=\bar{p}na^+$ would suggest violation of charge conservation!

Following Lee and Wick,[6] we consider first

$$H=H_{\mathrm{free}}+H_{\mathrm{st}}(\mathrm{SU}(3))+H_{\mathrm{wk}}, \tag{2}$$

where H_{wk} is the usual current-current weak interaction, while H_{free} is the free-particle Hamiltonian in which the masses of the different hadrons within the same SU(3) multiplet are considered to be the same, and the masses of all leptons are set equal to zero.

The significance of discrete operations in semiweak interactions can be best illustrated by writing the weak current J_λ in terms of quark fields $\psi_1(\vec{r},t)$, $\psi_2(\vec{r},t)$, and $\psi_3(\vec{r},t)$ as

$$J_\lambda = i\psi_2'^\dagger\gamma_4\gamma_\lambda(1+\gamma_5)\psi_1 + i\psi_e^\dagger\gamma_4\gamma_\lambda(1+\gamma_5)\psi_{\nu_e} + i\psi_\mu^\dagger\gamma_4\gamma_\lambda(1+\gamma_5)\psi_{\nu_\mu}, \qquad (3)$$

where $\psi_2' = \cos\theta\psi_2 + \sin\theta\psi_3$ and θ is the Cabibbo angle. The Hamiltonian for semiweak interaction (with appropriate coupling g) is

$$H(W^\pm) = H(a^\pm) = gJ_\lambda W_\lambda^* + \text{H.c.} = gJ_\lambda a_\lambda^* + \text{H.c.}, \qquad (4)$$

where W_λ (a_λ) are the operators describing the charged W (a-particle) fields; both W_λ^* (a_λ^*) and J_λ^* are related to their Hermitian conjugate operators $W_\lambda^\dagger$ $(a_\lambda^\dagger)$ and $J_\lambda^\dagger$ by $W_\lambda^* = W_\lambda^\dagger$ $(a_\lambda^* = a_\lambda^\dagger)$ and $J_\lambda^* = J_\lambda^\dagger$ for $\lambda \neq 4$ while $W_4^* = -W_4^\dagger$ $(a_4^* = -a_4^\dagger)$ and $J_4^* = -J_4^\dagger$.

Since we assume the CPT theorem, the discussion of $C_{\text{st}}P_{\text{st}} = C_{\text{wk}}P_{\text{wk}}$ can be more conviently discussed in terms of T_{st} and T_{wk}. We define

$$T_{\text{wk}} J_\mu T_{\text{wk}}^{-1} = -J_\mu^*(\vec{r}, -t),$$

$$T_{\text{wk}} W_\mu T_{\text{wk}}^{-1} = -W_\mu^*(\vec{r}, -t)$$

$$\text{or } T_{\text{wk}} a_\mu T_{\text{wk}}^{-1} = -a_\mu^*, \qquad (5)$$

and choose $T_{\text{st}} = T_{\text{wk}}$. The T_{st} operates on the elementary quark fields ψ_1, ψ_2, and ψ_3 as

$$T_{\text{st}}\begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\end{pmatrix} = \gamma_1\gamma_2\gamma_3 U\begin{pmatrix}\psi_1(\vec{r},-t)\\ \psi_2(\vec{r},-t)\\ \psi_3(\vec{r},-t)\end{pmatrix} = \gamma_1\gamma_2\gamma_3\begin{pmatrix}\psi_2'\\ \psi_1'\cos\theta + \psi_3'\sin\theta\\ \psi_1'\sin\theta - \psi_3'\cos\theta\end{pmatrix}, \qquad (6)$$

$$\psi_1'(\vec{r},t) = \psi_1(\vec{r},t), \quad \psi_2'(\vec{r},t) = (\cos\theta)\psi_2 + (\sin\theta)\psi_3, \quad \psi_3'(\vec{r},t) = -(\sin\theta)\psi_2 + (\cos\theta)\psi_3.$$

Here the γ matrices are in the Majorana representation, $U(\theta)$ is an element of SU(3) with explicit form

$$U = \begin{pmatrix}\theta & \cos\theta & \sin\theta\\ \cos\theta & -\sin^2\theta & \sin\theta\cos\theta\\ \sin\theta & \sin\theta\cos\theta & -\cos^2\theta\end{pmatrix},$$

while θ is the Cabibbo angle. For the leptons, the T_{wk} (or T_{st}) transformation exchanges[5,6] $e_L \leftrightarrow \nu_e$, and $\mu_L \leftrightarrow \nu_\mu$:

$$T_{\text{wk}}\psi_l T_{\text{wk}}^{-1} = \gamma_1\gamma_2\gamma_3\psi_{\nu_l}(\vec{r}, -t), \quad l = e, \mu. \qquad (7)$$

It is evident that Eqs. (3), (6), and (7) do lead to consistency with transformation of J_μ under T_{wk} as delineated in Eq. (5).

The semiweak vertices [Eq. (4)] have the desired transformation property under T_{wk}, while H_{wk} satisfies $T_{\text{wk}}H_{\text{wk}}(t)T_{\text{wk}}^{-1} = H_{\text{wk}}(-t)$. Since $U(\theta)$ is an element of SU(3), the choice of T_{st} $(= T_{\text{wk}})$ as set out in Eq. (6) in terms of the quark fields leads to $T_{\text{wk}}H_{\text{st}}(t)T_{\text{wk}}^{-1} = H_{\text{st}}(-t)$ where H_{st} is the SU(3)-invariant part of the strong-interaction Hamiltonian. Model examples of H_{st} and H_{wk} in terms of quark and a-particle fields, satisfying these conditions, can be taken from those of Ref. 2 and will not be discussed further here. Note that T_{wk} defined in the present context is the same as $C_{\text{wk}}T_{\text{wk}}$ of Lee and Wick.[6]

In a realistic situation, the Hamiltonian H of Eq. (2) is supplemented by electromagnetic and SU(3)-breaking Hamiltonians H_γ and H_{st}'. These interactions break the symmetry of the model under discussion, incurring mismatch to such hitherto exact relations like the condition $T_{\text{wk}} = T_{\text{st}}$.[7] The typical electromagnetic mismatch correction[5] to $T_{\text{wk}} = T_{\text{st}}$ is of order α. For H_{st}', we can characterize the mismatch correction to $T_{\text{wk}} = T_{\text{st}}$ by the estimate[8]

$$\theta \times [\text{SU(3) breaking}]. \qquad (8)$$

The Cabibbo angle θ is of order $\frac{1}{4}$, while the theoretical expectations are that SU(3) breaking is $\lesssim 10\%$ (such anomalies as the K-π mass difference are to be understood perhaps in terms of the higher mass-scale picture[9] proposed for quarks, triplets, and possibly $a^\pm$ as well). Hence (8) can be of order α also.

The particular form chosen in (8) is dictated by the fact that in the limit $\theta \to 0$, $T_{\text{st}} = T_{\text{wk}}$ holds even if SU(3) is broken [as long as SU(2) is good]. This is because in this limit the transformation T_{st} does not involve mixing between (p, n) and λ quarks. (8) is the simplest expression[8] which takes this into account. Note that (8) vanishes for either $\theta=0$ or [SU(3) breaking]=0; thus it represents a very lenient condition. However, the quark model makes it immediately obvious that $T_{wk}=T_{st}$ cannot be true exactly when both SU(3) is broken and $\theta\neq 0$.

As an illustration, let us consider again the classic CP-nonconserving decays $K_L^0\to 2\pi$. Our theory states that in the presence of H_γ and H_{st}', the observed CP nonconservation in the neutral K-meson complex is due to the presence of second-order terms

$$(\theta H_{st}')H_{wk}+H_\gamma H_{wk}, \tag{9}$$

where H_{wk} is the usual $\Delta S\neq\pm2$ weak interaction and is invariant under CP (or $T_{wk}=T_{st}$). The SU(3)-breaking H_{st}' conserves isospin, hence the first term of (9) contributes a mismatch to $T_{wk}=T_{st}$ characterized by the "coupling" parameter given by (8) (of order α) and observes the $\Delta I=\frac{1}{2}$ rule. The second term of (9) is of form $J_\mu K_\mu H_{wk}$ where J_μ observes the $\Delta I=0, 1$ rule, while by our assumption K_μ respects the $\Delta I=0$ rule. Hence this term allows for both $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ components to the $K_L^0\to2\pi$ modes—both contributing mismatch to $T_{wk}=T_{st}$ at level of the fine structure constant α. We therefore expect (8) to contribute to phenomenological parameters[10,11] ϵ and ϵ' at the order of $O(\alpha)$.

Remarks.—(a) The procedure of defining T_{st} [Eq. (6)] as an operation transforming the proton quark to a linear combination of neutron and lambda quarks $\psi_1\to\psi_2'$ is somewhat unusual. However, we have only used the freedom[6] that is available in the limit of SU(3) when electromagnetic interactions have been switched off [cf. Eq. (2)].

(b) Propositions (1) and (2) lead not only to a mismatch between T_{st} and T_γ but also between T_{st} and T_{wk} via the SU(3)-breaking strong interactions and Cabibbo-angle considerations. This picture is physically reasonable since imposition of the condition $W^\pm=a^\pm$ must necessarily entail some restriction on our ability to define T_{st} and T_{wk}—just as the existence of $a^\pm$ itself imposes a mismatch between T_{st} and T_γ.

(c) The quark model is used above only to give a simple and clear idea of the T_{wk} and T_{st} symmetries. One could equally well work directly with the currents J_μ, the properties of which can be specified by referring to Eq. (3).

(d) We have assumed that a^+ and a^- are both unitary singlets ($I=N=S=0$). In this case the observed $\Delta I=\frac{1}{2}$ rule for strangeness-changing nonleptonic decays has to be understood in terms of an ad hoc assumption of octet dominance. To the extent that we have made no commitments on whether neutral members to $a^\pm$ exist, our theory has no neutral currents.

(3) Experimental consequences.—The CP-nonconserving consequences of the present theory are in a large number of cases identical with those due to an isosinglet K_μ current itself. These have been summarized by Lee[2,5] and are compatible with data. The neutron electric dipole moment depends on the coupling of $\Delta S=0$ nonleptonic weak interactions. This latter is in principle estimable from the Cabibbo current-current Hamiltonian used here. However, there may be dynamical suppression of this coupling as is the case for the $\Delta T=\frac{3}{2}$ amplitude in Cabibbo current-current theory (also not understood).

The most relevant test of the theory remains whether W (=a) spin-1 particles exist. Simulaneous search for $W^\pm$ in $\nu_l(\bar\nu_l)+Z\to W^\pm+l^\mp+Z^*$ and $a^\pm$ (=$W^\pm$) in the strong process $\bar p+p\to a^++a^-+\cdots$ to identify their common mass M (>2.5 GeV according to Pepper et al.[1]) is needed. As pointed out by Lee,[5] large asymmetry in the energy distribution of $a^\pm$ is expected from the latter reaction. If $W=a$ is an SU(3) singlet, the decays $W\to\eta'(959)$[or $E(1420)$]$+l+\bar\nu_l$, $W\to\omega_S(\varphi_S)+l+\bar\nu_l$, $W\to f_S(1250)[f_S'(1500)]+l+\bar\nu_l$ [where subscript s denotes SU(3) singlet part of ω, φ, f, and f'] are expected to be competitive with the usual weak leptonic and nonleptonic decays of the W meson.

(4) L'Envoi.—It is important to emphasize that the propositions (1) and (2) cannot be combined into a single proposition since $W^\pm$ can be equal to $a^\pm$ without either particles' having strong interactions[12] with the known hadrons. In the latter model[13] $a^\pm$ communicates with hadrons via the usual electromagnetic intermediary leading to α (fine structure constant) type suppression of CP-nonconserving effects in $\eta\to(2\pi\gamma)$, $\gamma+p\to\pi^++n$, and the neutron dipole moment. On the other hand, as first pointed out to us by Goebel,[14] the cosmic-ray muon anomaly of the Utah experiment[15] can perhaps be better explained in terms of W mesons with strong interactions because of the larger production cross section by primary protons afforded by collision $p+p\to W^++W^-+$hadrons, etc.

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[6]T. D. Lee and G. C. Wick, Phys. Rev. 148, 1385 (1966). Unexplained notations here are the same as those of Lee and Wick.

[7]Since the leptons have no strong interactions, we can still define $T_{\rm st}=T_{\rm wk}$ to be rigorously satisfied for the leptonic part of the Hamiltonian. $T_{\rm wk}$ symmetry here is violated by the masses of the leptons and the electromagnetic interaction, though it could be valid to all orders of weak interactions. In general $T_{\rm wk}$ symmetry is essentially the $|\Delta I|=1$ rule to lowest order in weak interaction and for leptonic interaction with nonstrange particles [c.f. T. D. Lee, Nuovo Cimento 35, 945 (1965)].

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