THEORY OF *CP* NONCONSERVATION*

R. J. Oakes†
The Institute for Advanced Study, Princeton, New Jersey
(Received 24 May 1968)

A new current × current weak interaction is constructed from the neutral vector and axial-vector currents which preserves the hypothesis of conserved vector current, universality, lepton conservation, and *CPT* invariance, but violates *CP* conservation. The complete weak interaction Hamiltonian is shown to be in agreement with present experiments.

Sometime ago Cabibbo[1] constructed an elegant universal form of the $V-A$ current×current theory[2,3] of *CP*-conserving weak interactions from the charged vector and axial-vector currents whose hadronic parts had been postulated by Gell-Mann[4] to generate SU(3) ⊗ SU(3) under equal-time commutation. In this note we discuss how the neutral vector and axial-vector currents, which necessarily exist if the SU(3) ⊗ SU(3) algebra is to be physically meaningful, can be used to construct a neutral current×current weak interaction which, together with the charged current×current interaction discussed by Cabibbo,[1] is consistent with all present experimental information on weak interactions, including the observed violation of *CP* invariance.[5]

Consider the weak-interaction Hamiltonian

$$H=(G/\sqrt{2})[\tfrac{1}{2}\{J^{(+)}J^{(-)}+J^{(-)}J^{(+)}\}+J^{(0)}J^{(0)}], \quad (1)$$

where G is the Fermi constant and where $J^{(+)}=J^{(-)\dagger}$ and $J^{(0)}$ are, respectively, charged and neutral currents whose hadronic parts are composed from the vector and axial-vector octets.[4] The universal form of the charged current introduced by Cabibbo[1] is

$$J_\lambda^{(-)}=\cos\theta(V-A)_\lambda^{(1-i2)} + \sin\theta(V-A)_\lambda^{(4-i5)}+l_\lambda^{(-)}, \quad (2)$$

where $\theta\simeq 0.26$ is the Cabibbo angle and the charged leptonic current is

$$l_\lambda^{(-)}=\bar{\mu}\gamma_\lambda(1-\gamma_5)\nu_\mu+\bar{e}\gamma_\lambda(1-\gamma_5)\nu_e. \quad (3)$$

Within the vector and axial-vector octets there are four components from which the hadronic part of the neutral current $J^{(0)}$ can be built: the components labeled[4] 3, 6, 7, and 8. Heretofore, only the vector U-spin singlet $V^{[3+(1/\sqrt{3})8]}$, which is the hadron electromagnetic current, has been thought to play any role in nature. Here we shall investigate the possibility that the remaining neutral components 6, 7, and $3-\sqrt{3}\times 8$, which comprise the U-spin triplet, form the hadronic part of the neutral weak current $J^{(0)}$ in a universal manner. Specifically, we consider the following form for $J^{(0)}$:

$$J_\lambda^{(0)}=\cos\varphi(V+A)_\lambda^{(3-\sqrt{3}\times 8)} + i\sin\varphi(V+A)_\lambda^{(6-i7)} - i\sin\varphi(V+A)_\lambda^{(6+i7)}+l_\lambda^{(0)}. \quad (4)$$

The new angle φ is to be regarded as a fundamental quantity to be determined empirically. For the neutral lepton current we take

$$l_\lambda^{(0)}=\bar{\nu}_\mu\gamma_\lambda(1+\gamma_5)\nu_\mu+\bar{\nu}_e\gamma_\lambda(1+\gamma_5)\nu_e. \quad (5)$$

Possible variants of this theory will be analyzed elsewhere.

Some general features of this interaction are the following:

(i) It is of the current×current form[2,3] with both the charged and neutral vector hadron currents obeying the conserved vector current (CVC) hypothesis of Feynman and Gell-Mann[2] since they belong to the same octet as the electromagnetic current.

(ii) Lepton number is conserved separately for muons and electrons.

(iii) The interaction is universal[6] in the sense that the hadron and lepton parts enjoy the same algebraic properties. (The electromagnetic interaction is universal in this sense, also.)

(iv) The hadron part of $J^{(0)}$ can be obtained by a rotation of $(V+A)^{(3-\sqrt{3}\times 8)}$ about the 6 direction through the angle φ just as the hadron parts of $J^{(\pm)}$ can be obtained by a rotation of $(V-A)^{(1\pm i2)}$ about the 7 direction through the angle 2θ.

(v) The neutral current $J^{(0)}$ is $V+A$ while the charged currents $J^{(\pm)}$ are $V-A$. Thus, ν_μ and ν_e are four-component neutrinos which need not be massless. Their right- and left-handed components interact entirely differently.

(vi) The neutral current$\times$current interaction violates CP invariance but, of course, CPT remains valid. [For this reason the charged currents are symmetrized in Eq. (1).] To lowest order, the CP nonconservation occurs only in the $\Delta S=\pm1$ nonleptonic interactions, as will be discussed.

Next let us systematically examine the consequences of this neutral current-current interaction in the light of present experiments.

$\Delta S=0$ nonleptonic interactions.—To lowest order the $\Delta S=0$ nonleptonic part conserves CP. Therefore, neglecting higher orders in the Fermi constant, the neutron electric dipole moment is zero, in agreement with the present experimental upper limit of about[5] $10^{-22}e$ cm.

There are parity-nonconserving $\Delta S=0$ pieces in both the neutral and charged current$\times$current interactions which contribute to the observed parity mixing in nuclear levels.[7]

$\Delta S=\pm1$ nonleptonic interactions.—The strangeness-changing ($\Delta S=\pm1$) nonleptonic interaction contains a CP-conserving part proportional to $\sin\theta\cos\theta$ and a CP-nonconserving part proportional to $\sin\varphi\cos\varphi$. In both parts there are only $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ pieces so there is a "no $\Delta I=\frac{5}{2}$ rule" for the $K\to2\pi$ amplitudes[8]:

$$\sqrt{2}\langle\pi^0\pi^0|H|K^0\rangle-\langle\pi^+\pi^-|H|K^0\rangle=\sqrt{2}\langle\pi^+\pi^0|H|K^+\rangle. \quad (6)$$

The CP-nonconserving effects in $K\to2\pi$ can be conveniently discussed in terms of the complex parameters[9]

$$\eta_{+-}=\frac{\langle\pi^+\pi^-|H|K_L^0\rangle}{\langle\pi^+\pi^-|H|K_S^0\rangle}\simeq\epsilon_0+i\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}+\frac{i}{\sqrt{2}}\frac{\mathrm{Im}A_2}{\mathrm{Re}A_0}e^{i(\delta_2-\delta_0)} \quad (7)$$

and

$$\eta_{00}=\frac{\langle\pi^0\pi^0|H|K_L^0\rangle}{\langle\pi^0\pi^0|H|K_S^0\rangle}\simeq\epsilon_0+i\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}-i\sqrt{2}\frac{\mathrm{Im}A_2}{\mathrm{Re}A_0}e^{i(\delta_2-\delta_0)}, \quad (8)$$

where $A_I\exp(i\delta_I)$ is the amplitude for the K^0 to decay into an outgoing 2π state with isospin $I=0$ or 2 and ϵ_0 characterizes the eigenvectors of the mass matrix:

$$|K_{S,L}\rangle=\frac{(1+\epsilon_0)|K^0\rangle\pm(1-\epsilon_0)|\overline{K}^0\rangle}{[2(1+|\epsilon_0|^2)]^{1/2}} \quad (9)$$

corresponding to the (complex) eigenvalues M_L and M_S, respectively. By convention, we define $|\overline{K}^0\rangle=CP|K^0\rangle$ which differs from Wu and Yang[9] who require instead that A_0 be real. Their parameters ϵ and ϵ' in our notation are

$$\epsilon=\epsilon_0+i\,\mathrm{Im}A_0/\mathrm{Re}A_0 \quad (10)$$

and

$$\epsilon'=\frac{i}{\sqrt{2}}\frac{\mathrm{Im}A_2}{\mathrm{Re}A_0}e^{i(\delta_2-\delta_0)}. \quad (11)$$

To actually calculate these parameters we would need to know the mass matrix. However, we can estimate ϵ_0 and ϵ and show $|\epsilon'|\ll|\epsilon|$ as follows: Since the CP nonconservation is small, ϵ_0 can be expressed in terms of the difference of the off-diagonal mass-matrix elements and the difference of the eigenvalues:

$$2\epsilon_0(M_L-M_S)\simeq\sum\frac{\langle\overline{K}^0|H|n\rangle\langle n|H|K^0\rangle-\langle K^0|H|n\rangle\langle n|H|\overline{K}^0\rangle}{E_K-E_n+i\epsilon}. \quad (12)$$

We obtain a crude estimate of ϵ_0 and ϵ from the on-mass-shell contribution of the 2π intermediate state in Eq. (12) which gives

$$\epsilon\simeq\epsilon_0^*\simeq\frac{1}{\sqrt{2}}\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}e^{+\frac{1}{4}i\pi} \quad (13)$$

when the empirical mass difference is used. From the Hamiltonian we find

$$\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \simeq \frac{\sin\varphi\cos\varphi}{\sin\theta\cos\theta}\left(1+\frac{a_0'}{a_0}\right) \tag{14}$$

and

$$\frac{\mathrm{Im}A_2}{\mathrm{Re}A_0} \simeq -2\frac{\sin\varphi\cos\varphi}{\sin\theta\cos\theta}\frac{a_2}{a_0}, \tag{15}$$

where a_0, a_0', and a_2 are, respectively, the (real) reduced matrix elements corresponding to the two $\Delta I=\frac{1}{2}$ and the one $\Delta I=\frac{3}{2}$ independent amplitudes. The approximate validity of the $\Delta I=\frac{1}{2}$ rule, as evidenced in the suppression of the $K^+ \to \pi^+\pi^0$ decay rate, requires $|a_0| \simeq 20|a_2|$ and hence,

$$\left|\frac{\epsilon'}{\epsilon}\right| \simeq \left|\frac{\mathrm{Im}A_2}{\mathrm{Im}A_0}\right| = 2\left|\frac{a_2}{a_0+a_0'}\right| \sim \frac{1}{10} \tag{16}$$

provided there is no accidental cancellation between the two independent $\Delta I=\frac{1}{2}$ amplitudes. The result $|\epsilon'| \ll |\epsilon|$ is basically a consequence of the $\Delta I=\frac{1}{2}$ rule being valid to about the same degree for both the CP-conserving and CP-nonconserving amplitudes in this theory.

Since $|\epsilon'/\epsilon|$ is small we can write

$$\begin{aligned}\eta_{00}/\eta_{+-} &\simeq 1-3\epsilon'/\epsilon \\ &\simeq 1 \pm 3|\epsilon'/\epsilon| e^{i(\delta_2-\delta_0+\frac{1}{4}\pi)}\end{aligned} \tag{17}$$

which shows that while $\eta_{00} \neq \eta_{+-}$ in this theory they do not differ by much, say about 25 % if we take the estimate (16) as a guide. This disagrees with the early measurements[5] of $|\eta_{00}|$ but is consistent with the most recent result.[10]

Finally, from Eqs. (13) and (14) we estimate that

$$|\varphi| \sim 10^{-3} \tag{18}$$

to agree with the measurements[5] of $|\eta_{+-}|$ and $\mathrm{Re}\epsilon$.

$\Delta S = \pm 2$ nonleptonic interactions.—The neutral current×current interaction allows $\Delta S = \pm 2$ nonleptonic weak interactions in lowest order which, however, conserve CP. Thus there is a first-order contribution $\Delta M^{(1)}$ to the mass difference $\Delta M = M_L - M_S$ given by

$$\begin{aligned}\Delta M^{(1)} = \sqrt{2}G\sin^2\varphi \sum \langle \bar{K}^0|(V+A)^{(6+i7)}|n\rangle \\ \times \langle n|(V+A)^{(6+i7)}|K^0\rangle.\end{aligned} \tag{19}$$

The vacuum intermediate-state contribution to $M^{(1)}$ is easily found to be about $50\sin^2\varphi$ eV. The pion intermediate-state contribution can in principle be evaluated in terms of the K_{l3} form factors f_+ and f_-; however, the convergence of the integral over intermediate pion energy depends on the unknown asymptotic behavior of these form factors. If the effective cutoff is around a proton mass the contribution turns out the be comparable in magnitude with but opposite in sign to the vacuum contribution. [In general, the sign of the contribution of an intermediate state in Eq. (19) will be positive (negative) if the state is even (odd) under CP.] Although we are unable to compute the complete sum over intermediate states, the terms which we have estimated indicate that this first-order contribution to the mass difference $\Delta M^{(1)}$ is comparable in magnitude with the usual second-order contribution and with the observed mass difference $|\Delta M| \sim 5 \times 10^{-6}$ eV for $|\varphi| \sim 10^{-3}$, which is the estimate obtained above [Eq. (18)] from the observed CP nonconservation in $K_L \to 2\pi$.

There will also be $\Delta S = \pm 2$ nonleptonic decays possible in lowest order such as $\Xi \to N+\pi$, however, such amplitudes are proportional to $G\sin^2\varphi$ and hence are comparable with second-order weak interactions.

Semileptonic interactions.—The elastic scattering $\nu+p \to p+\nu$ can occur in lowest order through the neutral currents with an amplitude comparable with that for $\bar{\nu}+p \to n+l^+$, in apparent contradiction with the experimental upper limit[11] of 3 % for the ratio of these cross sections. However, recall that since the neutral current is $V+A$ rather than $V-A$ only the elastic scattering of right-handed neutrinos is large; the elastic scattering of the usual left-handed neutrinos still vanishes in lowest order. In the CERN neutrino experiment[11] the flux of these right-handed neutrinos was no doubt negligible since they are rather difficult to produce.

Right-handed neutrinos can be obtained from various strangeness-changing decays, e.g., $\Sigma^+ \to p+\nu+\bar{\nu}$ and $K^+ \to \pi^+ + \nu + \bar{\nu}$, but these branching ratios are suppressed by $\sin^2\varphi$ and thus are quite small ($\sim 10^{-6}$). The possible $\Delta S = 0$ decay modes, e.g., $\Sigma^0 \to \Lambda + \nu + \bar{\nu}$ and $\eta \to \pi^0 + \nu + \bar{\nu}$, are even more rare although they occur with amplitude $G\cos\varphi$ since they must compete with electromagnetic decay rates.

Leptonic interactions.—Another new feature of the neutral current×current theory is the occurrence of elastic (right-handed) neutrino-neutrino scattering in lowest order; but this seems virtually impossible to observe, although it might be of some theoretical interest.

In general, since the CP nonconservation occurs in the weak-interaction Hamiltonian, there are no large effects to be expected such as an asymmetry in $\eta \to \pi^+ + \pi^0 + \pi^-$ or $\eta \to \pi^+ + \pi^- + \gamma$, time-reversal noninvariance in nuclear γ emission, or the occurrence of $\eta \to \pi^0 + e^+ + e^-$.

It is a pleasure to acknowledge gratefully several stimulating discussions with colleagues here in Princeton. We have particularly benefited from the comments of Professor Francis Low and Dr. Mahiko Suzuki. We are indebted to Professor M. M. Block for an illuminating discussion on neutral lepton currents. We also wish to thank Professor C. Kaysen for his hospitality at The Institute for Advanced Study.

*Research sponsored by the Air Force Office of Scientific Research, Office of Aerospace Research, U. S. Air Force, under Grant No. 68-1365.

†Permanent address: Physics Department, Northwestern University, Evanston, Ill.

[1]N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).

[2]R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

[3]R. E. Marshak and E. C. G. Sudarshan, in Proceedings of the Padua-Venice Conference on Mesons and Recently Discovered Particles, September, 1957 (Società Italiana di Fisica, Padua-Venice, Italy, 1958), and Phys. Rev. 109, 1860 (1960).

[4]M. Gell-Mann, Phys. Rev. 125, 1067 (1962), and Physics 1, 63 (1964).

[5]For a survey of experiments bearing on CP nonconservation see the talk of J. Cronin, in Proceedings of the 1967 International Conference on Particles and Fields, Rochester, New York, 1967, edited by C. R. Hagen et al. (Interscience Publishers, Inc., New York, 1968).

[6]M. Gell-Mann and M. Levy, Nuovo Cimento 16, 705 (1958); Cabibbo, Ref. 1.

[7]For a survey see L. B. Okun, Comments Nucl. Particle Phys. 1, 181 (1967).

[8]E. C. G. Sudarshan, Nuovo Cimento 41A, 283 (1965).

[9]T. T. Wu and C. N. Yang, Phys. Rev. Letters 13, 501 (1964).

[10]D. Bartlett, R. Carnegie, V. Fitch, K. Goulianous, D. Hutchinson, T. Kamae, R. Roth, J. Russ, and W. Vernon, to be published.

[11]M. M. Block et al., in Proceedings of the Twelfth International Conference on High-Energy Physics, Dubna, U. S. S. R., 1964 (Atomizdat., Moscow, U. S. S. R., 1966).

PARITY-NONCONSERVING NUCLEAR FORCES*

Bruce H. J. McKellar†
Institute for Advanced Study, Princeton, New Jersey
(Received 3 May 1968)

The effective single-particle weak parity-nonconserving potential is recalculated including parity-nonconserving one-pion exchange forces and the two-body correlations induced by the hard cores in the nucleon-nucleon potential. The circular polarization of the γ ray from the 482-keV transition in ^{181}Ta is calculated to be $-(0.2 \pm 0.1) \times 10^{-4}$. The measured value is $-(0.06 \pm 0.01) \times 10^{-4}$, showing that the observed parity nonconservation in nonleptonic $\Delta S = 0$ transitions is in agreement with the Cabibbo theory of weak interactions.

The current-current theory of weak interactions predicts a weak force between nuclei which can in principle be detected through its parity-nonconserving effects.[1] The attempts to calculate such effects and the attempts to observe them have been recently reviewed by Okun.[2] The circular polarization measurements of Lobashov et al.[3] and Boehm and Kankeleit[4] are shown in Table I, and the asymmetry measurements of Abov et al.[5] and Warming et al.[6] are shown in Table II. Also shown in the tables are the results of calculations reported in the literature.[7-11] It is noticeable that the calculations overestimate the effect. It was pointed out by Adams[12] that the hard core of the nucleon-nucleon potential, in keeping the nuclei in the tail of the weak potential, may provide an explanation of this overestimate.

In the present Letter we show that this is in fact the case, and in taking the hard-core correlations into account we significantly improve the agreement with experiment. We also include in the calculation the strangeness-changing weak currents which induce a one-pion exchange, parity-nonconserving potential between nuclei.[13] Enhanced by its long range, and suppressed by the factor $\sin^2\theta_C$, the net effect is 25% of that of strangeness-conserving currents. The relative sign of the two contributions is not fixed by experiment, nor by any reliable theoretical arguments.[14] (The sign of G is taken as