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VECTOR-DOMINANCE MODEL COMPARISON OF π^+ PHOTOPRODUCTION WITH ρ^0 PRODUCTION BY PIONS

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A comparison of the reactions $\gamma p\rightarrow\pi^+n$ and $\pi^-p\rightarrow\rho^0n$ at 4 and 8 GeV/c has been made using the vector-dominance model. Although the ρ^0 data are insufficient to show the very narrow forward peak observed in the photoproduction data, agreement is obtained to within errors for $|t|\lesssim 0.1$ (GeV/c)^2. Taking interference effects into account, this agreement can be extended to $|t|\approx 1.5$ (GeV/c)^2 at 4 GeV/c, but only to 0.3 (GeV/c)^2 at 8 GeV/c.

The reactions

$$\gamma p\rightarrow\pi^+n \tag{1}$$

and

$$\pi^-p\rightarrow V^0n \tag{2}$$

(where V^0 is a mixture of ρ^0, ω, and φ) can be directly related to one another in the vector-dominance model[1] by time-reversal and isospin invariance as shown schematically in Fig. 1. The γ-ray–vector-meson couplings γ_V can in principle be obtained from the leptonic decays $V^0\rightarrow l^+l^-$; up to now only the decays $\rho^0\rightarrow e^+e^-$ and $\rho^0\rightarrow\mu^+\mu^-$ have been well measured, giving[2]

$$\gamma_\rho^{\,2}/4_\pi\approx 0.45 \tag{3}$$

with perhaps a 20% uncertainty.[3] The couplings γ_ω and γ_φ can be estimated using SU(3) with the usual $\omega\varphi$ mixing angle[4] ($\cos\theta=\sqrt{\tfrac{2}{3}}$):

$$\gamma_\rho^{\,-2}:\gamma_\omega^{\,-2}:\gamma_\varphi^{\,-2}=9:1:2. \tag{4}$$

Various modifications to the ratios have been proposed[5] but the $V^0=\rho^0$ amplitude of Fig. 1(a) is expected to be dominant, in which case the relation between processes (1) and (2) becomes[1,6]

$$\frac{d\sigma}{dt}(\gamma p\rightarrow\pi^+n)\approx\frac{\pi\alpha}{\gamma_\rho^{\,2}}\rho_{11}^{\,\mathrm{hel}}(t)\frac{d\sigma}{dt}(\pi^-p\rightarrow\rho^0n), \tag{5}$$

where we will take $\pi\alpha/\gamma_\rho^{\,2}=1/250$ and $\rho_{11}^{\,\mathrm{hel}}(t)$ is the helicity density matrix[7] giving the fraction of ρ mesons with helicity +1 at momentum transfer

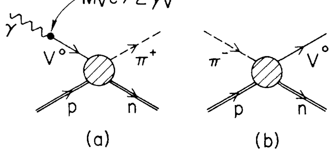

FIG. 1. Feynman diagrams showing the relationship between Reactions (1) and (2) in the vector-dominance model.

t. The factor $\rho_{11}^{\text{hel}}(t)$ is necessary since the incoming γ ray, and thus the virtual $V^0=\rho^0$ in Fig. 1(a), can only have helicity ±1.

Previous comparisons using Eq. (5) have been made.[8] In this Letter we compare in detail (with more statistics and a more careful analysis of the ρ^0 data than was used in the previous comparisons) the experimental data on Reactions (1)[9,10] and (2)[11,12] near 4- and 8-GeV/c incident momentum.

The evaluation of the right-hand side of Eq. (5) is made difficult by the background process $\pi^-p \to \pi^+\pi^-n$, where the $\pi^+\pi^-$ do not form a ρ^0, i.e., do not have $J^P=1^-$. Using data at all values of t, a fit to the $\pi^+\pi^-$ mass distribution was made using three-body phase space plus two Breit-Wigner curves, one for the ρ^0 and one for the f. The fraction of 4-GeV/c events fitted as ρ^0 events fluctuated by ±6% depending on the exact form taken for the Breit-Wigner resonance shape; the shape giving the best fit indicated that 40% of all $\pi^+\pi^-n$ events were ρ^0n.[11] The $\pi^+\pi^-$ mass distribution was examined for each t interval of interest; the fraction of events in the interval $700 \leq M_{\pi\pi} \leq 850$ MeV attributable to ρ^0 production (80 to 85%) was found to be independent of t (within statistics), and we have calculated $d\sigma/dt$ for ρ^0 production from the number of events with $M_{\pi\pi}$ in this region (normalized to the total ρ^0 cross section).

Previously published values[12,13] of the ρ^0 density matrices were evaluated with polar direction along the incident beam (Jackson direction) instead of along the ρ^0 direction of motion (helicity direction). One is tempted to argue that since we are primarily interested in the low-t region, the difference between the two frames cannot be large. However, numerical evaluation of the angle between the two frames shows a large effect, the angle increasing rapidly from 0 at $t=t_{\min}$ to about 45° at $|t|=0.1$ (GeV/c)^2 (independent of energy above 2 GeV) and then less rapidly to 90° at $|t|\approx 0.5$ (GeV/c)^2. In principle, the density matrix in the Jackson frame could be rotated by the angle between the two frames to obtain $\rho^{\text{hel}}(t)$: We have found that due to substantial off-diagonal-error matrix elements, the direct fit to the angular distribution in the helicity frame gives values which are slightly different from the results obtained from a rotation assuming uncorrelated errors. In what follows, we have used the values of $\rho_{11}^{\text{hel}}(t)$ obtained from direct fits to the data. The 8-GeV/c results for $\rho_{11}^{\text{hel}}(t)$ are shown in Fig. 2; the 4-GeV/c results will be published elsewhere.[11]

In order to minimize the non-ρ background, only events with $700 \leq M_{\pi\pi} \leq 850$ MeV were used to evaluate $\rho_{11}^{\text{hel}}(t)$; in this region 15 to 20% of the events are non-ρ. At 4 GeV/c there were sufficient data to check that $\rho_{11}^{\text{hel}}(t)$ was not being distorted by the non-ρ events in the ρ mass region. For this purpose $\rho_{11}^{\text{hel}}(t)$ was evaluated for events with $M_{\pi\pi}$ from 575 to 675 and 875 to 975 MeV; the resulting values for $\rho_{11}^{\text{hel}}(t)$ agreed with those for the ρ region to within statistics (typically 20 or 30% at individual t values) with no systematic differences observed. For this reason we do not believe that the background of non-ρ events presents a serious difficulty to the analysis.

The 5- and 8-GeV photoproduction data[9] are shown in Fig. 3, where the 5-GeV photoproduction data have been extrapolated to 4 GeV by assuming $d\sigma/dt$ to go as k^{-2} as indicated by a comparison of the Deutsches Elektronen-Synchrotron data[14] at 2.7 GeV with the Stanford Linear Accelerator Center 5-GeV data. The data are plotted as a function of $|t-t_{\min}|^{1/2}$, which at small angles is proportional to the production angle.[15] The photoproduction predictions obtained from the ρ^0 data using Eq (5) are also shown in Fig. 3. The errors shown in Fig. 3 are statistical only; when comparing the experimental data with the vector-dominance prediction one must keep in mind the 12% systematic uncertainty in the photoproduction data and the 15% systematic uncertainty in $\rho_{11}^{\text{hel}}(d\sigma/dt)$ for ρ^0 production as well as the uncertainty in γ_ρ^2.

General agreement between the vector-dominance prediction and the photoproduction data is obtained at both energies for $|t| \leq 0.1$ (GeV/c)^2. At larger momentum transfers the prediction from ρ^0 production falls below the observed $\gamma p \to \pi^+n$ cross section. Some of this discrepancy can be removed by considering the interference effects from the amplitude with $V^0=\omega$. These effects can be directly estimated from the ratio of

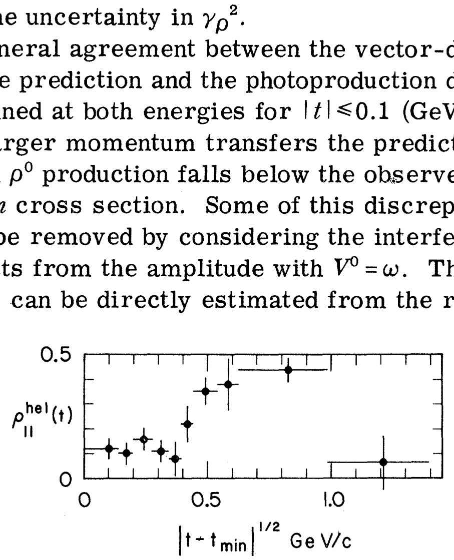

FIG. 2. Values of $\rho_{11}^{\text{hel}}(t)$ obtained by fitting the 8-GeV/c ρ^0-decay angular distributions in the helicity frame.

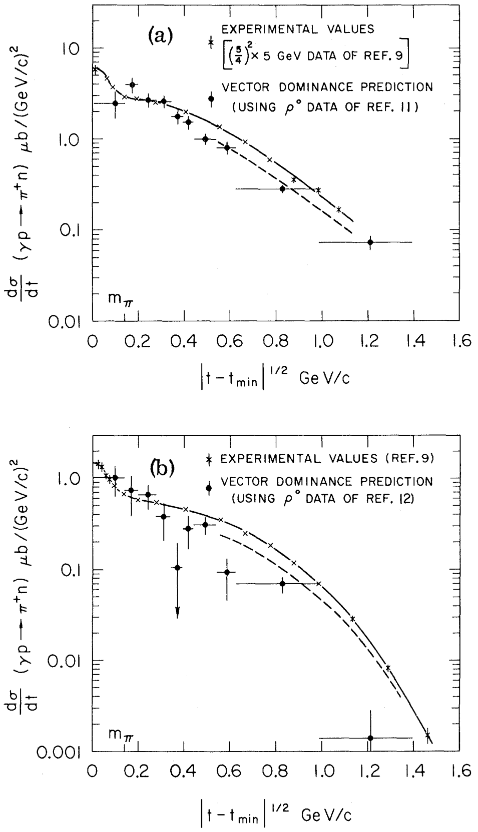

FIG. 3. Comparison of the two sides of Eq. (5) at (a) 4 GeV/c (the 5-GeV/c photoproduction data have been extrapolated to 4 GeV/c) and (b) 8 GeV/c. As discussed in the text, interference terms between the $V^0=\rho^0$ and $V^0=\omega$ amplitudes can be eliminated by taking $\frac{1}{2}[d\sigma(\gamma p\to\pi^+n)/dt+d\sigma(\gamma n\to\pi^-p)/dt]\approx\frac{1}{2}(1+R)\,d\sigma(\gamma p\to\pi^+n)/dt$, where R is the π^-/π^+ ratio for photoproduction of single π mesons from deuterium. For this reason the prediction based on the ρ^0 data should be compared with the dashed line which includes the factor $\frac{1}{2}(1+R)\approx 0.7$ as measured by Bar-Yam et al. (Ref. 17) at 3.4 GeV; this correction factor is expected to tend toward unity at small t.

cross sections

$$R=\frac{d\sigma(\gamma d\to p_s p\pi^-)/dt}{d\sigma(\gamma d\to n_s n\pi^+)/dt}\tag{6}$$

(where subscript s indicates a spectator nucleon). Assuming that the photoproduction amplitudes A_ρ and A_ω, corresponding to Fig. 1(a) with $V^0=\rho^0$ and $V^0=\omega$, are dominant for single-pion photoproduction,[16] isospin invariance gives

$$\frac{d\sigma}{dt}(\gamma p\to\pi^+n)=|A_\omega+A_\rho|^2\tag{7}$$

$$\frac{d\sigma}{dt}(\gamma n\to\pi^-p)=|A_\omega-A_\rho|^2.\tag{8}$$

Neglecting $|A_\omega|^2$, we then have

$$|A_\rho|^2=\frac{1}{2}\left(\frac{d\sigma}{dt}(\gamma p\to\pi^+n)+\frac{d\sigma}{dt}(\gamma n\to\pi^-p)\right)$$

$$\approx\frac{1+R}{2}\frac{d\sigma}{dt}(\gamma p\to\pi^+n).\tag{9}$$

Thus, the prediction for photoproduction shown in Fig. 3 based on ρ^0 production should be compared with the quantity shown on the right-hand side of Eq. (9).

The ratio R has been measured by Bar-Yam et al.[17] at 3.4 GeV. They found $R=0.35$ at $|t|=0.4$ $(\text{GeV}/c)^2$, increasing to 0.5 at $|t|=1.5$ $(\text{GeV}/c)^2$; for the A_ρ and A_ω amplitudes in phase, this gives $r=|A_\omega|/|A_\rho|=0.25$ and 0.17, respectively. Although the $V^0=\omega$ amplitude squared is then only 4% of the $V^0=\rho^0$ contribution, the interference term is nearly half as large as the ρ^0 term by itself. Using the experimental numbers for R, the coefficient $\frac{1}{2}(1+R)$ is typically 0.7 and the dashed curves in Fig. 3 correspond to this factor, as measured at 3.4 GeV/c, times the $\gamma p\to\pi^+n$ cross section. At small t the one-pion-exchange term ($V^0=\rho^0$) might be expected to dominate to the extent that R would be close to unity in this region.

The magnitude of the amplitude ratio r can be understood qualitatively on the basis of vector dominance. Experimental data on $\pi N\to\omega N$ are rather sparse,[18] but the cross section appears to be roughly a factor of 3 smaller than that for ρ^0 production with a somewhat wider t distribution than that of the ρ^0. If we arbitrarily assume that ρ_{11}^{hel} is roughly the same for ρ^0 and ω production and neglect the difference in t distributions, then the ratio r becomes [using Eq. (4)]

$$r=\frac{\gamma_\rho}{\gamma_\omega}\left(\frac{\rho_{11\omega}^{\text{hel}}}{\rho_{11\rho}^{\text{hel}}}\frac{\sigma_\omega}{\sigma_\rho}\right)^{\frac{1}{2}}\approx 0.2\tag{10}$$

in good agreement with the values quoted above.

The backward π^+ photoproduction data at 4.3 GeV/c of Anderson et al.[10] can also be compared with the 4-GeV/c ρ^0 data.[11] Both sets of data show a broad backward peak, but the ρ^0 helicity-one cross section is twice that predicted by Eq. (5). In the 8-GeV/c ρ^0 data[12] only one event is found in the backward direction with $M_{\pi\pi}$ between 700 and 850 MeV, compared with the four ρ^0's which might be expected [using the photoproduction data[10] and Eq. (5) with $\rho_{11}^{\text{hel}}=0.25$]. The vector-dominance comparison in the backward direction is thus inconclusive as a result of the poor statistics and the possibility of large effects from the ω amplitude; while the ρ^0 helicity-one cross section may have the appropriate order of magnitude in the backward direction, it appears to fall faster with increasing energy than expected.

The agreement over the wide range of t shown in Fig. 3 is the result of ρ_{11}^{hel} increasing rapidly at small t to counteract the $e^{-10|t|}$ falloff of $d\sigma/dt$ for ρ^0 production, the product yeilding a dependence close to the $e^{-3|t|}$ observed in photoproduction. A similar result has been obtained at lower energies.[19] Even after correcting for the $V^0=\omega$ interference there is a discrepancy at 8 GeV/c for large t. While some of this discrepancy seems to be the result of statistical fluctuations, the ρ^0 helicity-one cross section does appear to fall somewhat faster at large t than that for single-pion photoproduction. This discrepancy may indicate the need for corrections resulting from the virtual ρ in Fig. 1(a) being off the mass shell, or may simply be some background amplitude which eventually becomes important as t is increased.

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[3] Since $\gamma_\rho^2/4\pi$ is obtained from the partial width $\Gamma(\rho \rightarrow l^+l^-) = (\text{branching ratio}) \times \Gamma(\rho \rightarrow \text{all})$, a shift of $\Gamma(\rho \rightarrow \text{all})$ from the nominal value of 130 MeV to the Novosibirsk storage-ring value of 93 MeV would change $\gamma_\rho^2/4\pi$ from 0.45 to 0.63.

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[6] We have ignored the kinematic factor giving the ratio of phase space and incident flux, $|\vec{p}_\pi|^2/|\vec{p}_\gamma|^2$, which is very close to unity at the energies considered here.

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[16] The term with $V^0=\varphi$ is expected to be negligible because of the small coupling of the φ to nonstrange particles.

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[19] E. Malamud and P. Schlein have fitted the 2.7-GeV/c helicity ±1 ρ^0 cross section to the form $Ae^{-B|t|}$ and find $B=(3.64\pm0.32)$ (GeV/c)$^{-2}$ (private communication).