OCTET DOMINANCE IN NONLEPTONIC DECAYS*

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(Received 13 May 1968)

It is a well-known difficulty for the standard theory of weak interactions to explain the empirical $\Delta I=\frac{1}{2}$ rule that appears to be an excellent approximation for the strangeness-changing nonleptonic decays. Some time ago calculations based on current-algebra and soft-pion technique[1] have been performed showing that in this approximation one has an effective $\Delta I=\frac{1}{2}$ rule for some decays.[2]

In this note we want to point out a very simple feature of the standard weak-interaction theory which sheds some new light on the $\Delta I=\frac{1}{2}$ rule (and octet dominance) in the nonleptonic decays. We consider the vector boson theory, where the amplitude for the nonleptonic process $A\to B$ in the lowest order in weak interactions is given by

$$T=g^2\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2+m_w^2}\left(\delta_{\mu\nu}+\frac{q_\mu q_\nu}{m_w^2}\right)T_{\mu\nu},\quad T_{\mu\nu}=\int d^4x\,e^{iq\cdot x}\langle B|T[J_\mu(x)J_\nu(0)]|A\rangle, \tag{1}$$

where J_μ is the total hadronic weak current. Using the identity

$$q_\mu q_\nu T_{\mu\nu}=iq_\nu\int d^4x\,e^{iq\cdot x}\langle B|[J_0(x),J_\nu(0)]|A\rangle\delta(x_0)$$
$$+\int d^4x\,e^{-iq\cdot x}\langle B|[J_0(x),\partial_\mu J_\mu(0)]|A\rangle\delta(x_0)+\int d^4x\,e^{iq\cdot x}\langle B|T[\partial_\mu J_\mu(x)\partial_\nu J_\nu(0)]|A\rangle, \tag{2}$$

we may express Eq. (1) in the form

$$T=\frac{g^2}{m_w^2}\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2+m_w^2}\int d^4x\,e^{-iq\cdot x}\delta(x_0)\langle B|[J_0(x),\partial_\mu J_\mu(0)]|A\rangle+T_0, \tag{3}$$

where

$$T_0=g^2\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2+m_w^2}\int d^4x\,e^{iq\cdot x}\left\{\frac{1}{m_w^2}\langle B|T[\partial_\mu J_\mu(x)\partial_\nu J_\nu(0)]|A\rangle+\langle B|T[J_\mu(x)J_\mu(0)]|A\rangle\right\}. \tag{4}$$

Using Bjorken's technique[3] it is easy to see that T_0 given by Eq. (4) is at most logarithmically divergent. The worst divergence of T is thus isolated in the first term on the right-hand side of Eq. (3), and is quadratic. The fact that all nonleptonic decays diverge quadratically in the lowest-order weak interactions was first noticed by Halpern and Segre.[4] However, instead of seeking a way out of this divergence, we take here the less exotic attitude that the weak-interaction theory needs a cutoff.[5] For sufficiently large values of the cutoff, it is clear that the first term on the right-hand side of Eq. (3) will dominate the matrix element. Furthermore this term is proportional to the matrix element of the equal-time commutator $[J_0,\partial_\mu J_\mu]$, the so-called σ term. If we write

$$[J_0(x),\partial_\mu J_\mu(0)]_{x_0=0}=\sigma(0)\delta^3(x), \tag{5}$$

where σ is a sum of scalar and pseudoscalar densities, we may rewrite the dominant contribution to the nonleptonic amplitudes

$$T\simeq\frac{g^2}{m_w^2}\langle B|\sigma(0)|A\rangle\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2+m_w^2}. \tag{6}$$

Equation (6) in fact is a very important and simple feature of the conventional weak-interaction theory and leads to the following conclusions:

(a) In the usual quark model[6] σ is an octet,[7] and Eq. (6) therefore shows that in this model the conventional vector-boson theory leads to octet dominance. Furthermore this result is valid for all nonleptonic decays including both s- and p-wave hyperon decays. In this respect, as well as because of the fact that we do not have to treat pions as soft, this result goes further than the current-algebra result.[1]

(b) If the σ term is purely an octet, the small departures from $\Delta I=\frac{1}{2}$ observed in nonleptonic decays might find a rather novel explanation. By using Bjorken's arguments it is easy to see that the logarithmically divergent term in T_0 defined by Eq. (4) is in general not octet dominated. Hence the $\Delta I \geq \frac{3}{2}$ amplitude would be smaller than the $\Delta I = \frac{1}{2}$ matrix element by a factor proportional to $(\ln\Lambda)/\Lambda^2$, where Λ is the cutoff defined in suitable dimensionless units. Of course it is conceivable that the σ term may not be a pure octet so that this type of qualitative understanding of the $\Delta I \geq \frac{3}{2}$ effects may be wholly or partly incorrect.

(c) The fact that the dominant contribution to the nonleptonic amplitude is given in terms of the matrix elements of an octet σ is reminiscent of the tadpole model.[8] Equation (6) shows that one may write down an effective nonleptonic weak Hamiltonian

$$H_{\text{eff}} = G_{NL}\sigma, \tag{7}$$

where the coupling constant $G_{NL} \sim (g^2/m_w^2)\Lambda^2$. Effective Hamiltonians of the type (7) have been proposed by many authors[9] in the recent literature.

(d) So far we have considered the intermediate vector-boson theory of weak interactions. One might ask if the results (a)-(c) also follow from the current-current picture of weak interactions. Taking the usual limit $m_w \to \infty$ in Eq. (1), and writing $g^2/m_w^2 = G/\sqrt{2}$, one obtains on using Bjorken's technique the result that the nonleptonic amplitude in the current-current theory also diverges quadratically. However, instead of Eq. (6) we now find the dominant contribution to T to be

$$T \sim \frac{G}{\sqrt{2}}\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2} \times \int d^3x \langle B|[\dot{J}_\mu(x), J_\mu(0)]_{x_0=0}|A\rangle. \tag{8}$$

The equal-time commutator in Eq. (8) can be split into the σ term and an extra contribution

$$T \sim \frac{G}{\sqrt{2}}\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2}\{\langle B|\sigma(0)|A\rangle + \int d^3x \langle B|[\dot{J}_K(x), J_K(0)]_{x_0=0}|A\rangle\} \tag{9}$$

which in general may not transform as an octet. Only if the second term in Eq. (9) vanishes or transforms as an octet do we arrive at the same conclusions as in the vector boson theory.

(e) Finally, it is worthwhile pointing out that[10] the σ term in Eq. (6) is presumably proportional to m_π^2 so that for a massless pion, the quadratic divergence in Eq. (6) would disappear for all nonleptonic decays. This is the analog of the result obtained for the pion mass difference, where for a massless pion the worst divergence (logarithmic) also disappears[11] in the limit of a zero-mass pion. If we further make use of the recent algebra proposed by Lee,[12] it is simple to see from Eq. (4) that in the Bjorken limit, the logarithmic divergence in the lowest-order weak interactions would also vanish.

We would like to thank Professor R. E. Marshak for many discussions that led to this investigation. Thanks are also due to Professor S. Okubo, Mr. R. Mohapatra, and Mr. Subba Rao for their comments.

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*Work supported in part by the U. S. Atomic Energy Commission.

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