ICARUS: FURTHER CONFIRMATION OF THE RELATIVISTIC PERIHELION PRECESSION

Irwin I. Shapiro,* Michael E. Ash, and William B. Smith
Lincoln Laboratory,† Massachusetts Institute of Technology, Lexington, Massachusetts
(Received 3 May 1968)

The non-Newtonian perihelion precession for the asteroid Icarus has been deduced from optical observations and found to agree with the predictions of general relativity to well within the estimated uncertainty of 20%. If observations are made this June when Icarus passes close to the earth, this uncertainty could be reduced to 8%. With such new data a marginally useful bound could also be placed on the quadrupole moment of the sun's gravitational field.

Off and on since its discovery in 1949, the asteroid Icarus has intrigued physicists[1-3] because of its potential for disclosing relativistic contributions to orbital motion. The strikingly large eccentricity ($e \approx 0.83$) and moderately small semimajor axis ($a \approx 1.08$ A.U., period ≈409 days) of its orbit[4] insure that Icarus will be very susceptible to the relativistic perihelion precession, predicted to be 10 sec of heliocentric arc per century. For optical observations spread over a given time interval, this effect should actually be more noticeable for Icarus than for Mercury despite the latter's relativistic precession being 43 sec/100 yr. The explanation for this seeming paradox is rooted in three facts: (1) If two test particles have initial positions on the same heliocentric radial line and identical orbital elements except for a difference $\Delta\omega$ in the orientation of their perihelion positions, then the maximum difference in their heliocentric angular positions will be of the order of[5] $e\Delta\omega$; (2) since Icarus approaches the earth more closely than does Mercury, the effective accuracy of individual Earth-based observations, when translated into heliocentric positions, is higher for Icarus; and (3) Icarus, although faint, is essentially a point target (diameter ≈1 km) and is visible at night, whereas Mercury, with its perceptible disc (diameter ≈5000 km), goes through phase cycles and is only observed in daylight. Both of these last comparisons favor Icarus in regard to the accuracy of estimation of the target's position with respect to the stellar background.

Although only 71 photographic observations of Icarus have been made since its discovery[6]—compared with more than 10 000 meridian-circle and transit observations of Mercury spread over two centuries[7]—we nevertheless thought it of interest to ask how well Icarus' orbit follows the predictions of general relativity. To provide an answer, we analyzed these data with the aid of a digital computer to determine simultaneously the six initial conditions of the orbit plus the parameter λ defined as the coefficient of the relativistic terms in the equations of motion.[8] Thus $\lambda = 1$ corresponds to general relativity (i.e., the Schwarzschild line element) and $\lambda = 0$ to Newtonian theory. The motions and masses of all relevant perturbing bodies in the solar system were taken from previous work.[8,9] Our weighted-least-squares solution yielded $\lambda = 0.97 \pm 0.20$. (This value, of course, is also consistent with the Brans-Dicke prediction[10] when the inferences from the recent measurements of the visual oblateness of the sun[11] are included; the expected value of λ would then be about 1.04.[12]) The post-fit residuals for one solution were quite small. After the deletion of a few bad observations we find

$$\epsilon = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{O_i - C_i}{\sigma_i}\right)^2 = 0.68,$$

where N is the number of measurements and O_i, C_i, and σ_i are, respectively, the observed value, the computed value, and the estimated error for the ith measurement.[6] By contrast, the corresponding value for ϵ obtained from using a purely Newtonian theory is 0.93.

Naturally our result for λ has a far greater uncertainty than that found from analyzing the Mercury data.[7] Nevertheless, Icarus does seem to provide a more stringent test of the relativistic equations of motion than can be obtained from the literally tens of thousands of meridian-circle (optical) observations of either the sun, Venus, or Mars.[13]

Icarus will pass within about 0.04 A.U. ($\approx 6 \times 10^6$ km) of the earth at about 2100 hr UT on 14 June 1968. How will such an unusually close approach affect the accuracy of this test of the relativistic equations of motion? By considering the addition to the data set of (1) photographic observations (each with an uncertainty of 1 sec) made daily from 5 June to 30 June, and (2) radar observations (each with time-delay uncertainty of 20 μsec and frequency-shift uncertainty of 2

Hz, for transmission at 8 GHz) made on the five days surrounding the close approach, we find that the standard error in the estimate of λ would be reduced from 0.20 to 0.08. Without the radar data, the reduction is to 0.09. However, the radar data do make an impressive contribution to the improvement in the orbital element estimates.

Because of the high inclination ($i \approx 16°$) of Icarus' orbital plane to the apparent solar equator, we also have the possibility, in principle at least, of distinguishing the presence of a solar gravitational quadrupole moment from an error in the relativistic prediction.[2,3] The quadrupole moment, besides causing a perihelion precession, also causes a precession of the orbital plane about the sun's polar axis. This latter effect, which does not follow from the generalized Schwarzschild line element, is more easily discernible for moderately large values of i.[3] Unfortunately our error analysis shows that even inclusion of the data to be obtained at the June close approach could not quite allow us to distinguish reliably a solar gravitational quadrupole moment of the magnitude inferred from recent measurements of the sun's visual oblateness.[11] The uncertainty in our estimate of the quadrupole moment is about a factor of 2 too high. By the same token, these data alone do not allow us to distinguish reliably between general relativity and, for example, the Brans-Dicke theory when the free parameter s of the latter is set equal to 0.06.

Finally, we investigated the possibility of determining the mass of Mercury from its perturbations on the orbit of Icarus since the latter passes within about 0.1 A.U. of the former just prior to the close approach to Earth in June. We find that the uncertainty in this mass estimate will be about 20 times higher than that accompanying the determination of Mercury's mass from the measurements of time delays of radar signals traveling between the earth and Venus. Conversely, this result demonstrates that the uncertainty in Mercury's mass—the most poorly determined of the inner planet masses—will not influence significantly our estimate of λ.[14]

We thank S. Herrick for providing us with initial estimates of Icarus' orbital elements and B. Marsden for several stimulating discussions.

Note added in proof.—We recently discovered that G. Null and M. Warner of the Jet Propulsion Laboratory from an independent analysis of 57 of the Icarus observations have also found that ϵ decreased by about 30% when general relativity was substituted for Newtonian theory in the analysis.

---

*Also Department of Geology and Geophysics and Department of Physics, Massachusetts Institute of Technology, Cambridge, Mass.

†Operated with support from the U. S. Air Force.

[1]J. J. Gilvarry, Phys. Rev. 89, 1046 (1953), and Publ. Astron. Soc. Pacific 65, 173 (1953).

[2]R. H. Dicke, Astron. J. 70, 395 (1965).

[3]I. I. Shapiro, Icarus 4, 549 (1965).

[4]S. Herrick, Astron. J. 58, 156 (1953).

[5]Inclusion of higher-order terms increases the difference. (Note also that the eccentricity of Mercury's orbit is about 0.2.)

[6]Each photograph yields a value both for right ascension and for declination. A detailed discussion of these observations will be published separately.

[7]G. M. Clemence, Mercury (planet), Astronomical Papers prepared for use of American Ephemeris and Nautical Almanac, Vol. 11 (U.S. Government Printing Office, Washington, D. C. 1943), p. 1; R. L. Duncombe, Mercury (planet), Astronomical Papers prepared for use of American Ephemeris and Nautical Almanac, Vol. 16, Pt. 1 (U.S. Government Printing Office, Washington, D. C., 1958), p. 1.

[9]I. I. Shapiro, G. H. Pettengill, M. E. Ash, M. L. Stone, W. B. Smith, R. P. Ingalls, and R. A. Brockelman, Phys. Rev. Letters 20, 1265 (1968).

[10]C. Brans and R. H. Dicke, Phys. Rev. 124, 925 (1961).

[11]R. H. Dicke and H. M. Goldenberg, Phys. Rev. Letters 18, 313 (1967).

[12]This approximation estimate for λ is based on (1) a relativistic contribution of 0.92, obtained by setting the free parameter s [R. H. Dicke and P. J. Peebles, Space Sci. Rev. 4, 419 (1965)] of the Brans-Dicke theory to 0.06 (Ref. 11), and (2) a contribution from a solar gravitational quadrupole moment of 0.12, obtained by assuming that 8% of the advance of Mercury's perihelion is due to the quadrupole moment (Ref.11) and by noting that its relative contribution to the advance of Icarus' perihelion is 50% greater (Ref. 3). Of course, if general relativity were correct and if the solar gravitational quadrupole moment had the value Dicke inferred, then λ would equal 1.12.

[13]See, for example, B. Bertotti, D. Brill, and R. Krotkov, in Gravitation: An Introduction to Current Research, edited by L. Witten (John Wiley & Sons, Inc. New York, 1962), p. 42.

[14]The uncertainty in the mass of the earth plus moon, relatively important because of Icarus' close approach in June, will also not affect substantially our estimate of λ.