fields. Although we did observe results similar to those of Fig. 1, experimental difficulties forced us to devote most of our efforts to the longitudinal mode.

Note added in proof.—Borman, Gorelik, Nikolaev, and Sinitsyn[12] have measured the effect of an alternating magnetic field on the thermal conductivity of oxygen and have observed an effect related to molecular precession.

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VELOCITY OF SECOND SOUND NEAR THE λ POINT OF HELIUM

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We report measurements of the velocity of second sound near T_λ. It is found to vanish with the 0.386 power of $T_\lambda - T$. The results confirm the theoretical expression and permit a direct asymptotic comparison for the first time of independent but theoretically equal quantities.

We have measured the velocity of second sound, U_2, as a function of temperature near the λ transition of helium. The results, which extend from 2×10^{-4} to 4×10^{-1} deg below T_λ, are shown in Fig. 1, which displays on a log-log plot the square of the measured velocity as a function of the temperature difference $t = T_\lambda - T$. The straight line given by

$$U_p^{\,2} = 1203 t^{0.772} (\mathrm{m/sec})^2 \qquad (1)$$

represents the data well for $t < 0.08$°K, the exponent being determined with a nominal uncertainty of ±0.005. In spite of this agreement, however, (1) is almost certainly not the correct asymptotic form.

The second-sound velocity was determined by observing the resonant frequency of a rectangular (3×4×4 cm) cavity made of lavite and Perspex. The cavity is the same one, slightly modified, as described elsewhere.[1] Absolute velocities could be determined with an estimated error of 2 %, it being assumed that no systematic error, such as temperature-dependent end effect, was involved in the conversion from frequency to velocity.

The temperature of the helium was measured with a carbon resistance thermometer embedded in a copper block sunk into the floor of the cavity. Temperature differences and, by extrapolation, the origin of t could be determined with a precision of $\pm2\times10^{-5}$ °K, suitable for the medium resolution for which the experiment was intended.

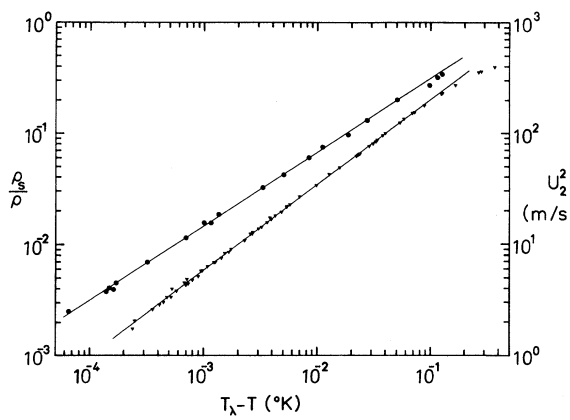

FIG. 1. Measured values of U_2^2 (triangles, right scale) shown as a function of $T_\lambda - T$ on logarithmic scales. Also shown (full circles, left scale) are values of ρ_S/ρ measured by Tyson and Douglass, Ref. 5. The straight lines represent the simple power expressions (1) and (3) in the text.

Absolute temperatures were determined within $\pm 1.5\times 10^{-4}$ by calibration against the helium vapor pressure. The second sound was generated and detected by means of carbon films on opposite sides of the cavity.

Apart from its intrinsic significance as an empirical property of liquid helium, the velocity of second sound is of particular interest since it is related by a precise phenomenological expression to a number of equilibrium properties, all of which have been measured close to T_λ. We thus have, probably for the first time, the opportunity for an accurate comparison of the apparent asymptotic behavior, near a cooperative transition, of independently measured but theoretically equated quantities.

Near the λ transition the usual equations of motion[2] of first and second sound become coupled by a term involving the diverging thermal expansion coefficient. The expression[3] for velocity $U_{\rm th}$ of the second-sound branch can be shown to become, near T_λ,

$$U_{\rm th}^2 = (\rho_s/\rho_n)TS^2/C_p, \qquad (2)$$

where ρ_s and $\rho_n = \rho - \rho_s$ are the superfluid and normal fluid densities, S and C_p the entropy and constant-pressure heat capacity per unit mass, respectively. Away from the λ point, where the coupling term can be neglected, the velocity is given by the more familiar expression obtained from (2) by replacing C_p with C_V, the constant-volume heat capacity. Actually (2) can be proven accurate to at least a part in 10^3 at any temperature; when the coupling is small and the expression should contain C_V the latter has, in any case, the same value as C_p.

The dependence of ρ_s/ρ on temperature near T_λ has been measured by Clow and Reppy[4] and by Tyson and Douglass.[5] The results of the latter have been reproduced in Fig. 1. They find for $t < 0.06°$K

$$\rho_s/\rho = 1.43t^{\xi} \qquad (3)$$

with $\xi = 0.666 \pm 0.006$, which we take as $\frac{2}{3}$. The specific-heat measurements of Buckingham, Fairbank, and Kellers[6] can be represented (for $t < 0.03$ deg) by

$$C_p = 1.30(\ln t^{-1} + 3.50)\ \text{J/g deg}. \qquad (4)$$

These results, together with others[7] further away from T_λ, can be integrated to obtain the entropy. In particular,

$$S_\lambda = S(T_\lambda) = 1.54 \pm 0.04\ \text{J/g deg}. \qquad (5)$$

Inserting these results in (2) we find the asymptotic form

$$U_\infty^2 = 6.06\times 10^3 t^{2/3}/(\ln t^{-1} + 3.50)\ (\text{m/sec})^2, \qquad (6)$$

where, for convenience in the analysis below, the coefficient has been multiplied by a factor 1.07. This factor is to normalize the various experimental results and is within the estimated 12% for the combined uncertainty in the absolute values of U_2^2 (4%) and the factors contributing in the theoretical expression (8%).

Examining first the empirical validity of (2) as an expression for U_2^2, we find that the measured quantities are in satisfactory agreement not only in absolute value—a test that can only be made with a precision of 12% (the actual absolute values obtained differ by 7%, accounting for the factor 1.07 above)—but also, and more accurately, in temperature dependence at all temperatures. Further from the λ point, satisfactory agreement with theory had already been demonstrated eight years ago by the results of Peshkov.[8] Figure 2 shows a deviation plot[9] of $U_{\rm th}^2$ (multiplied by the constant factor 1.07, as above) expressed as its percentage deviation from the function

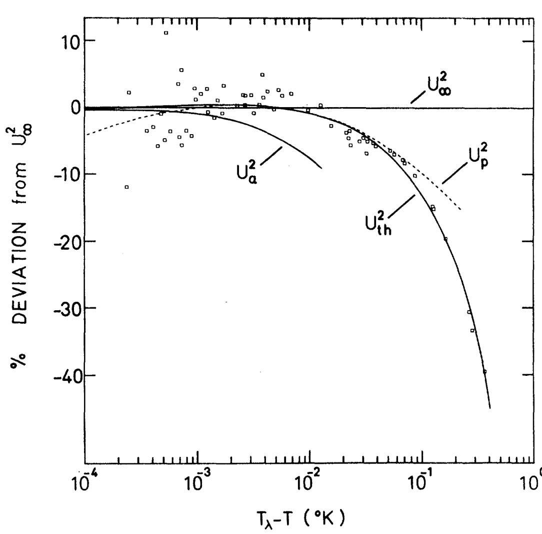

FIG. 2. Deviation plot of measured values of U_2^2 and various functions discussed in the text. The squares represent the same data as the triangles in Fig. 1. The value of each quantity at a temperature difference $t = T_\lambda - T$ is expressed as its percentage deviation from U_∞^2 [Eq. (6) in text] evaluated at the same t. The functions U_p^2, $U_{\rm th}^2$, and U_a^2 are defined in the text, Eqs. (1), (2), and (7), respectively.

U_∞^2, defined by (6). We see that for the whole temperature range it agrees within the experimental error with our measurements, similarly expressed as their percentage deviation from U_∞^2. We conclude that Eq. (2) as an expression for U_2^2 is fully supported by the experimental results, and henceforth we assume it to be exact. If this is so, it is clear that at least one of the empirical asymptotic expressions (1), (3), and (4) must be grossly in error, not as a numerical statement, but as a statement of an actual functional form.

The broken line in Fig. 2 represents the empirical expression U_p^2, Eq. (1). As it was chosen to do, it represents the observations well for $t < 0.08$°K. In fact it agrees with the measurements for nearly ten times further from T_λ than the asymptotic form U_∞^2 itself, which is only within experimental error for $t < 0.01$°K. Furthermore, even this range of agreement with U_∞^2 is due to a certain numerical "accident." According to (3) and (4), $U_{\rm th}^2$ can be represented for $t < 0.03$ deg by

$$U_\infty^2 \times (TS^2/T_\lambda S_\lambda^2) \times (\rho/\rho_n).$$

It so happens that the two factors in parentheses cancel each other to within 1 % over a range of t in which either factor alone has changed by nearly 10 %. This is illustrated in Fig. 2 by the curve U_a^2 which represents U_∞^2 multiplied by the first factor alone, i.e.,

$$U_a^2 = U_\infty^2 \times TS^2/T_\lambda S_\lambda^2. \qquad (7)$$

This function of course still has the same asymptotic form as $U_{\rm th}^2$.

Concerning the actual functional forms, it is clear that if (3) and (4) correctly describe ρ_S/ρ and C_p, then the asymptotic form of U_2^2 is given by U_∞^2, that is $U_2^2 \sim t^{2/3}/\ln t^{-1}$. This results, as in the case of ρ_S/ρ, in a "critical exponent" of $\frac{2}{3}$. This is far outside the uncertainty of the apparent empirical exponent 0.772 ± 0.005. It is clear, however, from Fig. 1 that a simple power describes the measurements of U_2^2 just as well as it does ρ_S/ρ. Thus the experimental evidence, by itself, cannot rule out quite different exponent values. For example, even retaining the logarithmic form for C_p, the results permit $\rho_S/\rho \sim (t \ln t^{-1})^{3/4}$ and $U_2^2 \sim (t^3 \ln t^{-1})^{1/4}$, giving exponent values of $\frac{3}{4}$. Theoretical evidence can of course restrict the possibilities considerably. Thus Josephson[10] has shown that if one accepts certain results of scaling arguments, $\rho_S/\rho \sim t^{(2-\alpha)/3}$ and therefore $U_2^2 \sim t^{(1+\alpha)/3}$, where $C_p \sim t^\alpha$. Still assuming $C_p \sim \ln t^{-1}$, one might then have $\rho_S/\rho \sim (t^2 \ln t^{-1})^{1/3}$ and $U_2^2 \sim (t/\ln t^{-1})^{2/3}$.

There is no novelty in the remark that the asymptotic form of a function may be an extremely bad approximation. Its significance, however, is vividly emphasized by the present example. While one may hope that this is a particularly unfortunate case, it nevertheless serves as a warning against the too-literal acceptance of empirically determined asymptotic forms.

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