NUCLEATION OF SUPERCONDUCTIVITY ABOVE H_{c3}

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It is shown theoretically for systems with a special geometry that nucleation of superconductivity can occur for magnetic fields above H_{c3}.

A material can become superconducting if one decreases the strength of an applied magnetic field below a certain value, called the nucleation field. Landau and Ginzburg[1] have shown that the value of this critical field for a bulk material, H_{c2}, equals $\kappa\sqrt{2}$ times the value of its thermodynamical critical field, H_{cb}. It was not until recently that Saint-James and de Gennes[2] discovered the existence of a larger nucleation field, called H_{c3}, by considering the possibility of nucleation at the surface of a semi-infinite material. It is obvious to question whether H_{c3} is a universal upper limit for nucleation to occur. In other words, is it conceivable to have a material with such a geometry that nucleation might occur above H_{c3}? An important contribution to this problem was made by Druyvesteyn,[3] who suggested that the process of nucleation may be favored by the presence of vacuum interfaces. A system with two intersecting vacuum interfaces might hence very well have a nucleation field above H_{c3}. For this reason we consider a material which (in a system of polar coordinates r, φ, z) occupies that region of space for which $|\varphi| \leq \alpha$, if α is a given angle. The applied magnetic field is parallel to the z axis.

An intuitive motivation for the assumption just mentioned is the following: The problem of nucleation resembles that of finding the ground-state energy of a particle in a magnetic field, the ground-state energy being inversely proportional to the nucleation field. Since Saint-James and de Gennes have shown that the ground state is localized near the boundary of the semi-infinite system, the wall may qualitatively be considered as attracting the particle. A particle inside the system we just defined is being attracted by two boundaries instead of one, and may thus become more tightly bound compared with the situation where just one wall is present. A serious objection against carrying the analogy too far, however, is the inequivalence of the boundary conditions in both problems. In spite of this, one can show that nucleation does indeed occur above H_{c3} if the system has a geometry as defined above.

Our proof is based on the variational formulation of the problem of nucleation. For this purpose we consider the difference in the Gibbs free energy ΔG between the superconducting and the normal phases. Nucleation becomes possible when the difference vanishes, i.e.,

$$\Delta G = \int d\vec{r}\,\kappa^{-2}(\nabla f)^2 + \int d\vec{r}\,\{(\vec{A}+\kappa^{-1}\nabla\Phi)^2 - 1\}f^2 = 0. \quad (1)$$

Here, $f(\vec{r})$ is the modulus of the Landau-Ginzburg order parameter and $\Phi(\vec{r})/\kappa$ is its phase. The vector potential $\vec{A}$ represents a homogeneous magnetic field parallel to the z axis. In polar coordinates we choose

$$\vec{A} = \tfrac{1}{2}Hr\vec{e}_{\varphi}. \quad (2)$$

Here H is the applied magnetic field, measured in units of $H_{cb}\sqrt{2}$. If one introduces the unit of length $(\kappa H)^{-1/2}$ into Eq. (1), it reduces to

$$\int d\vec{r}(\nabla f)^2 + \int d\vec{r}\left\{\left(\frac{\partial\Phi}{\partial r}\right)^2 + \left(\frac{1}{2}r + \frac{1}{r}\frac{\partial\Phi}{\partial\varphi}\right)^2 - E\right\}f^2 = 0. \quad (3)$$

The number E is defined as $\kappa\sqrt{2}\{H_{cb}/H\}$, where H_{cb} is the thermodynamic bulk critical field. Since the current density component normal to the vacuum interfaces must vanish, $f(\vec{r})$ and $\Phi(\vec{r})$ are subject to the following restrictions:

$$\left.\frac{\partial f}{\partial\varphi}\right|_{\varphi=\pm\alpha} = 0 \text{ and } \left.\frac{\partial\Phi}{\partial\varphi}\right|_{\varphi=\pm\alpha} = -\tfrac{1}{2}r^2. \quad (4)$$

The problem of nucleation is to determine the lowest value of E for which Eq. (3) can be satisfied. This corresponds to the largest magnetic field H_c for which the superconducting phase becomes stable. The procedure which we follow is analogous to that of de Gennes[4] for the semi-infinite system. It implies that one selects two continuous and differentiable functions $f(\vec{r})$ and $\Phi(\vec{r})$ which satisfy the conditions of Eq. (4). Substitution of these functions into Eq. (3) gives a value of E for which nucleation is certainly possible. The lowest value of E which we have succeeded

in finding this way corresponds to the following set of trial functions:

$$f(\vec{r}) = \exp\{-\tfrac{1}{2}(r/d)^2\} \quad (5)$$

and

$$\Phi(\vec{r}) = -\tfrac{1}{2}r^2\alpha F(\varphi/\alpha). \quad (6)$$

Here, d is an adjustable constant and F is a function which satisfies $F'(\pm 1) = 1$. Substitution of these expressions into Eq. (3) yields

$$E = Cd^2 + \frac{q^2}{d^2}, \quad (7)$$

with

$$C = \alpha^2 \int_0^1 F^2(x)dx + \tfrac{1}{4}\int_0^1 \{F'(x)-1\}^2 dx. \quad (8)$$

The lowest value of E is reached if one selects $d^4 = 1/C$, which leads to $E = 2\sqrt{C}$. In order to find the lowest value of E, we minimize the functional C with respect to F, which gives

$$F = \frac{1}{2\alpha}\frac{\sinh(2\alpha\varphi)}{\cosh(2\alpha)}.$$

Hence, the lowest value of E satisfies

$$E \leq \{1-\tanh(2\alpha)/2\alpha\}^{1/2}. \quad (9)$$

We have tried to improve this value of E further by letting the range of the trial function $f(\vec{r})$ be dependent on the polar angle φ. Explicitly,

$$d(\varphi) = \mu\{1 + p(1-\varphi^2/\alpha^2)^2\}.$$

It has turned out that the parameter p, which corresponds to the lowest value of E, is very small and that the corresponding reduction of E is less than 1%.

The value of E which corresponds to H_{c3} is 0.59. It follows from Eq. (9) that the nucleation field for superconductivity is certainly larger than H_{c3} if the angle 2α does not exceed 76°. Since the nucleation field H_{c2} corresponds to $E = 1$, it follows from Eq. (9) that nucleation occurs for a value of H_{c4} which satisfies

$$\frac{H_{c4}}{H_{c2}} \geq \left\{1-\frac{\tanh(2\alpha)}{2\alpha}\right\}^{-1/2}. \quad (10)$$

It follows from Eq. (10) that the nucleation field increases towards infinity at least like $\{\frac{2}{3}\alpha\sqrt{3}\}^{-1}$ if $\alpha \to 0$, so that nucleation can in principle occur for any field above H_{c3}, depending on the value of α. Another conclusion is that irregularities of the surface, like scratches, may cause nucleation to occur for fields above H_{c3}.

It has not been proven that a critical angle $\alpha < \pi/2$ exists below which the nucleation field exceeds H_{c3}. This question can only be solved by a rigorous analysis of the Landau-Ginzburg equations for our geometry.

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[2]D. Saint-James and P. G. de Gennes, Phys. Letters 7, 306 (1963).

[3]W. E. Druyvesteyn, private communication.

[4]P. G. de Gennes, Superconductivity of Metals and Alloys (W. A. Benjamin, Inc., New York, 1966), p. 199.

MEASUREMENT OF PHOTOABSORPTION OF THE SODIUM HALIDES NEAR THE SODIUM $L_{2,3}$ EDGE

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The optical absorption of the sodium halides (NaF, NaCl, NaBr, NaI) has been studied at energies about 30 to 75 eV, i.e., in the neighborhood of the sodium $L_{2,3}$ edge. The intense continuum of the synchrotron radiation emitted by the 6-GeV electron synchrotron has been used as the light source.

The investigation of transitions from inner shells to the conduction band of the sodium halides complements information on the electronic band structure gained from optical measurements in the region of fundamental absorption.[1-3] We have studied photoabsorption due to transitions from the $L_{2,3}$ shell of Na^+ in the four sodium halides (NaF, NaCl, NaBr, and NaI) with onset at photon energies of about 32 eV (~370-Å wavelength). Similar measurements have been performed by O'Bryan.[4] We have obtained improved results by using the synchrotron radia-