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MOTION OF NEGATIVE IONS ALONG QUANTIZED VORTEX LINES

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The motion of negative ions trapped in quantized vortex lines in rotating helium II has been measured down to 0.8°K by means of a time-of-flight apparatus. Trapped ions always move more slowly than free ions at the same temperature. The contribution of the extra retardation due to the vortex line appears to vary as $\exp(-\Delta/kT)$, where $\Delta/k=1.87$ °K between 0.8 and 1.5°K.

The interaction of ions with quantized vortices has proved a powerful tool in studying the properties of the vortices and, indeed, of the ions themselves.[1] The behavior of quantized vortex rings as evidenced by ion experiments has served to determine experimentally the magnitude of the quantized circulation.[2] The scattering and trapping of ions by vortex lines have yielded information about the motion of ions and vortex rings in the velocity field of the lines,[3,4] and the escape of trapped ions has served to determine the size of the ions.[5] The picture which emerges from all this confirms the general ideas one has of a quantized vortex line, but leaves the impression that with the exception of the quantization of circulation, the vortices behave entirely classically. It is necessary to look at the properties of vortices near the core in order to study the interesting region where many-body effects may become important. (Surprisingly enough, we know very little about the cores of classical vortices!) Unfortunately, such effects are expected to occur only within a few angstroms of the center of the vortex, and trapping experiments give information only down to the order of 100 Å. Further information can be inferred from vortex-creation experiments[6] or from vortex-wave experiments.[7] The motion of a trapped ion along a vortex line, however, offers a direct way to study the properties of a vortex core by means of a probe located directly on the site of interest. The diameter of a negative ion at the vapor pressure is approximately 32 Å and the characteristic length for the core is of order 1.4 Å. The ion therefore constitutes a fairly large discontinuity in a vortex line. In motion induced by an electric field in the direction of the line, the ion will be expected to encounter collisions related to those encountered when it is free, plus collisions owing to extra excitations or He^3 atoms which might concentrate near the vortex core, and finally excitations of the vortex line itself. Evidence for extra resistance encountered by trapped ions has been reported by Douglass[8] and by Domingo and Donnelly.[9]

The experimental cell and associated electronics are shown in Fig. 1. There are two regions in the cell: the drift space between $G3$ and $C1$, and the storage space between R and $G2$. These two regions are separated by the gating grids $G2$, $G3$ which are "open" during a pulse and "closed" between pulses. Ions are produced at S by means of an Am^{241} α source of strength 190 μCi and separated by means of a field between S and $G1$. Negative ions are then pulled towards collector $C2$ by an appropriate potential. The bottom gating grid $G2$ is always at a lower (less negative) potential than any other electrode in the storage region. Between pulses one has a continuous charge density in the storage region which is cut off sharply at $G2$. During a pulse, $G2$ is raised to a potential above $G3$ thus "opening" the gate. Simultaneously a pulse applied to $C2$ improves the pulse characteristics by making $G1$, R, and $C2$ equipotentials, so that all ions are forced through $G2$. The ions then enter the drift space and, experiencing the field between $G3$ and $G4$, are collected at $C1$. $C1$ is shielded from the effects of image charges by means of the Frisch grid $G4$. The field between $G2$ and $G3$ is large, so that the transit time through the gating grids is small. The pulse begins to record as soon as it crosses $G4$ and hence transit times are independent of the potential between $G4$ and $C1$. Accordingly the drift space is defined as the distance between $G3$ and $G4$ and is 4.93 cm. Guard rings on a resistive chain are provided to establish a uniform field in the drift region.

The ion current arriving at $C1$ is detected with a Keithley Model 417 high-speed electrometer whose output is a voltage proportional to the input current. This output is fed to a Hewlett-Packard 2212A voltage-to-frequency converter, and the trains of pulses from it are fed into an RIDL 400-channel analyzer operated in the time sequence storage mode. Textronix 161 pulse generators are used to provide pulses to $G2$ and $C2$, while at the same instant providing a trigger pulse to initiate the storage sequence in the analyzer. One pulse provides the RIDL memory with the time dependence of the current collected at $C1$. This process is repeated many times to improve the signal-to-noise ratio.

The pulse technique described above allowed us to detect the time of flight of free and trapped ions simultaneously. Examples of the output of the signal averager are shown in Fig. 2 taken with moderate and maximum damping of the electrometer. Since arrival times were independent of damping, we generally preferred to use maximum damping for reduction of noise. A series of subsidiary experiments were conducted to find a reliable pulsing technique, since under certain conditions of pulsing, anomalous times of flight could be recorded. The conditions used in this experiment above 1°K are shown on Fig. 1. A slightly modified pulse technique enhancing the trapped pulse was used below 1°K. One can see from Fig. 2 that the trapped-ion pulse is considerably narrower than the free-ion pulse. This is because the potentials around $G2$ form a well and the trapped ions are constrained on the line and cannot discharge on the grid wires. An investigation of the field dependence of this effect indicated that this "bunching" introduces negligible uncertainty in the transit times.

The apparatus, including low-level electronics, was mounted on the University of Oregon 54-in. rotating table which carries a He^3 cryostat. The externally pressurized hydraulic bearing on this table insures exceptionally smooth and precise rotations. Speeds up to 60 rpm were used.

Experiments have been carried out between 1.7 and 0.4°K. This Letter reports measurements down to 0.8°K. Below 0.8°K the results change qualitatively due to the appearance of quantized vortex rings instead of free ions and

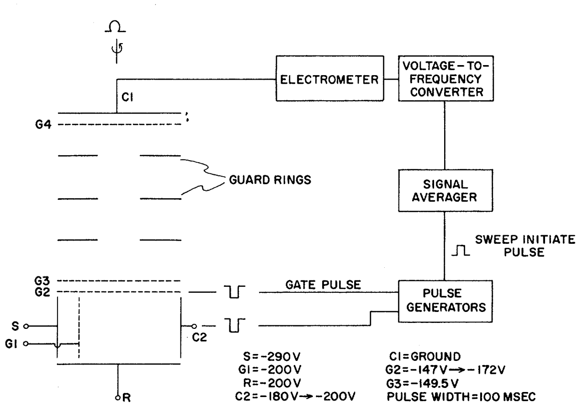

FIG. 1. Schematic diagram of the apparatus for measuring trapped-ion mobilities. The voltage conditions shown were used to obtain the data of Fig. 3.

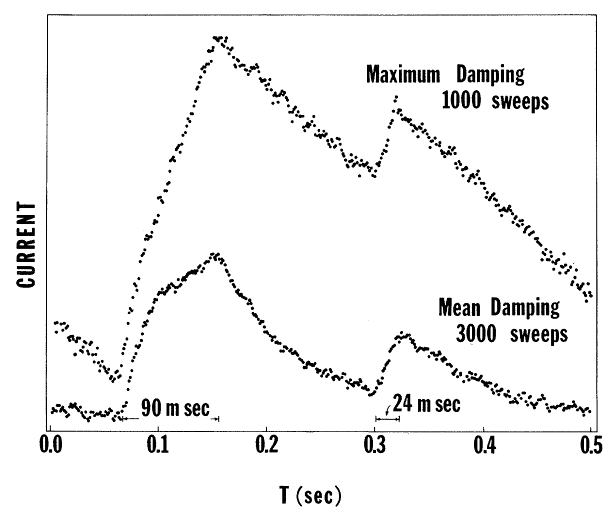

FIG. 2. Output of the signal averager for moderate and maximum damping of the electrometer. Note the relative widths of the free and trapped pulses. The ordinate is arbitrary and corresponds to electrometer current.

will form a separate continuing investigation. The interaction of rings with lines in this temperature range has already been investigated.[4]

Experiments conducted within the range of parameters available to us show that the free-ion and trapped-ion arrival times were independent of the rate of rotation Ω and of the current density. The Ω independence of trapped-ion times of flight means the motion along vortices is independent of vortex density. This in turn ensures that we are dealing with ion motion along individual vortices which extend from the bottom to the top of the apparatus, threading through the grids—a distance of nearly 7 cm. We have, therefore, established the existence of single, macroscopic, quantized vortex lines extending continuously over 7 cm. This observation supplies the evidence called for by Lin[10] to establish the macroscopic vortex picture of rotating He II.

The free-ion times of flight shown in Fig. 3 coincide, within the joint experimental errors, with the mobility data of Reif and Meyer[11]: The slope of the curve corresponds to $\Delta/k = 7.97 \pm 0.16$°K. The field dependence of free- and trapped-ion mobilities is nearly constant with a slight rise—10% in both—at low fields. This establishes that in the range 1.7 to 0.8°K trapped-ion motion is a true mobility phenomenon.

There is a possibility that the extra drag on the trapped ion is due to He^3 impurity atoms. Our well helium has a natural concentration of He^3 of approximately 1.4×10^{-7}. We added enough He^3 to bring this concentration up to 5×10^{-6} and were unable to detect any influence on free or trapped transit times. The transit times are also highly reproducible over many months. This indicates that our extra drag is a property of liquid helium and not of trace impurities.

If we examine the data of Fig. 3 we find that the trapped ions experience extra resistance at all temperatures from the "lifetime edge" at 1.7 °K downward. The difference in transit times is only 30% at 1.7°K, but at 0.8°K they are more than a factor of 10 different. If we assume that the scattering agents are dilute and independent (an assumption which may not be justified), then we can subtract the transit times and examine the difference. The difference varies as $\exp(-1.87/T)$ and suggests that the extra resistance along the vortex line may correspond to excitations having an effective energy gap of 1.87°K. Any theory of this effect must consider excess roton density near the core due to a shift in energy of

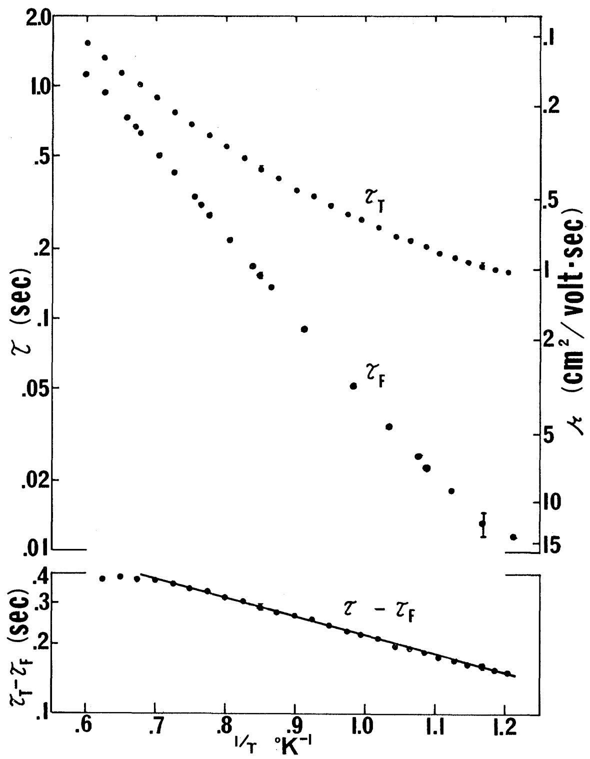

FIG. 3. The time of flight of free and trapped ions at a field of 30 V/cm. The difference in these times is plotted also. The straight line has a slope $\Delta/k = 1.87$°K.

rotons as they enter the rotating superfluid field near the core, i.e., a $\vec{p}\cdot(\vec{v}_n-\vec{v}_s)$ effect. At low enough temperatures vortex wave excitations will be expected to contribute to the extra resistance.

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NEW METHOD FOR LINEARIZING MANY-BODY EQUATIONS OF MOTION IN STATISTICAL MECHANICS

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A new criterion is proposed for linearizing many-body equations of motion which is well defined and is related to a stationary principle. The method is applied to the problem of correlation in a narrow s band and the results differ considerably from Hubbard's treatment of this problem.

One of the advantages of Green's function[1,2] and equation-of-motion methods[3,4] in statistical mechanics has been the fact that operators can be used which are not strictly fermion or boson operators. For example, the Green's-function method has been used for spins by Tyablikov[5] and others, and for atomic states in narrow energy bands, by Hubbard.[6] The above freedom brings with it disadvantages, however, because the truncation procedures are somewhat arbitrary. We propose here a prescription for truncating equations of motion which is well defined and which is related to a stationary principle.

We discuss the truncation scheme for a general many-body problem and in terms of the equation-of-motion method. We shall later apply it to the narrow s band considered in Hubbard I,[6] whose Hamiltonian is[7]

$$H=U\sum_i n_{i\uparrow}n_{i\downarrow}+\sum_{ij\sigma}t_{ij}c_{i\sigma}^{+}c_{j\sigma}, \tag{1}$$

where c's are creation and annihilation operators for Wannier sites, and the notation is as in Ref. 7. In the equation-of-motion method, we attempt to construct a creation or annihilation operator A for a quasiparticle, which satisfies in some approximation

$$[A,H]=\omega A. \tag{2}$$

Here, if we think of A as an annihilation operator, then ω is the quasiparticle energy. More generally, we might attempt to find a basis set of operators $\{A_i\}$ such that

$$[A_i,H]=\sum_j K_{ij}A_j. \tag{3}$$

If we obtain Eq. (3), we can then diagonalize K to give Eq. (2). Of course, for systems with interactions, Eqs. (2) and (3) are not in general satisfied by simple operators, so that one must make approximations. What is usually done is to replace some operators on the right-hand side by their expectation values. The simplest example is the Hartree-Fock approximation, for which we use only one-fermion operators and for which in our example we have, using $c_{i\sigma}$ as the basis set,

$$K_{ij}{}^{\sigma}=t_{ij}+U\langle n_{-\sigma}\rangle. \tag{4}$$

Higher approximations have been worked out by many authors, and we mention especially Suhl and Werthamer's[4] work in which they show how to include all three-fermion operators in the set $\{A_i\}$ and to truncate expressions with five-fermion operators. However, in Hubbard's[6] work it was shown that the most important correlations in a narrow energy band are those on a single atomic site, so that he proposed decoupling according to particular states of occupancy of atomic sites rather than according to the number of fermion operators involved. In particular, for the case of the narrow s band he singled out one type of three-fermion operator, $n_{-\sigma}c_{i\sigma}$. The result of Hubbard's first calculation was criticized by Harris and Lange,[8] who showed that certain moments of the spectral function[8,2] were not reproduced correctly. This seems to be a result of the ambiguities in truncating equations of motion with one but not all three-fermion operators.

Our proposal is to use the following prescription for determining the K_{ij} self-consistently: Let us commute or anticommute, according to the statistics, both sides of Eq. (3) with another member of the set and then take either the expec-