SPIN-PARITY ANALYSIS OF THE B MESON*

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From the decay distribution and the polarization of the ω in $B\to\omega\pi$, the B is found to have either $J^P=1^+$ or $J^P=2^+3^-4^+\cdots$. The observed polarization of the B appears more plausible for the $J^P=1^+$ hypothesis.

An ω-π resonance with $M\approx 1.22$ BeV was discovered by Abolins et al.[1] and dubbed the B meson; doubts raised in later studies[2,3] of the B in $\pi^\pm p\to p\pi^\pm\omega$ were dispelled by observation of the B in $\bar{p}$-p annihilation.[4] The strong decay $B\to\omega\pi$ implies $I^G=1^+$. Therefore, the decay modes $B\to\pi\pi$ and $B\to K\bar{K}$ are allowed for $J^P=1^-3^-\cdots$ and forbidden for $J^P=0^-1^+\cdots$ or $J^P=2^+4^+\cdots$. Since neither the $\pi\pi$ or $K\bar{K}$ mode has been observed, $J^P=1^-3^-\cdots$ can probably be ruled out. No data have been published,[5] however, to support the assignment $J^P=1^+$ favored by model builders. In a study of the angular distribution and polarization of the ω from $B\to\omega\pi$, we find that the B must have either $J^P=1^+$ or $J^P=2^+3^-\cdots$. However, for $J^P=2^+3^-\cdots$, the B is produced with a degree of alignment ($|J_Z|$ nearly equal to J along the "exchange axis") which we believe rather unlikely to occur in a peripheral reaction.

From an exposure of the 72-in. Berkeley hydrogen bubble chamber to a 5-BeV/c π^- beam, we obtained 55 000 four-prong events. From these we selected a sample compatible by ionization and kinematics with

$$\pi^-p\to p\pi^-\pi^-\pi^+\pi^0 \quad (8255 \text{ events}). \qquad (1)$$

We required, for acceptance, a one-constraint fit with $\chi_{1C}^2<6.3$ for (1) and no fit (with a $\chi_{4C}^2=40$ cutoff) for $\pi^-p\to p\pi^-\pi^-\pi^+$. From this sample, the final sample, compatible with

$$\pi^-p\to p\pi^-\omega, \quad \omega\to\pi^-\pi^+\pi^0 \quad (1310 \text{ events}), \qquad (2)$$

was selected by requiring $\chi_{2C}^2-\chi_{1C}^2<3$, where χ_{2C}^2 is the χ^2 for the two-vertex, two-constraint (fixed m_ω) fit to (2). The method of selection is nearly equivalent to requiring the $(\pi^-\pi^+\pi^0)$ mass from fit (1) to be within $\sqrt{3}\times$(measurement error) of the ω mass. We chose this method to avoid a bias against events with kinematical configurations resulting in higher errors on $M(3\pi)$. The $(\pi^-\pi^+\pi^0)$ mass distributions for samples (1) and (2) are shown in Fig. 1(a). We estimate that the non-ω background accounts for about 300 out of the 1310 events in the final sample.

The $(\omega\pi^-)$ mass distribution [Fig. 1(b)] shows a prominent B peak, centered at $M(\omega\pi^-)\approx 1.24$ BeV, with a width ≈200 MeV. We cannot give here a complete account of the data in the $\omega p\pi^-$ channel. We would like, however, to point out that objections[2,3] raised in earlier $\pi^\pm p\to pB^\pm$ experiments do not seem to apply to the present data:

(1) We have not found the anomaly in the ω Dalitz plot reported by Goldhaber et al.[2]

(2) The $\omega\pi^-p$ Dalitz plot (not shown) shows that the lower part of the B band does have structure associated with crossing N^{*0} bands, but that a comparable population exists in the upper half of the B band [$M^2(p\pi)>3.3$ BeV^2].

(3) The t distribution of B events (after back-

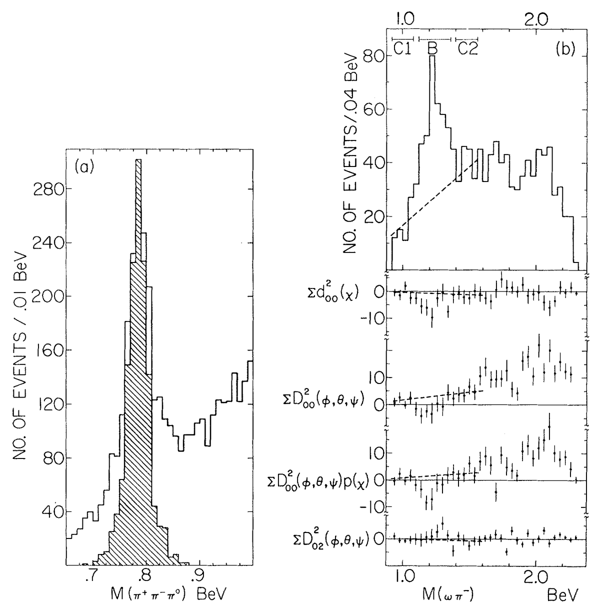

FIG. 1. (a) Mass spectrum of $(\pi^+\pi^-\pi^0)$ from Reaction (1). Hatched area shows events accepted as Reaction (2). (b) Mass $(\omega\pi^-)$ spectrum and unnormalized moment distributions. The control regions C_1 and C_2 used in the background subtration go from 1.40 to 1.52 BeV, while the resonance region B extends from 1.12 to 1.36 BeV. The dashed lines indicate the linear interpolation of background from the control regions.

ground subtraction) is shown in Fig. 2. B production is not confined to small momentum transfers, although a sharp forward peak is observed. For small $|t|$ the distribution is $\sim\exp(At)$ with a slope $A\sim 4$ BeV^{-2}. We conclude from (2) and (3) that the "Deck" interpretation of the B, suggested by Chung et al.,[3] does not seem valid at this energy. We emphasize, however, that our data are affected to some extent by overlap and possibly by interference with N^{*0} production. We point out in particular that we do not consider estimates of the B mass and width from this experiment to be meaningful.

To explain the method and the notation, we give a brief account of the theory[6] relevant to the spin-parity analysis of the B. From rotational invariance, the amplitude[7] for the sequential decays ($B\to\omega\pi$, $\omega\to 3\pi$) from a state with definite J, J_z is

$$\mathfrak{M}(JJ_z)\propto\sum_\lambda F_\lambda D_{J_z\lambda}^{J*}(\varphi\theta 0)D_{\lambda 0}^{1*}(\psi\chi 0), \quad (3)$$

where we have used the following notation: $\lambda=\omega$ helicity (B frame); $(\varphi,\theta)=$(azimuthal, polar) angles of $\hat{p}_\omega$ (B frame), referred to axes $\hat{x}\hat{y}\hat{z}$, for which we make the usual choice (in B frame, $\hat{z}$ along $\pi_{\rm in}^-$, $\hat{y}$ normal to production plane); (ψ,χ) = angles of $\hat{n}$, the normal to the ω-decay plane (ω frame), referred to axes $\hat{x}_H\hat{y}_H\hat{z}_H$, defined (in B frame) by $\hat{z}_H=\hat{p}_\omega$, $\hat{y}_H\propto\hat{z}\times\hat{z}_H$. With the normalization

$$\sum_\lambda |F_\lambda|^2=1, \quad (4)$$

Eq. (3) gives for the normalized distribution in $(\theta\varphi\chi\psi)$

$$W=(4\pi)^{-2}\sum_{LM\mu} t_{LM}^{\;*}D_{M\mu}^{L*}(\varphi\theta\psi)\tau_{L\mu}^{\;*}(\chi), \quad (5)$$

where

$$t_{LM}^{\;*}=(2L+1)\sum_{ab}\langle Ja|JLbM\rangle\rho_{ab},$$

$$\tau_{L\mu}^{\;*}=3\sum_{\alpha\beta}\langle J\alpha|JL\beta\mu\rangle F_\alpha F_\beta^{\;*}d_{\alpha 0}^{\;1}(\chi)d_{\beta 0}^{\;1}(\chi).$$

Parity conservation in $B\to\omega\pi$ requires that $F_{-\lambda}=\epsilon F_\lambda$ with $\epsilon=1$ for $J^P=0^-1^+\cdots$, $\epsilon=-1$ for $J^P=1^-2^+\cdots$. Two methods to determine the parity follow from this relation: (1) One may look for longitudinal ω polarization ($F_0\neq 0$), which is done most simply by studying the $\cos\chi=\hat{n}\cdot\hat{p}_\omega$ distribution, given by

$$W(\chi)=(\tfrac{3}{2})[|F_0|^2\cos^2\chi+|F_1|^2\sin^2\chi]. \quad (6)$$

The presence of a $\cos^2\chi$ term would rule out $J^P=1^-2^+\cdots$ (since $F_0=0$ for this series), while the absence (within errors) of a $\cos^2\chi$ term would rule out only $J^P=0^-$, since F_0 could be small for $J^P=1^+2^-\cdots$. (2) If the data are consistent with $F_0=0$, the parity may be determined by finding the ω-polarization plane. For $J^P=1^+2^-\cdots$, the ω polarization should vary from complete polarization in the $\hat{z}$-$\hat{p}_\omega$ plane for $J_z=0$ to (nearly) complete polarization along $\hat{z}\times\hat{p}_\omega$ for $J_z=\pm J$. For $J^P=1^-2^+\cdots$, the opposite polarizations should occur. A practical method is to compare (for fixed even L, fixed M) the coefficients of $D_{M0}^{L*}(\varphi\theta\psi)$ and of $D_{M\pm 2}^{L*}(\varphi\theta\psi)$; the two terms have the same dependence on B polarization, and the second term is proportional to $F_{\pm 1}F_{\mp 1}^{\;*}=\epsilon|F_1|^2$. Since we find no significant φ dependence or $L>2$ moments in our data, we give below only the formulas for the $L=2, M=0$ moments [obtained from Eq. (5)]:

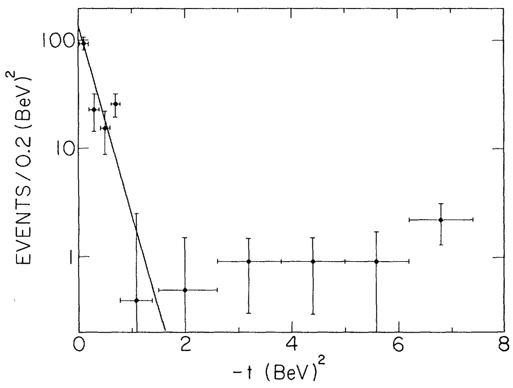

FIG. 2. Subtracted t distribution for B events. The same control and resonance regions as in the moment analysis are used, and the background subtraction is handled in the same fashion.

$$A_{00}^{\;2}\equiv 5\langle D_{00}^{\;2}(\varphi\theta\psi)\rangle = Q_J\left\{\frac{|F_0|^2}{2|F_1|^2}+1-\frac{3}{J(J+1)}\right\}, \quad (7a)$$

$$S_{00}^{\;2}\equiv 5\langle D_{00}^{\;2}(\varphi\theta\psi)\mathcal{P}(\chi)\rangle = Q_J\left\{1-\frac{3}{J(J+1)}\right\}, \quad (7b)$$

$$A_{0\pm 2}^{\;2}\equiv 5\langle D_{0\pm 2}^{\;2}(\varphi\theta\psi)\rangle = Q_J\{(\tfrac{3}{8})^{1/2}\epsilon\}, \quad (7c)$$

where we put[8]

$$Q_J=\frac{5J(J+1)}{4J(J+1)-3}x;$$

$$x=\left\{1-\frac{3\langle J_z^{\;2}\rangle}{J(J+1)}\right\}2|F_1|^2. \quad (7d)$$

In (7b) the projection operator $\mathcal{P}$ was introduced to select the contribution from transversely polarized ω's:

$$\mathcal{P}(\chi) \equiv \tfrac{5}{2}\sin^2\chi - 1. \tag{7e}$$

We discuss next the method used to subtract background: We simply subtract from each observed distribution (or moment) in the B region the background calculated by linear interpolation between control regions below and above the B. As partial justification for the implied neglect of interference between resonant and background amplitudes, we note that the subtracted angular distributions (not shown) satisfy, within errors, all symmetries expected for the decay of a pure J^P state. We emphasize, however, that all errors quoted in Table I are purely statistical (errors in control-region data and correlations between moments are included); no allowance has been made for systematic errors arising from interference and from the choice of background interpolation procedure.

We come finally to the results displayed in Fig. 1(b) and summarized in Table I. Figure 1(b) shows various unnormalized moments[9] versus $\omega\pi^-$ mass (the dashed lines indicate the linear interpolation used to estimate background); Table I(A) shows the same moments summed over the two control regions and over the B region [the three regions are defined in Fig. 1(b)]; also shown in Table I(A) are the moments for the B region after subtraction of the background and normalization to one event. Moments not shown vanish within errors. We note the following:

(A) The distribution in $\cos\chi = \hat{n}\cdot\hat{p}_\omega$ is nearly pure $\sin^2\chi$. From the moment $\langle 5d_{00}{}^2(\chi)\rangle = 3|F_0|^2 - 1 = -0.70 \pm 0.25$, we get $|F_0|^2 = 0.10 \pm 0.08$. This result, consistent with $F_0 = 0$, rules out only $J^P = 0^-$. We note (for $J^P = 1^+$) that the result disagrees, by about 2.7 standard deviations, with $|F_0|^2 = \frac{1}{3}$, which is required for pure S-wave decay; the result disagrees also with the value $|F_0|^2 = 1$ given by a simple quark-antiquark model computation.[10] We have looked for additional symptoms of $F_0 \neq 0$ [e.g., we looked at $\langle D_{01}{}^2(\varphi\theta\psi) \times d_{10}{}^2(\chi)\rangle$ which is proportional to $\mathrm{Re}(F_0 F_1{}^*)$], but we have found no result more significant than the one quoted.

(B) The moment $S_{00}{}^2 = 5\langle D_{00}{}^2(\varphi\theta\psi)\mathcal{P}(\chi)\rangle = -0.83 \pm 0.28$ means that the angular distribution of transversely polarized ω's favors the equatorial plane; crudely this means[11] that either $J = 1, J_z \approx 0$ or $J > 1, J_z \approx \pm J$. The sign of $A_{02}{}^2 = 0.38 \pm 0.19$ means that the ω's are polarized (mainly) in the $\hat{z}$-$\hat{p}_\omega$ plane; it appears therefore that either $J^P = 1^+$ or $J^P = 2^+3^-\cdots$. For a quantitative test we can look [Table I(B)] at the two values of x—defined in (7d)—computed from the two moments $S_{00}{}^2$ and $A_{02}{}^2$ using Eqs. (7); the two values agree, within the large experimental errors, for $J^P = 1^+$ or $J^P = 2^+3^-\cdots$, but disagree by about 3.5 standard deviations for the opposite parity choices.

(C) From the weighted average of x for each surviving J^P, we obtain the following information about the polarization of the B: (1) For $J^P = 1^+$, using $2|F_1|^2 = 0.90 \pm 0.08$ we can calculate the diagonal elements of the spin matrix; we find $\rho_{00} = 0.65 \pm 0.11$. (2) For $J^P = 2^+3^-4^+\cdots$, the

Table I. Summary of B-decay data. (All errors are statistical.)

A. Moments of decay distribution.

Weight, f_α	Weighted events, $\sum_i f_{\alpha i}$ Control regions C1	C2	B Region	B moments, subtracted, normalized.
1	65±8	158±13	350±19	(182±22 evts)
$5\,d_{00}^2(\chi)$	-11±19	-25±27	-154±39	$3\lvert F_0^2\rvert-1$ = -.70±.25
$5\,D_{00}^2(\phi\theta\psi)$	30±19	93±30	-31±40	A_{00}^2 = -.68±.28
$5\,D_{00}^2(\phi\theta\psi)\mathcal{P}(\chi)$	14±20	51±31	-103±41	S_{00}^2 = -.83±.28
$5\,\mathrm{Re}\,D_{02}^2(\phi\theta\psi)$	-5±12	-15±18	54±31	A_{00}^2 = .38±.19

B. Parity test, B polarization.

J^P	x_1 (a)	x_2 (a)	χ^2 (b)	$x_{av.}$	$\langle J_z^2\rangle^{1/2}$
1^+	0.83±.28	.31±.16	2.7	.43±.14	(ρ_{00}=.65±.11)
2^-	-1.17±.40	.43±.22	11.7	-	-
3^+	-0.83±.28	.47±.24	11.5	-	-
(∞)	-0.66±.22	.50±.25	11.1	-	-
1^-	0.83±.28	-.31±.16	11.7	-	-
2^+	-1.17±.40	-.43±.22	2.7	-.60±.20	1.79±.11
3^-	-0.83±.28	-.47±.24	1.1	-.62±.19	2.54±.15
(∞)	-0.66±.22	-.50±.25	0.3	-.58±.17	(.73±.04)J

(a) $x = [1-3\langle J_z^2\rangle/(J^2+J)]\,2|F_1|^2$. x_1 from S_{00}^2, x_2 from A_{02}^2.

(b) $\chi^2 = [(x_1 - x_2)/\delta(x_2 - x_1)]^2$.

values of $\langle J_z^2\rangle^{1/2}$ [shown in Table I(B)] effectively summarize the information available. These values (computed from x with $2|F_1|^2 \equiv 1$) exhibit the rather extreme degree of alignment of the B required by the data for the $J^P = 2^+3^- \cdots$ assignments.

(D) We have attempted to determine, for the $J^P = 1^+$ assignment, the amount of D-wave decay amplitude. Since we find our data to be consistent with a very wide range of the ratio $|D/S|^2$ (about 0.03 to 3), we omit a detailed presentation.

In conclusion we note that our data are equally consistent with $J^P = 1^+$ and with $J^P = 2^+3^- \cdots$, so that any attempt to rule out $J^P = 2^+3^- \cdots$ is pure speculation; the following remarks are speculative but possibly relevant:

(1) In our experiment B production is rather peripheral (Fig. 2). We find it hard to believe that the extreme alignment required by the assignments $J^P = 2^+3^-4^+ \cdots$ could arise in a peripheral process. This is nevertheless a prejudice, not an argument.

(2) As mentioned previously, the assignments $J^P = 3^-5^- \cdots$ are unlikely because of the apparent absence of $\pi\pi$ and $K\bar{K}$ decay modes.

(3) If one of the assignments $J^P = 2^+4^+ \cdots$ should turn out to be correct, it would mean that the B is incompatible with a quark-antiquark model, since $q\bar{q}$ states with $I^G = 1^+$ and $J^P = 2^+4^+ \cdots$ are not possible.

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[5]We have not seen the paper by H. Foster et al., quoted in Proceedings of the International Conference on Elementary Particles, Heidelberg, Germany, 1967, edited by H. Filthuth (North-Holland Publishing Company, Amsterdam, The Netherlands, 1968), p. 33.

[6]An excellent summary of methods useful in spin-parity analysis is given by J. D. Jackson, High Energy Physics (Gordon and Breach Publishers, Inc., New York, 1965). Our discussion is merely a specialization of well-known techniques to the ω-π system.

[7]Equation (3) includes the effects of parity conservation in $\omega \to 3\pi$. An irrelevant factor, showing the dependence on the Dalitz variables for $\omega \to 3\pi$, has been left out.

[8]$\langle J_z^2\rangle$ is defined by $\langle J_z^2\rangle = \mathrm{Tr}(J_z^2\rho)$.

[9]The unnormalized moment of a function f_α is $M_\alpha = \sum_i f_{\alpha i}$. We compute errors and correlations from $\langle \delta M_\alpha \delta M_\beta\rangle = \sum_i f_{\alpha i} f_{\beta i}$.

[10]J. L. Uretsky, private communication.

[11]For a graphic demonstration of the anomalous behavior of $J=1$, see J. D. Jackson, Classical Electrodynamics (John Wiley & Sons, Inc., New York, 1962), Fig. 16.1, p. 552.

PROPERTIES OF THE g MESON*

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We have observed the g meson in $\pi^+\pi^-$, $\pi^-\pi^0$, and $\pi^-\pi^-\pi^+\pi^0$ mass spectra and have measured masses, widths, and some branching ratios. The angular distributions and total cross sections presented strongly indicate a J^P of 3^- for the g meson.

The g meson was discovered by Goldberg et al.,[1] and by Forino et al.[2] in a $\pi^+\pi^-$ state and later by Deutschmann et al.[3] and Crennell et al.[4] in a $\pi^-\pi^0$ system. It is likely that the g^- was observed in the missing-mass experiments of Maglić and coworkers.[5] Recently others have reported evidence of a 4π state[6,7] at the same energy and conflicting evidence has been given concerning a possible $\omega^0\pi$ decay mode.[8,9] We report here further confirmation of the existence of this state and evidence for a spin and parity assignment of 3^- from an analysis of 340 000