duce in radiative corrections. Detailed statements of the conditions under which the ETC exist have been provided, as have precise statements of the relation between the latter and the divergences. A more complete derivation will be given elsewhere.

One of us (D.G.B.) thanks Brandeis University for hospitality while part of this work was performed.

---

*Work supported by the U. S. Atomic Energy Commission under Contract No. RLO 1388B.

†Work supported by the U. S. Air Force, Office of Aerospace Research, under Office of Scientific Research Grant No. 368-67.

[1]J. D. Bjorken, Phys. Rev. 148, 1467 (1966).

[2]R. Jost and H. Lehmann, Nuovo Cimento 5, 1598 (1957).

[3]It is possible, in particular models, for the (00) indices to be degenerate, with $C^{\mu\nu\lambda\sigma} \sim g^{\mu\nu}\tilde{C}^{\lambda\sigma}$. In general, however, the C_{00ik}'s can be components of arbitrarily high rank tensors, $C_{00ik\theta\theta00\cdots}$, as we shall see later. In the present context we restrict ourselves to foruth-rank structure.

[4]T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters 18, 1029 (1967).

[5]T. D. Lee, to be published.

[6]This result was established independently by the authors [S. Deser, in Lectures of the Summer Institute of the Niels Bohr Institute, Copenhagen, Denmark, 1967 (to be published)]; M. Halpern and G. Segré, Phys. Rev. Letters 19, 611, 1000(E) (1967); and G. C. Wick and B. Zumino, Phys. Letters 25B, 479 (1967).

[7]These terms are also related to ETC, namely, to the higher-order derivatives of the δ function which may occur in the ETC. Throughout, when referring to ETC, we mean the lowest-order derivative term of the commutator which is allowed to occur, e.g., $[j^0, \vec{j}] \sim \nabla\delta(\vec{r})$ and $[\partial^0 j^i - \partial^i j^0, j^l] \sim \delta(\vec{r})$.

[8]The actual calculations in this case require generalization of the conditions given in the quadratic case for the asymptotic behavior of the $\psi_i^{(n)}(s)$. The relevant behavior of the $\mathfrak{M}_i$ is $k^{-4}(-\nu^2/k^2)^n$ in this case. It is amusing that in some of our models, including algebra of fields (with Yang-Mills ETC), $\mathfrak{M}_1$ has terms $\sim k^{-4}(\nu^2/k^2)$ which cancel parts of $\mathfrak{M}_2$, exemplifying the possibility of nonkinematical structures of the type we have discussed.

MISUSES OF THE FINITE-ENERGY SUM RULES*

Jeffrey E. Mandula and Richard C. Slansky
California Institute of Technology, Pasadena, California
(Received 22 April 1968)

It is shown that any model in which the scattering amplitude is given by finitely spaced trajectories of direct-channel resonances does not yield Regge asymptotic behavior in the direct channel. Several difficulties associated with the use of Regge asymptotic behavior at low values of the energy through the finite-energy sum rules are noted.

(I) The finite-energy sum rules (FESR),[1,2] or generalized superconvergence relations, relate a finite integral of the imaginary part of a scattering amplitude to the crossed-channel Regge poles controlling the high-energy scattering. The FESR are easily derived from the superconvergence of $A(t,s)-\sum_i R_i(t,s)$, the amplitude minus the necessary number of leading Regge trajectories. The zeroth-moment sum rule for the scattering of equal-mass scalar particles is

$$\frac{1}{\pi}\int^s ds' \,\mathrm{Im}\,A(t,s') \cong \sum_i \beta_i(t) s^{\alpha_i(t)+1}. \tag{1}$$

Certain factors have been absorbed into $\beta_i(t)$. The absolute value of the difference of the two sides of Eq. (1) goes to zero in the limit $s\to\infty$. Note that for $t \leq 0$ the integral traverses part of the s-channel physical region.

In practice the FESR are used in two distinct ways. For very large s they are used to determine the parameters of a representation for $\mathrm{Im}\,A(t,s)$ (other than the Regge representation), which is assumed to be valid over the entire range of integration. For values of s currently accessible experimentally, they are used to obtain information about the Regge parameters from the low-energy data.

The most obvious and simple parametrization of $\mathrm{Im}\,A(t,s')$ when s is large is a sum of direct-channel poles. Such a model is particularly attractive if the Regge trajectories rise indefinitely, since one can then identify the Regge poles with the direct-channel resonances, and the

FESR become "bootstrap" equations.[3] In this "high-energy resonance model" s is assumed to be arbitrarily large, and so the FESR are arbitrarily accurate. The FESR are sufficiently powerful so that they may be used to check the internal consistency of the model, i.e., whether a sum of direct-channel resonances can give Regge asymptotic behavior.

In a rising-trajectory model the FESR are integral equations for the Regge residue functions (or coupled equations in a non–self-conjugate bootstrap). In Sec. II we show that these equations have no solutions, so long as there exists a nonzero minimum spacing between the trajectories. Thus the FESR imply that rising-trajectory models of the direct-channel singularities cannot give Regge asymptotic behavior in that channel, and so the use of the FESR to determine parameters in such models is inconsistent.

In Sec. III we discuss applications of the FESR where the value of s is assumed small enough that $\operatorname{Im} A(t, s')$ can be evaluated directly from the experimental data. It is usually assumed that the Regge sum is reasonably accurately parametrized in terms of the experimentally established trajectories. However, with this prescription there are cases where the FESR are badly violated experimentally and conclusions based on the FESR are not valid. We discuss the $B^{(+)}$ amplitude of pion-nucleon scattering as an example.

(II). A sad theorem.—In this section we use the FESR to show that finitely spaced Regge trajectories of resonances do not yield Regge asymptotic behavior. We will first prove this result in a trivial model with a single trajectory, and then describe how the argument generalizes to arbitrary two-body scattering with any number of finitely spaced trajectories.

The example is a model of the elastic scattering of two identical self-conjugate spin-zero bosons. The dynamical assumptions are that in any channel the amplitude is given as the sum of an infinite number of narrow resonances which lie on a single Regge trajectory, and that in any channel the high-energy asymptotic behavior is just the Regge asymptotic behavior resulting from the same trajectory, but in the crossed channel. The asymptotic behavior assumption is expressed through the FESR, Eq. (1).

To use the FESR, we must evaluate the left-hand side of Eq. (1). The s-variable discontinuity produced by a single rising trajectory of resonances in the narrow resonance approximation is

$$[A(s',t)]_s = \sum_n [2\alpha(s_n)+1]\tilde{\beta}(s_n)\delta(s'-s_n) \times P_{\alpha(s_n)}\left(1+\frac{2t}{s'-4m^2}\right), \qquad (2)$$

where s_n is the $(\text{mass})^2$ of the nth resonance. The same trajectory function occurs because we have assumed a crossing-symmetric model, and so the s- and t-channel Regge poles are the same. The residue functions have been distinguished because $\tilde{\beta}(s)$ should include the coupling of inelastic channels to the Regge pole.

The FESR are supposed to become exact as $s \to \infty$, but in a narrow-resonance approximation they obviously cannot, because the right-hand side is a smooth function of s, while the left-hand side is the sum of step functions. Within a narrow-resonance approximation, Regge asymptotic behavior must apply to a smoothed-out amplitude, and we represent this by smearing out the integrand on the left-hand side:

$$[A(s',t)]_s = [2\alpha(s')+1]\tilde{\beta}(s')\frac{d\alpha(s')}{ds'} \times P_{\alpha(s')}\left(1+\frac{2t}{s'-4m^2}\right). \qquad (3)$$

For each value of t, the FESR, with the above approximations, is an integral equation for the residue function $\tilde{\beta}(s)$. Because these equations are just approximations to the FESR, we should not demand that the functions $\tilde{\beta}(s)$ which solve them at each t be exactly the same, but only approximately so. We express this by requiring that for each t the asymptotic s dependence of the solution $\tilde{\beta}(s)$ be the same, and so replace $\tilde{\beta}(s')$ by $\tilde{\beta}(s')e^{\rho(s',t)}$ in Eq. (3); $|\rho(s',t)|$ is bounded, but otherwise arbitrary.

Having built into the FESR the requirement that the solutions be approximately consistent, we shall show that they are not. The argument depends only on the way that the s and t dependence are separated on each side, and so we will not hesitate to absorb into $\beta(t)$ and $\tilde{\beta}(s)$ functions of t and s, respectively.

We examine the FESR asymptotically in s (keeping $s \gg t$). Using the appropriate asymptotic form of the Legendre function, we get

$$e^{\rho(s,t)}\beta(s)I_0[2\alpha(s)(t/s)^{\frac{1}{2}}] = \tilde{\beta}(t)s^{\alpha(t)}. \qquad (4)$$

The asymptotic s dependence of the Regge term (the right-hand side) is t dependent, and so the left-hand side must exhibit this property also. This requires $\alpha(s)$ to rise more quickly than $\sqrt{s}$, which allows us to use the asymptotic form of I_0 to write Eq. (4) as

$$e^{\rho(s,t)}\beta(s)e^{2\alpha(s)(t/s)^{1/2}}=\tilde{\beta}(t)s^{\alpha(t)}. \tag{5}$$

Although we maintain $t \ll s$, we may take s so large that t can also be in the asymptotic region, where we must use the same functional form for $\alpha(t)$ as for $\alpha(s)$. But there is no function α which yields the same asymptotic behavior on both sides of Eq. (5), and so the FESR cannot be satisfied.

Although the argument has been presented for a crossing-symmetric model with no spins and a single trajectory, it is much more general. If the s and t channels contain different kinds of particles, there are two FESR's, one in each channel. Exactly the same argument shows that there are no trajectory functions α_s and α_t which allow both sides of both FESR's to have the same asymptotic behavior. Unequal-mass kinematics has no effect on the argument. It simply requires the replacement of $s'-4m^2$ by $4q^2$ in the argument of the Legendre function in Eq. (3). Since $4q^2$ grows like s' for large s', the same asymptotic form of the FESR, Eq. (4), obtains.

So long as there is a nonzero minimum spacing between them, additional trajectories do not affect the argument, since at any finite energy both sides of the FESR are replaced with a finite sum of terms, each characterized by its own trajectory and residue functions. The asymptotic s dependence of the right-hand side is still dominated by the highest trajectory, because no finite sum of slower growing contributions can equal the fastest growing term. This trajectory must grow more quickly than $\sqrt{s}$, and also dominate the left-hand side of the FESR, if its asymptotic s dependence is to be t dependent. Thus the many-trajectory model is reduced to a one-trajectory model, and the argument following Eq. (5) applies.

Spins alter some details of the proof, but do not affect its structure. When spins are present, the argument may be applied to each of the invariant amplitudes separately. The trajectory functions on each side of the FESR are modified by the addition of a function (different on each side) bounded by the total spin in the process. This extra bounded function, which depends only on the same variable as the trajectory function to which it is added, cannot affect the asymptotic dependence of the trajectory functions, and so cannot modify the conclusion of the proof.

Thus Regge asymptotic behavior, as expressed through the FESR, is incompatible with any model in which the scattering amplitude is given by finitely spaced trajectories of direct-channel resonances. The FESR cannot, then, be consistently used to determine the parameters characterizing such a model.

III. Regge behavior at low energy.—This section is devoted to a brief discussion of the range of validity of Regge asymptotic behavior. There have been several recent attempts to use Regge behavior at fairly small s through the FESR. There are scant theoretical grounds for estimating the accuracy of the results, and one must simply query the extant data.

If s is small enough, $\operatorname{Im}A(t,s')$ can be constructed from the experimental data, and the FESR can then be used to relate this low-energy data to the parameters in the sum over Regge poles in Eq. (1). In practice, the leading trajectories are identified by fitting high-energy experiments. The experimental errors are large enough so that the analyses warrant only a few Regge poles, and this expression is used at these low values of s.

The sum of crossed-channel Regge poles is an asymptotic representation of the amplitude, and although it may be sensible to retain only the leading terms, it is certainly not obvious that the right-hand side of Eq. (1) is well approximated by the leading Regge trajectories for low s. For example, a Regge pole lying one unit below the leading poles introduces a correction of order $1/\cos\theta_t$; in pion-nucleon scattering, this correction is of order one, since the upper limit on the phase-shift analyses is about 2-BeV lab energy of the pion. Thus it is not obvious a priori that the prescription of extrapolating only the experimentally established trajectories to low s does not introduce a large error into the sum rule.

In πN scattering,[4] the sum rules for $A'^{(+)}$ and $B^{(-)}$ are reasonably well satisfied, but the $A'^{(-)}$ and $B^{(+)}$ sum rules are rather badly violated. The $B^{(+)}$ amplitude presents an amusing example. As a function of s, the integral of the data starts large and positive (the nucleon term), dips to about two-thirds the original value [the $\Delta(1238)$], then increases to a large positive number. The P and P' residues are negative[5]; so the Regge sum starts at zero and grows negative as a function of s. For s chosen not far above the $\Delta(1238)$, Gatto[6] notes that the Regge term is small compared with the nucleon or $\Delta(1238)$, and so he neglects it. His sum rule is a superconvergence relation for $B^{(+)}$ with the integral cut off at s. Thus it looks like a FESR except that the Regge sum is set equal to zero. Experimentally, this is much better than the FESR with just the P and P', although it is far from perfect if one uses phase shifts[7] instead of the narrow-width resonance approximation.

This example shows that the FESR are not universally valid, even at the upper limit of present phase-shift analyses, and contains a lesson about models for fitting the experimental data below the Regge region. The criticism based on the FESR against the interference model (the amplitude parametrized as a sum of crossed-channel Regge poles plus a sum of direct-channel resonances) is valid only if the FESR are well satisfied. In fact, Gatto's sum rule suggests the interference model as correct for the $B^{(+)}$ amplitude. However, the accuracy of the FESR for $B^{(-)}$ with just the ρ trajectory would suggest the Dolen-Horn-Schmid[2] prescription for parametrizing the $B^{(-)}$ amplitude. (They parametrize the amplitude as a sum of the crossed-channel Regge poles plus direct-channel resonances minus the average of the resonances.) Neither parametrization is adequate to describe all of πN scattering below the Regge region. Chiu and Stirling[8] have already shown that the interference model is inadequate. It is unwise to assume that Regge asymptotic behavior is more accurate at low energies when expressed in terms of the FESR than when expressed directly in terms of scattering amplitudes.

It is a pleasure to thank Dr. Padmanabhan Babu, Dr. Claudio Rebbi, and Professor Fredrik Zachariasen for stimulating conversations.

---

*Work supported in part by the U. S. Atomic Energy Commission. Prepared under Contract No. AT(11-1)-68 for the San Francisco Operations Office, U. S. Atomic Energy Commission.

[1]K. Igi, Phys. Rev. Letters 9, 76 (1962).

[2]R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1768 (1968).

[3]S. Mandelstam, Phys. Rev. 166, 1539 (1968); D. J. Gross, Phys. Rev. Letters 19, 1303 (1967); and P. G. O. Freund, Phys. Rev. Letters 20, 235 (1968).

[4]We use the notation of V. Singh, Phys. Rev. 129, 1889 (1963).

[5]See, for example, W. Rarita, R. J. Riddell, C. B. Chiu, and R. J. N. Phillips, Phys. Rev. 165, 1615 (1968).

[6]R. Gatto, Phys. Rev. Letters 18, 803 (1967).

[7]C. Lovelace, CERN Report No. TH 837, 1967 (unpublished); and P. Bareyre, C. Bricman, and G. Villet, Phys. Rev. 165, 1730 (1968).

[8]C. B. Chiu and A. V. Stirling, CERN Report No. TH 840, 1967 (unpublished).

IMPOSSIBILITY OF FINITE CHARGE RENORMALIZATION IN SIXTH ORDER*

C. R. Hagen and M. A. Samuel†
Department of Physics and Astronomy, University of Rochester, Rochester, New York
(Received 2 April 1968)

Making use of a recent result obtained by Jackiw in the context of the renormalization group we examine the question of whether there exists the possibility of obtaining finite charge renormalization by means of an eigenvalue condition on the bare coupling constant. It is shown that to sixth order in e_0 a cancellation of the divergences encountered in perturbation theory cannot occur.

Quantum electrodynamics has long enjoyed a relatively privileged position in the study of relativistic field theories. Although this is in large part a consequence of the impressive successes which it has achieved in predicting experimentally observable quantities, its usefulness has been further enhanced by the fact that the divergences of this theory are at least partially understood, a circumstance which has suggested that it might well be the most promising candidate for a completely finite field theory. In particular, the freedom which one has available in the choice of gauge is known to imply the possibility of eliminating the divergences in Z_2 (the electron wave-function renormalization) while the Ward identity implies $Z_1 = Z_2$ and the consequent finiteness of the vertex renormalization in the same gauge. In addition, the problem of the photon mass has been shown to resolve itself provided that sufficient care is taken to ensure the gauge invari-