

tems.⁷

In a subsequent experiment with less statistical accuracy we have measured Reaction (2) in a Li^6 target and have found two levels with binding energies compatible with those given above for the two lowest levels. The data from Li^6 are inconclusive with regard to the third peak mentioned above.

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NEW DYNAMIC EFFECT IN MUONIC ATOMS*

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For certain nuclear species, it is possible to use the studies of muonic x-ray transitions to investigate nuclear Coulomb monopole transition matrix elements. The case of $0^+ - 0^+$ vibrational excitation is treated in detail as an example.

It has been found that the studies of muonic atoms are a rich source of nuclear information. In the following, we detail a further use of muonic x-ray investigations. For a select group of atoms, it is possible to obtain direct information on the monopole Coulomb excitation of nuclear states with the same quantum numbers as the ground state. Studies of the $2s-2p$ x-ray intensities and energies determine the sign and magnitude of the Coulomb matrix element and accurately locate the energy of the excited nuclear state. The direct Coulomb excitation of 0^+ excited states in even-even nuclei cannot be studied readily except through electron-scattering experiments, but muonic investigations give one a further experimental tool.

The muon-atomic levels of a given nuclide can readily be found when the nucleus can be treated as static. As was shown more than a decade ago by Wilets and by Jacobsohn,¹ there are many species for which dynamic quadrupole effects must be taken into account in order to determine accurately the energies of atomic levels. Recent measurements² have shown agreement with these theories. Further dynamic effects occur whenever an excited nuclear level with the proper quantum numbers is almost degenerate with mu-

onic energy differences. The case of particular interest to us here occurs when an excited state of the same spin and parity as the nuclear ground state lies at an excitation energy which nearly coincides with the muonic $2s-1s$ energy difference. The simplest case, and the one to which we will restrict ourselves, is that of even-even nuclei with excited 0^+ (vibrational) states. If the coincidence noted above occurs, then the $2s$ atomic state and nuclear ground state α is nearly degenerate with the atomic $1s$ state and the nuclear excited state β . The levels are repelled because of the Coulomb transition matrix element between the two states and a doubling of the "2s atomic" state occurs. The $2s-2p$ x-ray spectrum serves as the detector of the nuclear transition matrix element and the nuclear excitation energy. This information is of prime value in determining the collective character of the excited nuclear state.

Although we have not made a thorough search of possible nuclei, two cases that appear to fit the necessary criteria are ^{68}Zn and ^{82}Kr , both of which are stable and have 0^+ ground states and 0^+ excited states at 1.63 MeV (Zn) and 2.19 MeV (Kr).³ These energies agree closely with the corresponding muonic $2s-1s$ energy differences

Table I. Muonic atom energies. The values of the 2s and 1s energies were computed for the Ford-Wills charge distribution and include vacuum polarization effects. The $2p_{3/2}$ and $2p_{1/2}$ energies were obtained by interpolation from the table in Ref. 4. All energies are given in keV.

| State | 1s | 2s | 2s-1s | $2p_{1/2}$ | $2p_{3/2}$ |
|------------------|---------|--------|--------|------------|------------|
| ^{68}Zn | -2218.8 | -596.2 | 1622.6 | -642.7 | -635.4 |
| ^{82}Kr | -3052.6 | -841.1 | 2211.5 | -931 | -916 |

shown in Table I. The energies of the atomic states were computed for a nuclear charge distribution employed by Ford and Wills.⁴ The matrix element for the Coulomb excitation of the state β from the state α is given by

$$\begin{aligned} \langle H' \rangle &\equiv \langle 1s, \beta | H' | 2s, \alpha \rangle \\ &= Z e^2 \int \varphi_{1s}^*(\vec{x}) \langle \beta | \frac{\rho(\vec{r})}{|\vec{r}-\vec{x}|} | \alpha \rangle \varphi_{2s}(\vec{x}) d^3x, \end{aligned} \quad (1)$$

where φ_{2s} and φ_{1s} are the atomic wave functions and $\rho(\vec{r})$ is the nuclear-charge distribution operator normalized to $\int \rho(\vec{r}) d^3r = 1$. If the nuclear excited state corresponds to an incompressible liquid-drop two-phonon quadrupole vibration, then the matrix element can readily be shown to be given by⁵ ($\hbar = c = 1$)

$$\begin{aligned} \langle H' \rangle &= \frac{3(10)^{1/2}}{8\pi} \frac{Z e^2}{R} \frac{1}{2(BC)^{1/2}} \\ &\times \int_0^R \varphi_{1s}^*(\vec{x}) \varphi_{2s}(\vec{x}) d^3x, \end{aligned} \quad (2)$$

where R is the effective nuclear radius (taken below to be $R = R_{\text{rms}} = 1.2A^{1/3}$ fm), and B and C are the constants which appear in the kinetic and potential energies of the harmonic oscillator. We use⁵

$$\frac{1}{2}(BC)^{-1/2} \approx 3.24A^{-7/6} \quad (3)$$

as suggested by the semiempirical mass formula for light and medium nuclei. A comparison with electron scattering in Ni indicates that this estimate may be high (by as much as a factor of 5) for $E2$ transitions.⁶ For light nuclei, we have approximately

$$\begin{aligned} &\int_0^R \varphi_{1s}^*(\vec{x}) \varphi_{2s}(\vec{x}) d^3x \\ &\approx \varphi_{1s}(0) \varphi_{2s}(0) 4\pi R^3/3. \end{aligned} \quad (4)$$

Since we have $|\varphi_{ns}(0)|^2 = Z^3/(\pi n^3 a_\mu^3)$, where $a_\mu \approx 255$ fm is the muonic Bohr radius, we find for

light nuclei

$$\langle H' \rangle = \frac{5^{1/2} \times 3.24 e^2}{4\pi} \left(\frac{r_0}{a_\mu} \right)^3 \frac{Z^4}{A^{1/2}}, \quad (5)$$

where $r_0 = 1.2$ fm. The $Z^4 A^{-1/2}$ behavior of Eq. (5) suggests that it is advantageous to use heavy nuclei. Unfortunately, the vibrational energy decreases whereas the 2s-1s transition energy increases with A , so that a match occurs only for a limited region of nuclei. For heavy nuclei, the variation of $(BC)^{-1/2}$ is less rapid than given by Eq. (3) due to the increasing importance of the Coulomb energy relative to the surface energy. This increases H' . However, approximation (4) tends to overestimate the matrix element. The ratio of the left-hand side of Eq. (4), based on numerical integration, to the approximate value given by the right-hand side is 0.29 for Zn and 0.22 for Kr. With the use of Eqs. (2) and (3), the numerically evaluated matrix elements are

$$\begin{aligned} \langle H' \rangle &= 2.0 \text{ keV for } ^{68}\text{Zn} \\ &= 2.9 \text{ keV for } ^{82}\text{Kr}. \end{aligned} \quad (6)$$

Although the magnitudes of these matrix elements are not large, we believe that their effects are measurable with modern techniques if the estimates of $\langle H' \rangle$ are of the right order of magnitude.

The off-diagonal matrix elements, $\langle H' \rangle$, produce an admixture of the states $|1s, \alpha\rangle$ and $|2s, \beta\rangle$. In calculations presented here, the 2×2 Hamiltonian submatrix involving only these states is diagonalized. This leads to energy eigenvalues

$$E_{\pm} = E_{2s, \alpha} - \frac{1}{2}\Delta \pm \epsilon, \quad (7)$$

where

$$\epsilon \equiv [H'^2 + (\frac{1}{2}\Delta)^2]^{1/2}, \quad (8)$$

and

$$\Delta \equiv E_{2s, \alpha} - E_{1s, \beta}. \quad (9)$$

The energies $E_{2s, \alpha}$ and $E_{1s, \beta}$ include the diagonal Coulomb interaction, which can give a difference in muon binding of the two nuclear states.

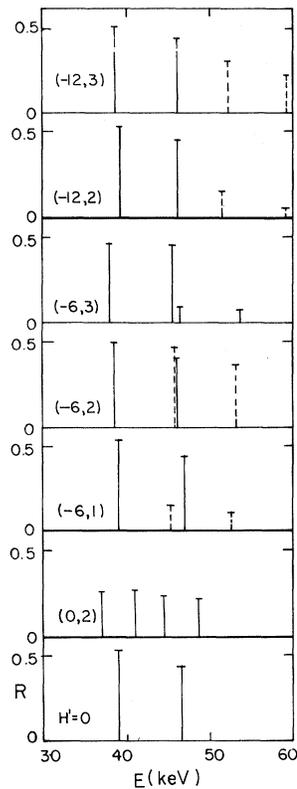


FIG. 1. Typical x-ray spectra for muonic transitions from $|\pm\rangle$ to $2p$ states for ^{68}Zn . The bottom figure is for $\langle H' \rangle = 0$, the upper ones for various values of $(\Delta, \langle H' \rangle)$ in keV. The intensities of the lines shown dashed have been multiplied by 10. The most likely case is that corresponding to $(-6, 2)$.

The eigenstates can be written as

$$\begin{aligned} |+\rangle &= a|2s, \alpha\rangle + b|1s, \beta\rangle, \\ |-\rangle &= b|2s, \alpha\rangle - a|1s, \beta\rangle, \end{aligned} \quad (10)$$

with

$$a^2 = 1 - b^2 = \frac{1}{2}(1 + \Delta/2\epsilon). \quad (11)$$

The electromagnetic transition rates are completely dominated by the muon. These transition matrix elements connect components of the wave functions for which the nuclear state does not change. As the muons cascade down through Bohr orbits, the nuclear states are not excited until the "2s state" is reached. The relative populations, P , of the + and - states are

$$P_+ : P_- = a^2 : b^2. \quad (12)$$

Each of these states can decay to the p states, $|2p_{3/2}, \alpha\rangle$ and $|2p_{1/2}, \alpha\rangle$. The branching is pro-

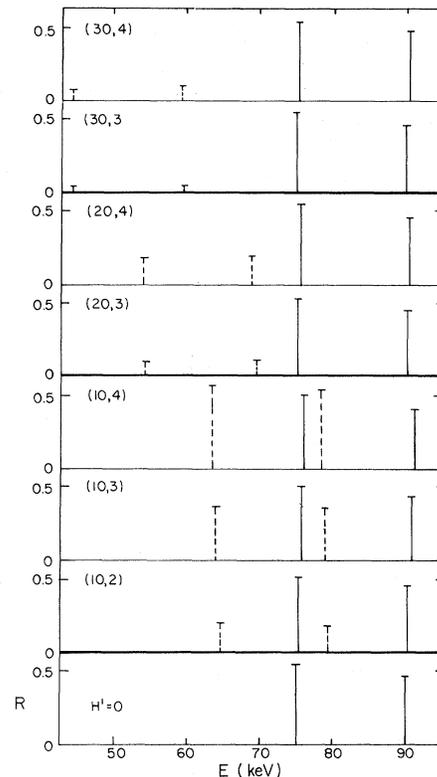


FIG. 2. Similar to Fig. 1 for ^{82}Kr . The most likely case is expected to be that for $(\Delta, \langle H' \rangle) = (20, 3)$.

portional to the cube of the x-ray frequency and to the statistical weight $(2j+1)$ of the final state.

The uncertainties in the nuclear energies ($\approx \pm 10$ keV), and hence Δ , are greater than the estimated magnitude of H' . Calculations are presented in Figs. 1 and 2 for a range of reasonable values of Δ and H' . From a measurement of the energies and intensities of the x rays, one can determine both Δ and H' .

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POLARIZATION OF TRITONS SCATTERED FROM ^4He †

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Absolute values for triton polarizations have been measured for the first time. We suggest an analyzer for polarized tritons with energies of 7 MeV and higher. A scattered beam of 10-MeV tritons with a polarization of 88% from a cross section of greater than 600 mb/sr is reported.

The polarizations of tritons elastically scattered from ^4He at several energies and angles have been measured using double-scattering techniques. Absolute polarization values are deduced from the measurements of three asymmetries at appropriate energies and angles. The large polarization values given here are in contrast to our earlier attempts^{1,2} to measure polarizations in scattering from heavier targets. One other result³ known to us reported a single asymmetry measurement from which no absolute values can be extracted. The present experiment indicates general agreement with recently published values⁴ of phase shifts.

A schematic diagram of the experimental apparatus is shown in Fig. 1. The primary target consists of a 3.5-cm-diam cylinder with 0.013-mm-thick Havar⁵ windows. The target is filled with helium to a pressure of 10 atm. Collimators are arranged so that tritons which scatter from the Havar windows are not accepted by the secondary target. Typical triton energy losses in penetrating one-half of the primary target are 1 MeV. The energy spread of the beam in the helium is about 300 keV. Typical primary beam currents of accelerated tritons are 1.5 μA .

The tritons scattered from the primary target are focused onto the secondary target with a three-element magnetic-quadrupole lens system. The lens system has an inside diameter of 10 cm and can be used to focus up to 18-MeV tritons with an angle of acceptance of 4×10^{-3} sr. The

scattered particles are focused onto a 6-mm \times 12-mm collimator in the secondary chamber. The distance from the primary target to the secondary target is about 2 m. The lens system and secondary chamber can be rotated to positive or

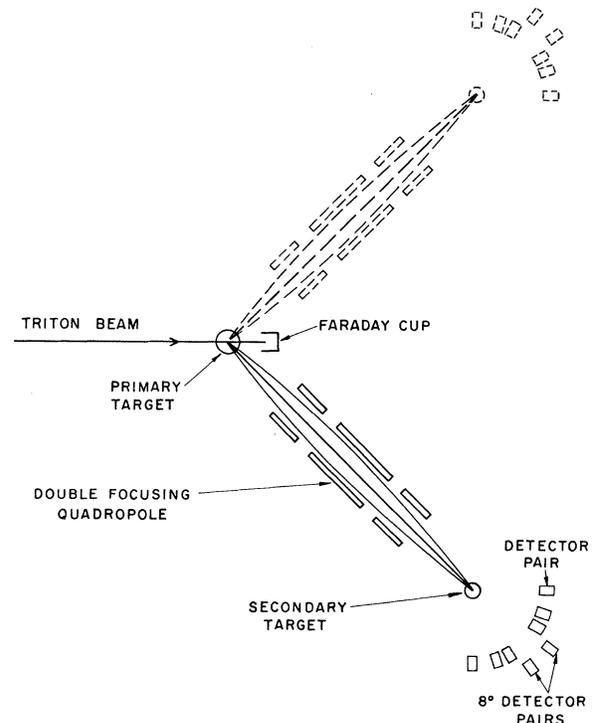


FIG. 1. A schematic diagram of the experimental apparatus.