[15]K. A. Wickersheim, Phys. Rev. 122, 1376 (1961).

[16]This Hamiltonian corresponds to the one previously deduced by P. M. Levy, Phys. Rev. 135, A155 (1964).

[17]As one of the electrons has no orbital moment and therefore has no preferred set of axes of quantization, it is not possible to obtain antisymmetric exchange, for $j_1 = j_2$, as was done in Eq. (4), by rotating the axes of quantization.

LOCAL-MOMENT FORMATION AND THE KONDO EFFECT*

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Recently the Wolff model[1] of local-moment formation in dilute alloys has been extensively investigated[2,3] at nonzero temperatures. The conduction electrons of the host metal are taken to interact with each other through a zero-range potential v and with the single impurity through a zero-range potential V which leads to the formation of a virtual level in the absence of v. A Bethe-Salpeter equation for the triplet vertex function, in which the kernel is given by the product of two single-particle propagators, and the Dyson equation for the propagator, in which the mass operator is given in terms of the vertex function, are solved simultaneously. For v greater than a critical v_C,[2] the vertex function is divergent if zero-order propagators (in v) are used; in the self-regulating solutions of Suhl[2] and Levine and Suhl,[3] the broadening of the resonance through the electron-electron interaction keeps the zero-frequency vertex finite, though close to a pole. In the approximation solved by Suhl[2] the susceptibility was shown to increase roughly as $\beta = (kT)^{-1}$, which implies the formation of a local magnetic moment at the impurity site. The numerical solution of the exact equations[3] found a susceptibility which increased with β, though not as fast as β, over an appreciable temperature range and flattened out to a constant at low temperatures. This Letter will show that these results, within the framework of this theory, imply that the resistivity due to the impurity exhibits a Kondo-like behavior, i.e., approximately $\sim \ln T$ over an appreciable temperature range.[4] Extrapolations of the numerical results obtained by Levine and Suhl[3] show identical behavior.

Suhl's equations[2] can be written in terms of the band-averaged propagator $h(i\omega)$ and the vertex function $\gamma(i\omega)$[5]:

$$\gamma(i\omega) = S(i\omega)[1 - S(i\omega)]^{-1},$$

$$S(i\omega) = -\frac{v}{\beta}\sum_{iv} h(iv)h(iv + i\omega), \tag{1}$$

$$\Sigma(i\omega) = \frac{3}{4}\frac{v}{\beta}\sum_{iv}\gamma(iv)h(iv + i\omega). \tag{2}$$

γ, S, Σ, and h are analytic in the complex frequency plane except for a cut along the real axis so that on the physical sheet, we have

$$\gamma(z) = \int_{-\infty}^{\infty}\frac{d\omega'}{\pi}\frac{\mathrm{Im}\gamma(\omega' + i\epsilon)}{\omega' - z}, \tag{3a}$$

$$h(z) = \int_{-\infty}^{\infty}\frac{d\omega'}{\pi}\frac{\mathrm{Im}h(\omega' + i\epsilon)}{\omega' - z}. \tag{3b}$$

In the temperature range where $\gamma \equiv \gamma(0)$ {in these units, the dc susceptibility is $\chi = \gamma[g\mu_B S(0)]^2/v$} is large, we expect $\gamma(\omega)$, which is nearly singular at $\omega = 0$, to be much more rapidly varying than $S(\omega)$ for small ω. Therefore, a fairly good approximation to $\gamma(\omega)$ is obtained by keeping only the leading terms in the expansion of S about $\omega = 0$, giving

$$\mathrm{Im}\gamma(\omega + i\epsilon) = \frac{1}{\zeta}\frac{\omega}{\omega^2 + \Gamma^2}, \tag{4}$$

$$\zeta = \frac{d\mathrm{Im}S(\omega + i\epsilon)}{d\omega}\bigg|_{\omega=0} = \frac{v}{\pi}[\mathrm{Im}\,h(0)]^2,$$

$$\Gamma = \frac{1}{\zeta(1+\gamma)}. \tag{5}$$

For $\gamma \gg 1$, Eq. (4) is already in its asymptotic region at the limit of its validity, $\omega \sim 1/\zeta$. The expression for ζ in terms of $\mathrm{Im}h(0)$ is valid at low enough temperatures such that $f'(\omega) = -\delta(\omega)$, where f is the Fermi distribution function.

Using Eqs. (3),

$$\Sigma_i \equiv \mathrm{Im}\Sigma(0 + i\epsilon)$$

$$= \tfrac{3}{4}v\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\frac{\mathrm{Im}\gamma(\omega + i\epsilon)\,\mathrm{Im}h(\omega + i\epsilon)}{\sinh\beta\omega}. \tag{6}$$

$\mathrm{Im}\gamma$ has most of its strength at frequencies on the order of Γ, which, for $\gamma \gg 1$, is small compared with the range of the frequency variations in Imh. In addition, the contribution of the asymptotic part of Imγ is negligible due to the exponential convergence of Eq. (6). Hence

$$\Sigma_i = \frac{3}{4}\frac{v\mathrm{Im}h(0)}{\pi\xi}\left\{\frac{1}{\xi} - \left[\psi\left(\frac{\xi}{2}+1\right) - \psi\left(\frac{\xi}{2}+\frac{1}{2}\right)\right]\right\},$$
$$\xi = \beta\Gamma/\pi, \tag{7}$$

where $\psi(x)$ is the digamma function. With the initial resonance at the Fermi surface,[6] $h_0(0+i\epsilon)$ (the band-averaged propagator in the absence of v) and $h(0+i\epsilon)$ are pure imaginary and we have

$$\Delta + [\mathrm{Im}h(0)]^{-1} = \Sigma_i, \tag{8}$$

Δ the width of the original virtual level. Equations (5), (7), and (8) can be solved for Im$h(0)$.

According to the usual multiple-scattering theory for dilute solutions of randomly distributed impurities,[7] the inverse lifetime for an electron at the Fermi surface is given by the concentration times Im$T_{kk}(0)$, where $T_{kk'}(\omega)$ is the single-impurity T matrix for the scattering of an electron from k to k'. From Eq. (8) of Suhl,[2] the T matrix in the band-averaged model is

$$T(\omega) = \frac{V+\Sigma(\omega)}{1-[V+\Sigma(\omega)]F(\omega)}, \tag{9}$$

where $F(\omega)$ is the band average of the electron propagator ignoring all impurity effects.[2] After some algebra, the result for the lifetime is

$$\frac{1}{\tau} = \frac{1}{\mathrm{Im}F(0)}\,\frac{1-A\Sigma_i/\Delta + (\Sigma_i/V)^2}{(1-A\Sigma_i/\Delta)^2 + (\Sigma_i/V)^2}; \tag{10}$$

A is the strength of the virtual level.[2] Im$F(0)$ is the lifetime in the absence of electron-electron interactions and for a resonance at the Fermi surface represents the unitarity limit for the lifetime.

If τ is slowly varying near the Fermi surface, the resistivity is proportional to $1/\tau$. The $(\Sigma_i/V)^2$ terms in Eq. (10) are very small for reasonable values of V ($V/\Delta = 5$-10) and have not been included in the numerical calculations. For $A=1$, the case treated in previous calculations of the susceptibility,[2,3]

$$\frac{\rho}{\rho_{\mathrm{unitarity}}} = \frac{1}{1-\Sigma_i/\Delta} = -\Delta\mathrm{Im}h(0). \tag{11}$$

When β/γ is constant, Σ_i (which is always negative) and ρ are constant, with ρ less than the unitarity limit. When β/γ increases with β, the term in brackets in Eq. (7) decreases and therefore $|\Sigma_i|$, determined self-consistently, decreases, causing ρ to increase. When γ is constant and β/γ large, the term in brackets in Eq. (7) equals $1/2\xi^2 \sim 1/\beta^2$, so that[8] $|\Sigma_i|$ goes as T^2 and ρ goes to the unitarity limit. Since the temperature variation of γ is spread out over several decades in β,[3] the resistivity increases to the unitarity limit over several decades and simulates a ln$T$ behavior over a large part of this range. A numerical solution of Eqs. (5), (7), and (8), using values of γ from Levine and Suhl,[3] is shown in Fig. 1, where the ln$T$ behavior is quite clear. The temperature range over which ρ equals the unitarity limit coincides exactly with the range over which γ is constant.

Of course, the resistivity can be obtained directly by analytic continuation, without any of the assumptions used above. We have extrapolated to the real axis the values of $h((2n+1)\pi kTi)$, $n=0, 1, 2, \cdots$, found in the Levine and Suhl[3] solution to estimate an exact Im$h(0)$. Since we only know $h(z)$ on the imaginary axis in the upper half plane, this extrapolation is actually a continuation through the cut on the real axis into the lower half plane of the unphysical sheet. Since h has singularities on the unphysical sheet,[9] we used Padé approximants with poles in the lower half plane to do the extrapolation. This should be the most realistic way of simulating the analytic structure of the continued h. The extrapolated

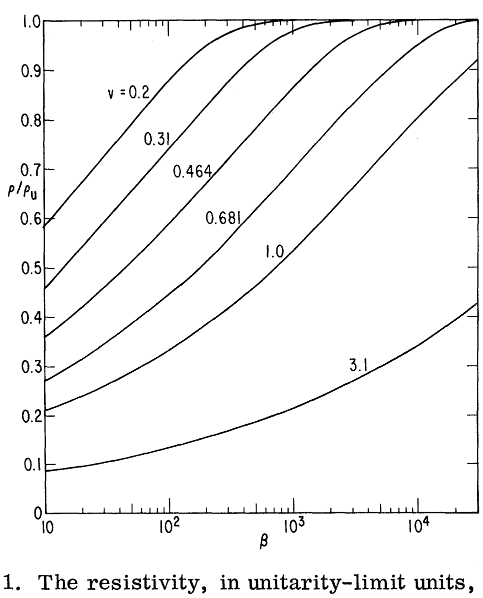

FIG. 1. The resistivity, in unitarity-limit units, versus β calculated in the approximate theory [Eqs. (5), (7), and (8)]. The parameters used in this solution are $A=1$, $\Delta=0.1$, $v_c=0.31$.

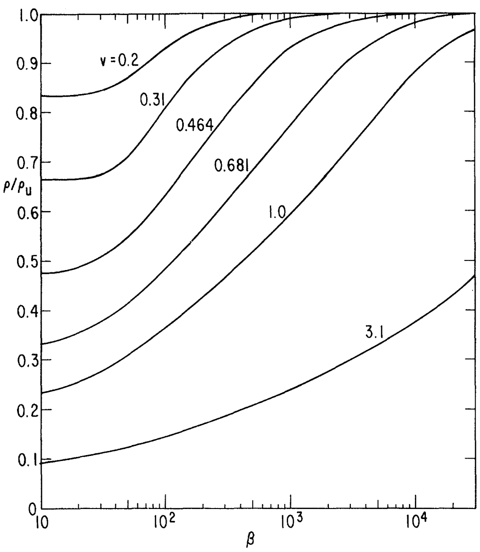

FIG. 2. The resistivity, in unitarity-limit units, versus β calculated in the exact theory of Levine and Suhl (Ref. 3). The parameters used in this solution are $A=1$, $\Delta=0.1$, $v_c=0.31$.

values of $\mathrm{Im}\,h(0)$ are determined to a few percent by the h's with $n\leq 4$, and were insensitive to the h's for higher values of n.

The extrapolated results for the resistivity are shown in Fig. 2. These exhibit the same behavior as a function of temperature as the results shown in Fig. 1, though they are 10-20% higher than the resistivity derived from Eq. (7) except when both are bounded above by the unitarity limit. The worst differences are for small v and β, where γ is not $\gg 1$. In both cases, the resistivity reaches the unitarity limit when γ becomes independent of temperature.

Comparing these results with those for the s-d exchange Hamiltonian,[4] which assumes a hard local moment, we see that the resistivity curves are fairly similar in shape. It is interesting that Hamann's solution[4] of the s-d exchange model shows behavior for ρ and χ similar to that shown here.

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[1]P. A. Wolff, Phys. Rev. 124, 1030 (1961).

[2]H. Suhl, Phys. Rev. Letters 19, 442 (1967). Our $\gamma(\omega)$ differs by a factor of $-v$ from the one used here.

[3]M. Levine and H. Suhl, Phys. Rev. (to be published).

[4]J. Kondo, Progr. Theoret. Phys. (Kyoto) 32, 37 (1964); A. A. Abrikosov, Physics 2, 5 (1965); H. Suhl and D. Wong, Physics 3, 1 (1967); D. R. Hamann, Phys. Rev. 158, 570 (1967). Curves for the conductivity, $\sigma=1/\rho$, correctly continued through the famous Kondo temperature can be found in Suhl and Wong.

[5]The usual methods of many-body field theory at finite temperatures, as given in A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinski, Quantum Field Theory in Statistical Mechanics, translated by R. A. Silverman (Prentice-Hall, Inc., Englewood Cliffs, N. J., 1963), will be used.

[6]With the resonance not at the Fermi surface $\mathrm{Re}\Sigma(0)\neq 0$ and the solution of Dyson's equation is much more difficult.

[7]J. M. Luttinger and W. Kohn, Phys. Rev. 109, 1892 (1958).

[8]We notice that Σ_i has a local enhancement factor $(1+\gamma)^2$ due to spin fluctuations, e.g., as found in P. Lederer and D. L. Mills, Phys. Rev. 165, 837 (1967).

[9]In Ref. 2, the singularities are explicitly exhibited as a square-root cut Δ below the real axis.