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FLUX FLUCTUATIONS AND CRITICAL CURRENTS IN A SUPERCONDUCTING RING NEAR T_c †

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Measurements of current fluctuations and critical currents as a function of temperature have been made in a ring of superconducting tin with a current resolution corresponding to about 0.1 quanta of magnetic flux in the cylinder and a temperature resolution of a few microdegrees. The fluctuations occur between neighboring quantum states at a rate which increases rapidly with increasing temperature. The critical currents do not follow the temperature dependence predicted by theory.

The details of the transition from normal to superconducting state of a metal[1] have remained largely unexplored until recently. Questions concerning, for example, the intrinsic width of the resistive transition,[2] the existence and influence of fluctuations,[2-7] and the decay of persistent currents[3] have only recently been approached. In order to investigate these problems, we constructed an apparatus which allows us to measure a small fraction of a quantum of flux through a ring of macroscopic dimensions. The temperature of the ring can be regulated at any desired temperature in the liquid-helium range to within a few microdegrees. We report here the use of this apparatus to make measurements of fluctuations in the flux through the ring and measurements of its critical current.

The apparatus is shown schematically in Fig. 1. One end of a superconducting transformer is placed around the sample so that any current which flows in the sample induces a current in the transformer. Part of the other end of the transformer is placed around a superconducting-flux detector[8] which detects the flux generated by the current. The flux detector involves the use of rf fields which could affect the measurements. Therefore this end of the superconducting transformer and the rf coils were wound in two halves such as to cancel the rf currents in the transformer. For additional shielding from rf fields the sample was surrounded with a copper can. The sample rings are formed by evaporating a 1-mm wide strip, 1000 Å thick, onto a sapphire tube of $\frac{3}{16}$ in. diam. The results described below were obtained with a ring which was cut so that a section about 8 μ long was reduced to a width of about 3 μ. The sapphire cylinder is thermally connected to a copper block, the temperature of which is electronically regulated to provide the stability mentioned above. The entire assembly of Fig. 1 is inside of a vacuum container which is immersed in liquid helium. The entire system, including the Dewar flasks, is then sur-

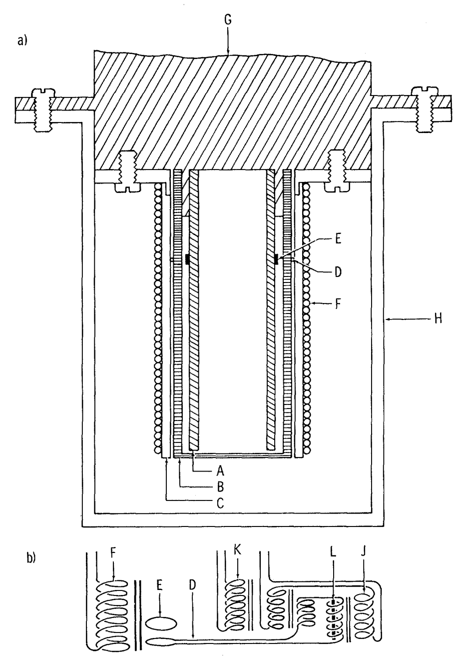

FIG. 1. Schematic diagrams of (a) the sample holder and (b) the transformer and field coils. A, sapphire tube; B, copper can; C, epoxy coil form; D, superconducting transformer; E, sample loop; F, field coil; G, temperature-regulated copper block; H, lead-coated copper can for magnetic shielding; J, rf coils for flux detector; K, low-frequency feedback coil; and L, flux detector.

rounded by copper screening for rf shielding and by two Mumetal shields which reduce the static field to approximately 3×10^{-5} G.

The fluctuations appear as in Fig. 2(a) in which it can be seen that they consist of jumps between two neighboring quantum states. The range of temperatures over which these fluctuations can be observed is limited by the frequency response of the detection scheme (~100 Hz) at the high-temperature end and by the patience of the observer at the low-temperature end. In the data of Fig. 2 the ambient field was set in such a way that half-integral flux quanta threaded the ring. Under this condition equal amounts of time are spent in the two states. If the ambient field is set closer to one state, then more time is spent in the state.

We have attempted to interpret our data in terms of an expression for the number of jumps per second of the form[2]

$$N=N_0e^{-F_0/kT}.$$

F_0 represents the free-energy barrier to the passage between quantum states and should depend on $|T_c-T|$ to some power n. N_0 represents a characteristic frequency of the noise which is driving the system. By choosing appropriate values of T_c, the data can be made to fit the expression equally well for $n=2$ or 3. However, the value of N_0 in these cases is 45 and 115 Hz; so there is considerable question about this interpretation of the results.

Considerable effort has been made to establish that the observed fluctuations are intrinsic and not driven by external sources. Repeated sweeps synchronized with the line voltage and averaged with a multichannel analyzer showed no correlation with the line voltage. Thus, the fluctuations were not being driven by stray 60-cps fields. Any low-frequency (within the band pass of the instrument) noise of sufficient amplitude to drive the system between two quantum states should be directly observable when the sample is normal. No such noise is present. High frequencies (above about 1 kHz) are excluded by the copper can. Furthermore, the jumping rate at a given temperature is unaffected by changes in the noise level in the room.

The currents which can be induced by the application of a magnetic field are shown in Fig. 3. A

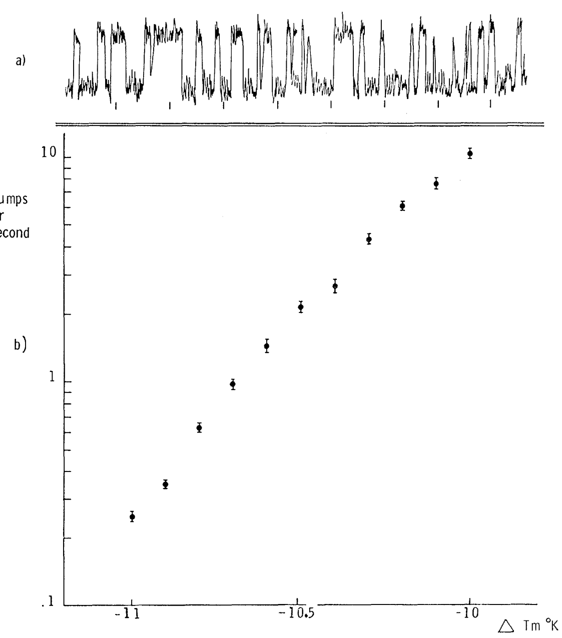

FIG. 2. (a) Flux in a sample ring as a function of time at constant temperature and with no externally applied variations in field. Marks indicate 1-sec intervals. (b) Number of jumps per second as a function of temperature.

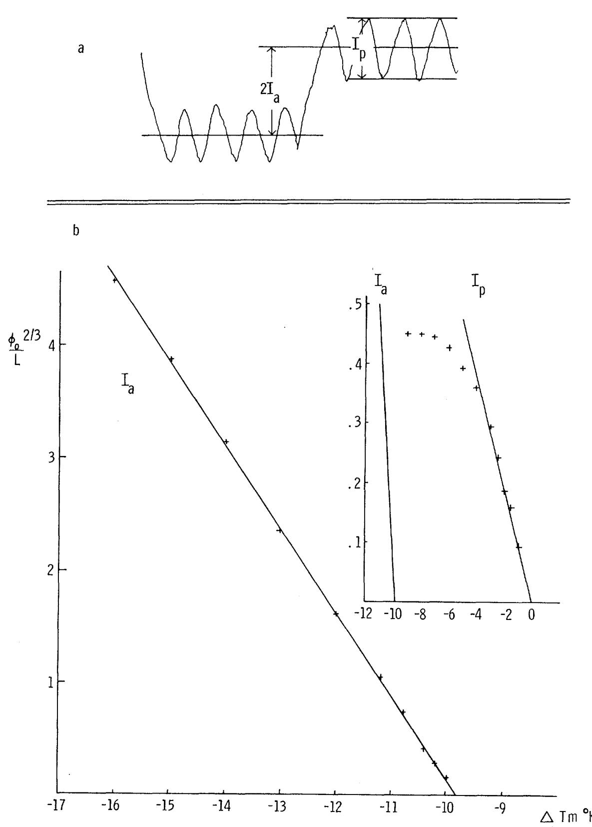

FIG. 3. (a) Sample current as a function of time during application of a triangle-function magnetic field. (b) I_a and I_p to the $\frac{2}{3}$ power in units of $(\varphi_0/2L)^{2/3}$ versus temperature.

triangle-function field at a frequency of 1 Hz is applied with coil F of Fig. 1. The same function with appropriate phase and amplitude is also applied to coil K so that when the sample is in the superconducting state, no current flows in the transformer. Thus, no bucking field is generated by the transformer. With the sample in the normal state, then, current will flow in the transformer and a triangle-function signal will be obtained. Now an appropriate amplitude of the original triangle function is added to this signal in a differential amplifier so that no output is obtained with the sample in the normal state. Thus, the output is proportional to the current which flows in the sample, and when it is superconducting, the field is from coil F alone. The curve of Fig. 3 was obtained in this way and is an average of 100 periods of the triangle function obtained with the multichannel analyzer. A single sweep of the field without such averaging shows clearly that the current increases to a point at which it abruptly decreases by an amount corresponding to 1 quantum of flux, after which it immediately begins again to follow the applied field. The point at which the current decreases is random to within a fraction of a flux quantum, so that the averaging transforms the jumps and ramps into something resembling a sine wave as seen in Fig. 3. The average point at which the current decreases is, of course, a function of temperature and must be related to what is usually referred to as the critical current.

The mean-field theory of superconductivity[9] predicts that the critical current will vary as $(T_c-T)^{3/2}$. Therefore, our data are plotted as current to the two-thirds power versus temperature. If we plot the average value of the "sine wave" I_a in this fashion, it falls very accurately on a straight line. Thus the average peak supercurrents I_c will depart radically from such a curve at small currents, and instead, will follow the relation

$$I_c-\frac{\varphi_0}{2L}\propto(T_c-T)^{3/2}.$$

φ_0 is the flux quantum, and L the inductance of the sample. This relation can be understood by considering that the current always decreases in steps of $\varphi_0/2L$. Thus the average peak current must be greater than I_a by this amount.

Under the conditions that $I_a=0$, I_b can still remain finite as shown in the insert of Fig. 3. It can be seen there that $I_b^{2/3}$ also approaches a linear dependence on temperature, although with a different slope and a different intercept from those of $I_a^{2/3}$. The T_c's chosen for the fluctuation data as described above fall between these two intercepts with higher values of n corresponding to higher values of T_c.

In the presence of the observed fluctuations we expected the values of I_c and I_a at a given temperature to depend on the rate at which the current is varied. Indeed, it is observed that as the frequency of the triangle function is increased, the plots of $I_a^{2/3}$ shift in the direction of higher currents with higher intercepts of the temperature axis but nonetheless remain straight lines. The voltage corresponding to the rate of flux transfer into the ring for the data of Fig. 3 is 4×10^{-15} V. This is several orders of magnitude lower than that which was used in the experiments on straight pieces of superconductor[5,6] and may indicate that the results of this experiment would approach those if sufficiently high frequencies could be used.

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