the spectral amplitude near the threshold $4\mu^2$ in analogy with our result.

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ESTIMATES OF REGGE-CUT EFFECTS*

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(Received 8 April 1968)

In inelastic channels (charge exchange, etc.) at $t=0$, moving-cut effects are found to be $\lesssim 5\%$ of the Regge-pole effects. This result is based on a dynamical approximation. Our findings clarify the success of relations among total cross sections derived by taking into account Regge poles alone.

A wide range of high-energy scattering data in and near the forward direction may be understood on the basis of Regge-pole exchange in the crossed channel. The exchange of two or more Regge poles can generate cuts in the complex angular-momentum plane.[1,2] Although cancellation of these cuts is not ruled out, it is thought to be unlikely.[1,2] While the theoretical situation points to the existence of such cuts, there has been no estimate of their actual contribution to physical processes described in current phenomenology by Regge poles alone. The success of Regge-pole theory would be hard to understand if these cuts contributed substantially to such processes. One might think that because of the mildness of the logarithmic factor that distinguishes a cut from a pole, the former could easily be accommodated in phenomenology. This, however, is not so. Cuts of the type suggested by Mandelstam[1] are not factorizable in general. A large set of experimentally valid relations among total cross sections[3,4] is known to rely essentially upon the factorizability of the leading contributions to the forward amplitudes at high energies. It is the purpose of this paper to explore the magnitude of Regge-cut contributions to the forward and near-forward scattering amplitudes and the conditions under which these contributions could be such as not to violate the total cross-section relations of Refs. 3 and 4. The main result, based on certain—in our view reasonable—approximations, will be that in the Pomeranchuk (P) channel (i.e., internal quantum numbers in the t channel are those of the vacuum) the cut contributions can be sizable, namely up to 15-30% of the pole contribution. In "inelastic" channels (such as charge exchange, hypercharge exchange, etc.) the cut contributions appear to be extremely small, typically $\lesssim 5\%$ of the pole contributions.

It is well known that the cuts originally suggested by Amati, Fubini, and Stanghellini[5] (AFS) do not contribute to the physical amplitude but to its continuation to an unphysical Riemann sheet. However, in virtue of the crossing symmetry of the S matrix, these cuts, as first pointed out by Mandelstam,[1] have counterparts on the physical sheet of the amplitude. The remarkable fact is that the physical cuts appear in the same position as the AFS cuts and that (in perturbation theory at least) the coefficients of $s^{\alpha(t)}(\ln s)^{-1}$ in the AFS and in the physical cuts are essentially the same.[6] We will therefore obtain our estimate of the magnitude of the cut contributions by actually calculating the relatively simple AFS cut contributions to the unphysical amplitude and then realizing that these contributions are essentially equal to those of the corresponding Mandelstam cuts to the physical amplitude.[7] In this way we should at least obtain a good order of magnitude estimate of the cut effect.

We consider, for simplicity, $\pi^+\pi^+$ scattering in the forward direction. For the discontinuity arising from the exchange of two Regge poles, R_1 and R_2, we have

$$\mathrm{Im}A_{\mathrm{cut}}^{PP}(s,0)=\frac{(1-\frac{1}{2}\delta_{RR_2})}{8\pi s}\int_{-s}^{0}dt\,\mathrm{Re}[A_{\mathrm{pole}}^{R_1}(s,t)A_{\mathrm{pole}}^{*R_2}(s,t)], \tag{1a}$$

where

$$A_{\mathrm{pole}}^{R_i}(s,t)=f_i(t)\{\exp[i\pi\alpha_i(t)]+\tau_i\}\left(\frac{s}{s_0}\right)^{\alpha_i(t)} \tag{1b}$$

are the single-pole contributions ($\tau_i\equiv$ signatures).

In the Pomeranchuk channel the exchange of two Pomeranchuk (P) poles yields

$$\mathrm{Im}A_{\mathrm{cut}}^{PP}(s,0)=s\sigma_{\mathrm{el}}^{P}(s),$$

where $\sigma_{\mathrm{el}}^{P}(s)$ is the P-pole contribution to the total elastic $\pi^+\pi^+$ cross section. Similarly

$$\mathrm{Im}A_{\mathrm{pole}}^{P}(s,0)=s\sigma_{\mathrm{tot}}^{P}(s).$$

If we assume the P-pole contributions to σ_{tot} and σ_{el} in both cases to be the dominant ones, then we may in a rough first approximation set

$$\sigma_{\mathrm{tot}}^{P}(s)\approx\sigma_{\mathrm{tot}}(s) \tag{2a}$$

and

$$\sigma_{\mathrm{el}}^{P}(s)\approx\sigma_{\mathrm{el}}(s). \tag{2b}$$

We thus conclude that

$$\left|\frac{A_{\mathrm{cut}}^{PP}(s,0)}{A_{\mathrm{pole}}^{P}(s,0)}\right|\approx\frac{\sigma_{\mathrm{el}}(s)}{\sigma_{\mathrm{tot}}(s)}. \tag{3}$$

Needless to say, this result is not restricted to $\pi^+\pi^+$ scattering but would obtain for any elastic scattering process. In all cases where experimental data are available ($\pi^\pm p, K^\pm p, pp, \bar{p}p$), the right-hand side of Eq. (3) is of the order of 0.15-0.3 for energies of 10 to 30 BeV. Whereas this is sufficiently small to make our approximation (2) meaningful, it also shows that the cut contribution is sizable.

As an example of a cut in an "inelastic" channel, we next consider the ρP cut in $\pi^+\pi^-\rightarrow\pi^0\pi^0$. We want to calculate the ratio

$$\epsilon_\rho^{\rho P}(s,0)=\frac{\mathrm{Im}A_{\mathrm{cut}}^{\rho^+P}(s,0)_{\pi^+\pi^-\rightarrow\pi^0\pi^0}}{\mathrm{Im}A_{\mathrm{pole}}^{\rho^+}(s,0)_{\pi^+\pi^-\rightarrow\pi^0\pi^0}}=\frac{\mathrm{Im}A_{\mathrm{cut}}^{\rho^0P}(s,0)_{\pi^+\pi^+\rightarrow\pi^+\pi^+}}{\mathrm{Im}A_{\mathrm{pole}}^{\rho^0}(s,0)_{\pi^+\pi^+\rightarrow\pi^+\pi^+}}=\frac{2\sigma_{\mathrm{cut}}^{\rho P}(s)}{\sigma_{\pi^-\pi^+}(s)-\sigma_{\pi^+\pi^+}(s)}, \tag{4}$$

where $\sigma_{\pi^\mp\pi^+}$ are the $\pi^\mp\pi^+$ total cross sections and $2\sigma_{\mathrm{cut}}^{\rho P}(s)$ is the ρP-cut contribution to $\sigma_{\pi^-\pi^+}-\sigma_{\pi^+\pi^+}$.

Reggeized ρ- and P-pole universality,[6] or alternatively, the quark model[7] may be used to obtain the ρ and P residues in $\pi^+\pi^+$ scattering from the known residues of these poles in πp scattering. To wit,

$$\beta_{\pi^+\pi^+\rightarrow\pi^+\pi^+}^{\rho^0}(t)=2\beta_{\pi^+p\rightarrow\pi^+p}^{\rho^0}(t) \tag{5}$$

and

$$\beta_{\pi^+\pi^+\rightarrow\pi^+\pi^+}^{P}(t)=\tfrac{2}{3}\beta_{\pi^+p\rightarrow\pi^+p}^{P}(t). \tag{6}$$

To have a concrete case we shall take $s=20$ BeV^2. For this energy we proceed to calculate the integral (1) in two different approximations:

(A) We use the parametrization of Chiu, Phillips, and Rarita[8] for both the P and ρ trajectories and residues. Using (5) and (6) and inserting into (1), we find in this case

$$\sigma_{\mathrm{cut}}^{\rho P}(s=20\ \mathrm{BeV}^2)\approx0.07\ \mathrm{mb}. \tag{7a}$$

(B) In this approximation we use Ref. 8 for all the parameters of the ρ pole, but we assume the P pole to be fixed at $l=1$ [i.e., $\alpha(t)\equiv1$].[9] The P-residuum function $\beta_{\pi^+p\rightarrow\pi^+p}^{P}(t)$ is then determined from a fit to the diffraction pattern in πp scattering.[10]

We find now

$$\sigma_{\rm cut}^{\rho P}(s=20\ {\rm BeV}^2)\approx 0.10\ {\rm mb}. \qquad (7b)$$

The ρ-universality relation $\sigma_{\pi^-\pi^+}-\sigma_{\pi^+\pi^+}=2\times(\sigma_{\pi^-p}-\sigma_{\pi^+p})$ and the experimental value $(\sigma_{\pi^-p}-\sigma_{\pi^+p})_{s=20\ {\rm BeV}^2}=1.93\pm0.09$ mb,[11] when inserted along with the values (7a) or (7b) into Eq. (4), give

$$\epsilon_\rho^{\rho P}(s=20\ {\rm BeV}^2, t=0)=0.035 \qquad (8a)$$

and

$$\epsilon_\rho^{\rho P}(s=20\ {\rm BeV}^2, t=0)=0.05, \qquad (8b)$$

respectively, very small values indeed. From universality, SU(3)-symmetry, and exchange-degeneracy considerations it is obvious that this result is not restricted to the ρP cut in $\pi\pi$ scattering but that similar small values are to be expected for ϵ_f^{fP}, $\epsilon_{A_2}^{A_2P}$, $\epsilon_\omega^{\omega P}$, etc., in all kinds of inelastic processes at $t=0$. Other cuts formed by two Regge poles, neither of which is the Pomeranchuk pole, are negligible because of their energy dependence. Our result (8) is obtained at $t=0$. By continuity, it is nevertheless clear that, for energies up to 30 BeV at least, these cut contributions will be negligible in the near-forward region. Of course, as energy increases, the larger exponent of the cut will ultimately increase its role away from $t=0$. Away from $t=0$, even in the simple model considered above, the quantitative estimate of cut effects becomes much more involved [the formula corresponding to (1a) becomes much more complicated]. We shall therefore refrain here from discussing this problem in any quantitative detail.

Our finding that cut contributions are smaller than pole contributions at $t=0$ makes it plausible that higher cuts (generated by the simulataneous exchange of three or more poles) will be even less important there.

We now wish to discuss briefly the problem raised in the introduction about cut effects on the relations among total cross sections of Refs. 3 and 4. To do this we shall distinguish between two cases. An example of the first case is the weak Johnson-Treiman relation,

$$(\sigma_{K^-p}-\sigma_{K^+p}-\sigma_{K^-n}+\sigma_{K^+n}) -(\sigma_{\pi^-p}-\sigma_{\pi^+p})=0. \qquad (9)$$

The only cut that contributes to this combination of cross sections is the $P\rho$ cut. The Pomeranchukon being predominantly a unitary singlet, Eq. (1) now immediately tells us that if the ρ-pole contribution to the left-hand side of Eq. (9) vanishes, so will the $P\rho$ cut contribution. The crucial point in obtaining this result was the fact that Eq. (9) contains only meson-baryon (MB) cross sections. Thus besides the $P\rho$ cut contributions to both parentheses in Eq. (9) being small (see above), they cancel each other so that this relation is expected to be exactly valid even in the presence of cuts. If, however, a total cross-section relation contains both BB (or $\bar{B}B$) and MB cross sections, then because of the factor $\frac{2}{3}$ between the P contribution to the MB and BB amplitudes, no cancellation will occur and cut effects will violate the relation. As an example let us consider the effect of the PP cut on the relation

$$\sigma_{BB}=\tfrac{3}{2}\sigma_{MB}. \qquad (10)$$

If we call P and C the P-pole and cut contributions (in a suitable normalization), respectively, then we have

$$\sigma_{MB}=6P+36C$$
$$\sigma_{BB}=9P+81C \qquad (11)$$

and the relation (10) is violated by

$$\Delta=\frac{\sigma_{BB}-\frac{3}{2}\sigma_{MB}}{\sigma_{BB}}=\frac{27C}{\sigma_{BB}}\approx\frac{27}{36}\frac{\sigma_{{\rm el}\,MB}}{\sigma_{{\rm tot}\,BB}}\approx\frac{27}{36}\frac{2}{3}\frac{\sigma_{{\rm el}\,MB}}{\sigma_{{\rm tot}\,MB}}=\frac{1}{2}\frac{\sigma_{{\rm el}\,MB}}{\sigma_{{\rm tot}\,MB}}. \qquad (12)$$

Comparing with Eq. (3) we see that, so to speak, only "50% of the cut" violates relation (10). Similar considerations can be made for the other relations of Refs. 3 and 4 that involve both BB and MB cross sections.

To conclude, at present energies it appears that cut contributions to forward-scattering amplitudes can be important in the Pomeranchuk channel but are smaller in other (inelastic) channels. In both cases they appear to be smaller than Regge-pole contributions.

One of us (P.G.O.F.) wishes to thank Dr. Ivan Muzinich for an illuminating discussion on the nonfactorizability of Mandelstam-type cuts.

*Work supported in part by U. S. Atomic Energy Commission.

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