described by

$$\frac{dp^\lambda}{d\tau}=\frac{e}{m_\Sigma c}p^\sigma F_\sigma{}^\lambda+\frac{p^\lambda}{|\vec{p}|}\frac{d|\vec{p}|}{d\tau} \text{ for } \lambda=1,2,3,$$

$$\frac{dp^4}{d\tau}=\frac{e}{m_\Sigma c}p^\sigma F_\sigma{}^4+\frac{|\vec{p}|}{p^4}\frac{d|\vec{p}|}{d\tau}.$$

Here the first terms arise from the Lorentz force and the second terms from the momentum loss due to ionization.

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LEPTONIC DECAYS OF NEUTRAL VECTOR MESONS AND ρ-ω INTERFERENCE*

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The effect of ρ-ω interference on the leptonic decay of photoproduced neutral vector mesons is shown to be much larger than previously estimated. The experimental determination of the leptonic branching ratio of the ρ is shown to be sensitive to the amount of interference of the ρ and ω.

In this Letter we investigate the resonance shape of the leptonic decay of a photoproduced vector meson and the interference effects of the leptonic decay of two vector mesons of nearly equal mass, specifically the ρ^0 and ω. Several recent experiments have measured photoproduction of lepton pairs with the invariant mass of the lepton pairs in the region of the ρ mass.[1,2] In none of these experiments has a resonance at the ω mass been observed superimposed on the observed ρ resonance, even though the mass resolution of Asbury et al.[2] (±15 MeV) should have been sufficient to resolve the ω if it were present to the extent of about 10%. It has generally been assumed as a result of these experiments that the effect of the ω was small (of the order of or less than 10%). This result did not seem odd because the effect of the ω alone expected from unbroken SU(3) with an ω-φ mixing angle of $\sin\theta=(\frac{1}{3})^{1/2}$ is about 10%. However, we will show that if the unbroken SU(3) predictions for the ω branching ratio are correct[3] $[\Gamma_{\rho\to ll}/\Gamma_{\omega\to ll}\approx 9/1$ with $\sin\theta=(\frac{1}{3})^{1/2}]$, the ω will have a very large effect (of the order of 60%) on the analysis of these experiments. This is because of the effect of ρ-ω interference.

We may write the amplitude for the process

$$\gamma+A\to V^0+A\to l^++l^-+A$$

(see Fig. 1) as

$$\mathfrak{a}=\frac{em_v^2}{2\gamma_v}\frac{1}{-m^2}A\frac{1}{m^2-m_v^2+im_v\Gamma_v}\frac{em_v^2}{2\gamma_v}\frac{1}{m^2}\times\langle ll|V|\gamma\rangle, \qquad (1)$$

where A is the (complex) amplitude for $V^0+A\to V^0+A$ and m is the invariant mass of the lepton pair. We have included here a propagator $(k^2-m^2)^{-1}$, where $k^2=0$, for the "initial" vector meson.[4] This leads to a resonance shape for the decay of

$$|m^2-m_v^2+im_v\Gamma_v|^{-2}m^{-8}\sum_{\text{final states}}|\langle ll|V|\gamma\rangle|^2.$$

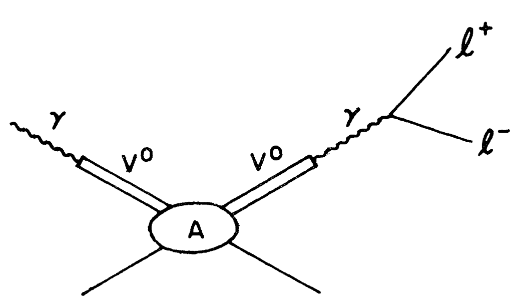

FIG. 1. Feynman diagram for photoproduction of lepton pairs on complex nuclei via a vector meson.

Note especially the factor m^{-8}. A factor m^{-4} is from the "initial" vector-meson propagator and another factor m^{-4} is from the "final" photon propagator. Asbury et al.[5] report that the m^{-4} factor from the meson propagator gives an improved fit to the resonance shape of the process $\gamma+A\rightarrow\rho^0+A\rightarrow\pi^++\pi^-+A$. The final state in this experiment is a pion pair and thus tests only the m^{-4} factor from the meson propagator. The m^{-4} factor from the photon propagator must be included whenever the final state is a lepton pair. These factors produce a very pronounced asymmetry of the resonance shape with heavy weighting in the low-mass regions. At present, there is some question as to whether it is proper to include the m^{-4} from the meson propagator.[6] However, the conclusions we shall draw regarding $\Gamma_{\rho\rightarrow ll}/\Gamma_{\omega\rightarrow ll}$ are only weakly dependent upon this question.

Since the ρ and ω have nearly equal masses, they will interfere with one another when they both decay into leptons and have the same final state including polarizations. If we temporarily exclude the factor m^{-8} and the square of the photon-lepton-pair matrix element integrated over the experimentally relevant phase space, and assume that both the ρ and ω are diffraction produced, the resonance shape for leptonic photoproduction through the ρ and ω channels and their interference is

$$R(m^2)=m_\rho^{\ 2}\Gamma_\rho^{\ 2}\left|\frac{1}{m^2-m_\rho^{\ 2}+im_\rho\Gamma_\rho}+\frac{m_\omega^{\ 4}}{m_\rho^{\ 4}}\frac{\gamma_\omega^{\ -2}}{\gamma_\rho^{\ -2}}\frac{e^{i\varphi}}{m^2-m_\omega^{\ 2}+im_\omega\Gamma_\omega}\right|^2. \tag{2}$$

We have assumed that there is no difference—except for the possibility of a phase difference of φ—between ρ-nucleus scattering and ω-nucleus scattering, and we have normalized $R(m^2)$ such that $R(m_\rho^{\ 2})=1$ when the ω contribution is absent. We have plotted

$$R_n(m^2)\equiv(m_\rho/m)^nR(m^2)$$

for $n=0$, 4, and 8 for the case $\gamma_\omega^{\ -2}/\gamma_\rho^{\ -2}=0$ in Fig. 2(a), and for the case $\gamma_\omega^{\ -2}/\gamma_\rho^{\ -2}=\frac{1}{9}$ and $\varphi=0°$ in Fig. 2(b). $R_8(m^2)$ is the expected resonance shape with the Ross-Stodolsky m^{-4} factor[4] and $R_4(m^2)$ is the expected resonance shape without it. The ratio $\gamma_\omega^{\ -2}/\gamma_\rho^{\ -2}=\frac{1}{9}$ is an unbroken-SU(3) prediction with $\sin\theta=(\frac{1}{3})^{1/2}$. The ratio of the area under $R_8(m^2)$ between 620 and 840 MeV for ρ-ω interference to that for the ρ alone is 1.58. For $R_4(m^2)$ the ratio is 1.68; for $R(m^2)$ the ratio is 1.77. We see that the effect of the interference term is large, but is not strongly dependent upon the power of the mass multiplying the Breit-Wigner resonance shape.

The photon-lepton-pair matrix element squared when integrated over all available phase space is proportional to m^2. In this case the experimentally observed yield shapes would correspond to $R_6(m^2)$ or $R_2(m^2)$. However, the matrix-element-phase-space m dependence must be calculated for each experimental arrangement. As we noted above, the ratio of the areas under the resonance curves is not strongly dependent on the power of m multiplying the Breit-Wigner resonance shape.

The apparent divergence of the ρ-ω tail for $m\ll m_\rho$ will not remain when Γ_ρ is corrected for

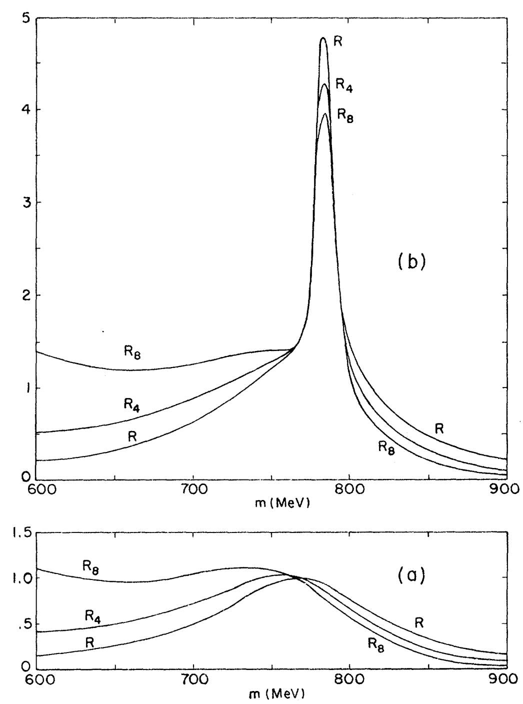

FIG. 2. (a) Graph of the functions $R_8(m^2)$, $R_4(m^2)$, and $R(m^2)$ for the ρ alone: $\gamma_\omega^{\ -2}/\gamma_\rho^{\ -2}=0$. (b) Graph of the functions $R_8(m^2)$, $R_4(m^2)$, and $R(m^2)$ for ρ-ω interference: $\gamma_\omega^{\ -2}/\gamma_\rho^{\ -2}=\frac{1}{9}$, $m_\omega=783.3$ MeV, $\Gamma_\omega=12.2$ MeV. For both graphs $m_\rho=765$ MeV, $\Gamma_\rho=128$ MeV, and $\varphi=0°$.

phase space effects, i.e.,

$$\Gamma_\rho(m)=\frac{m_\rho}{m}\left[\frac{m^2-4m_\pi^{\ 2}}{m_\rho^{\ 2}-4m_\pi^{\ 2}}\right]^{3/2}\Gamma_\rho(m_\rho).$$

In addition, the effect of the ρ-ω tail will not be seen as a "breakdown" of quantum electrodynamics for $m \ll m_\rho$ since the Bethe-Heitler cross section is proportional approximately to m^{-8} for nearly symmetric pairs. Thus, the ρ-ω tail will affect only the normalization of photoproduction experiments.

In the foregoing, we have presented two distinct ideas.

First, the shape of the lepton-pair resonance should have an m^{-8} factor if the Ross-Stodolsky m^{-4} factor is correct.[4] Thus the shape of the lepton-pair resonance may be a sensitive test of the Ross-Stodolsky m^{-4} factor, i.e., does the resonance shape agree with $R_4(m^2)$ (no Ross-Stodolsky factor) or does it agree with $R_8(m^2)$? If this test is made well away from the resonance (either above or below), the presence or absence of the ω interference is almost irrelevant. The factor m^{-4} from the "final" photon propagator is present in all cases of lepton decay and suppresses the $\rho \to ll$ background near the φ resonance. Another possibly more sensitive test of the m^{-8} factor might be obtained by observing the interference terms between Bethe-Heitler pair production and pair production via Compton diagrams where the observation is made well below the ρ resonance. Such experiments have already been done.[2,7] At an invariant lepton mass of about 380 MeV, for example, the Compton amplitude due to the low-mass tail of the ρ is enhanced by a factor of 4 due to the Ross-Stodolsky factor and another factor of 4 due to the photon-propagator factor.

The second distinct idea discussed is the large contribution to the lepton-pair yield due to the ρ-ω interference term. The contribution is large (60-80%) even if the ω yield is very small (10%). It is generally assumed[8] that the bubble-chamber data showing that ω photoproduction is about 10% of ρ^0 photoproduction lead to the conclusion that it is not surprising that no effect of the ω peak or of the ρ-ω interference term is observed. Contrary to this, our work shows that even for such a small ω contribution the expected effect of ρ-ω interference terms is to increase the yield in the region of the ρ mass by typically 60%. Also this effect is not very dependent on the mass factor described above.

Let us, finally, consider the experimental situation of ρ, ω leptonic branching. The experimental determinations of the ρ branching ratio $B_\rho = \Gamma_{\rho\to ll}/\Gamma_\rho$ from photoproduction of lepton pairs by de Pagter et al.[1] $[B_\rho = (5.9\pm1.5)\times10^{-5}]$ and by Asbury et al.[2] $[B_\rho = (6.5\pm1.4)\times10^{-5}]$ neglected in their analysis the contribution of the ω alone and ρ-ω interference. Asbury et al.[2] have designed their apparatus with a resolution of ±15 MeV specifically in an attempt to observe the decay $\omega\to e^+ + e^-$ or ρ-ω interference effects. They believe that a large interference effect would have been seen, and consequently they have set a limit of 5% on the $\omega\to e^+ + e^-$ contribution. This limit is below the 10% value expected from the unbroken-SU(3) ratio of $\frac{1}{9}$. If the unbroken-SU(3) ratio of $\frac{1}{9}$ is correct, the values of B_ρ and γ_ρ^{-2} in the two photoproduction experiments should be reduced by a factor of about 1.6 to get the true values (i.e., those which would occur in the absence of the ω).

There should be no ρ-ω interference term contributing to B_ρ in the colliding-beam experiment $e^+ + e^- \to \pi^+ + \pi^-$ of Augustin et al.[9] whose result is $B_\rho = (6.2\pm1.0)\times10^{-5}$, or of Auslander et al.[9] The latter report $\sigma_\rho = 1.2\pm0.2$ μb and, using $\sigma_\rho = 12\pi m_\rho^{-2}B_\rho$, we obtain $B_\rho = (4.8\pm0.8)\times10^{-5}$. The four experimental values for B_ρ mentioned above are in excellent agreement with each other. But why is this so? Since we will have interference between the ρ and the ω in photoproduction of lepton pairs but not in colliding beams (except for the small $\omega\to\pi^+ + \pi^-$ mode about which a Letter is in preparation), we have two possibilities [we assume $\varphi = 0°$ in Eq. (2)]:

(i) We retain the unbroken-SU(3) prediction of $\Gamma_{\rho\to ll}/\Gamma_{\omega\to ll} = 9/1$. Then the colliding-beam experiments and the photoproduction experiments should disagree because of the interference terms. Since they agree, one or more of the experiments may have suffered a large statistical fluctuation. If the Orsay colliding-beam experiment is correct, then $B_\rho = (6.2\pm1.0)\times10^{-5}$ as stated in their report. If the photoproduction experiments are correct, then one must reduce their results by a factor of ≈1.6 to allow for ρ-ω interference. In this case $B_\rho \approx 3.8\times10^{-5}$. It is difficult to see why, in this latter case, the ρ-ω interference term does not produce an effect large enough to be easily seen in the resonance shape of an experiment with a mass resolution of ±15 MeV, such as that of Asbury et al.[2] This is not seen in their published results. The value we obtain from Auslander et al.[9] is in reasonable agreement with our prediction for the possibility in which the photoproduction experiments are assumed correct.

(ii) We may assume that the ratio of $\Gamma_{\rho\to ll}/\Gamma_{\omega\to ll}$ given by unbroken SU(3) is too small. This is in the direction indicated by the results of Oakes and Sakurai[10] who find that the unbroken-SU(3) prediction of $\Gamma_{\rho\to ll}/\Gamma_{\omega\to ll}=9/1$ must be modified to $\Gamma_{\rho\to ll}/\Gamma_{\omega\to ll}=9/0.65$. This result reduces the magnitude of the effect of the ω by a factor of ≈ 0.8. We would then predict that if the photoproduction experiments[1,2] are correct, $B_\rho \approx 4.8\times 10^{-5}$ in excellent agreement with Auslander et al.[9] However, even in this case the ω should be visible superimposed upon the ρ resonance in the photoproduction experiment of Ref. 2.

We next consider the experimental consequences of the assumption that $\Gamma_{\rho\to ll}/\Gamma_{\omega\to ll}$ is considerably larger than 9:1. We note that while a value of, say, $\theta\approx 0$ would increase the predicted value of $\Gamma_{\varphi\to ll}$, this quantity has not yet been measured. A value of $\theta = 0$ would produce no disagreement with the present upper limits on $\Gamma_{\varphi\to ll}$.[11,12] One must note, however, that the total cross sections for photoproduction of the ρ and the ω appear to be in agreement with unbroken SU(3) in the diffraction region,[13] i.e., $\sigma_\rho:\sigma_\omega \approx 9:1$. The agreement of the bubble-chamber data on photoproduction of ρ and ω with the 9:1 rule and the violation of this rule in leptonic decay may be patched up by assuming that the ω-nucleon cross section is larger than the ρ-nucleon cross section. This would allow a $\Gamma_{\rho\to ll}/\Gamma_{\omega\to ll}$ ratio of more than 9 and a $\sigma_\rho/\sigma_\omega$ ratio of about 9.

Another point should be mentioned. There is the possibility that $\varphi\neq 0°$. As an example of a nonzero phase angle between ρ-nucleus and ω-nucleus scattering, $R(m^2)$ for $\varphi = 210°$ is plotted in Fig. 3.

Finally, we wish to note that the question of the ρ-ω interference term has a large effect on the test of e-μ universality in the timelike region.[12,14] If an experiment on $\rho^0\to\mu^+ + \mu^-$ is compared with an experiment on $\rho^0\to e^+ + e^-$, one must know the magnitude of ω production in the two experiments and the magnitude of γ_ω^{-2} (which is not known experimentally). Alternatively, in order to have equal ρ-ω interference effects for the muon and electron experiments, two experiments may be compared using the same incident beam and target particles at the same energy, as was done by Weinstein[14] comparing the experiments of Refs. 1 and 2. For example, a measurement of B_ρ from π-produced ρ's has been made.[15] However if one compares

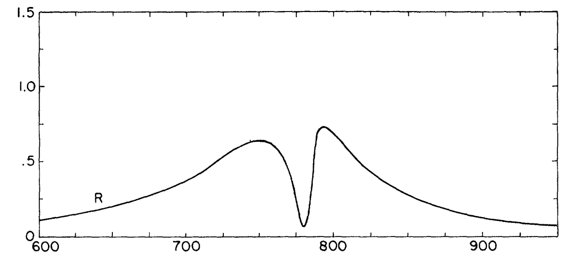

FIG. 3. Graph of the function $R(m^2)$ for $\gamma_\omega^{-2}/\gamma_\rho^{-2}=\frac{1}{9}$, $m_\omega = 783.3$ MeV, $\Gamma_\omega = 12.2$ MeV, $m_\rho = 765$ MeV, $\Gamma_\rho = 128$ MeV, and $\varphi = 210°$.

the decay of π-produced ρ's to γ-produced ρ's as was recently done by Wehmann et al.,[12] discrepancies of up to 60% can result from the differences of the interference terms in the two experiments.

We wish to thank Professor S. D. Drell for his valuable discussions on the possible values of φ.

After this paper was written, we were informed that Michel Davier at Stanford Linear Accelerator Center was independently working on this problem and had obtained values of ρ-ω interference terms in essential agreement with ours.

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STUDY OF STRUCTURE IN THE "$A_2(1300)$" REGION IN π^-p INTERACTIONS AT 6 GeV/c*†

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In a bubble-chamber experiment we have observed a splitting of the "$A_2(1300)$" similar to that observed in the CERN missing-mass experiment. Analysis of the $K_1^0K_1^0$ system indicates that there are two different resonances in the "A_2" region.

Recent data on the mass spectrum of negative bosons (X^-) produced in the reaction $\pi^-p \to pX^-$, where X^- can decay into one or three charged particles plus possible neutrals, showed structure in the mass region of the $A_2(1300)$ meson.[1,2] This structure consisted of a splitting of the A_2^- into two peaks with masses (m) and full widths (Γ) of $m_1 = 1274 \pm 16$, $\Gamma_1 = 29 \pm 10$ and $m_2 = 1320 \pm 16$, $\Gamma_2 = 35 \pm 10$ MeV. In this note, we report the observation in a bubble-chamber experiment of a similar splitting of the A_2^- peak in the reaction $\pi^-p \to p\pi^-$ + missing mass (MM) at 6 GeV/c. In addition, investigation of the $K\bar{K}$ mass spectrum in the A_2 region reveals a narrow $K_1^0K_1^0$ enhancement centered at the higher A_2^- peak (m_2) with a width of 21_{-6}^{+10} MeV. These observations suggest that there are two resonances with different spin and parity in the "A_2" mass region.

The data were obtained from a 6-GeV/c π^-p exposure of 230 000 pictures in the Brookhaven National Laboratory (BNL) 80-in. liquid-hydrogen bubble chamber. About $\frac{3}{5}$ of the events were measured by the BNL flying-spot digitizer and the rest by conventional measuring machines. The size of the event samples for the four reactions studied are shown below:

Reaction	Number of events
(1) $\pi^-p \to p\pi^-$MM (elastic events excluded)	10 319
(2) $\to nK_1^0K_1^0$	183
(3) $\to K_1^0K_1^0$ + neutrals	206
(4) $\to pK^-K_1^0$	199

The protons in Reaction (1) have been unambiguously identified for this analysis. This requirement is equivalent to an effective four-momentum transfer cut to the protons $t_{p\to p}$ of ≤ 2 GeV^2. We also required visible K_1^0 decays in Reactions (2)-(4).

Figure 1(a) is a display of the effective mass of X^- as a function of the laboratory proton momentum (P_p) and angle (θ_p) at 6 GeV/c. For the purpose of our subsequent analysis of the X^- mass spectrum in the "A_2" region, we define limits of P_p in region II identical to those in the 1965 CERN missing mass-spectrometer experiment at 6 GeV/c.[1] Adjacent regions I and III are chosen with the same range of P_p as that of region II [see Fig. 1(a)]. We have also selected a region "M" which simulates the "tapered-absorber" selection of the same experiment. Figures 1(b)-1(d) are effective-mass spectra in 40-MeV bins of the X^- (π^- + MM) in regions I, II, and III, respectively. A marked enhancement centered about 1.3 GeV with a width of about 100 MeV is clearly seen in Fig. 1(c). The mass and width of this enhancement are similar to the accepted values for the conventional "A_2" meson.[3] We observe little or no "A_2" production in either region I, probably because of the large Deck-like background at low $t_{p\to p}$ values, or region III, because of lack of A_2 production at high $t_{p\to p}$ values. A study of the experimental mass resolution (Γ_{exp}) of π^- + MM yields 12 ± 3 MeV in region I, 10 ± 2 MeV in region II, and 14 ± 3 MeV in region III.[4] We therefore present the X^- effec-