MEANS OF MEASURING THE EARTH'S VELOCITY THROUGH THE 3° RADIATION FIELD

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The velocity of the earth's motion with respect to the rest frame of the 3°K cosmic background radiation can be determined by measuring the anisotropy of the radiation. Partridge and Wilkinson[1] attempted to detect this anisotropy at $\lambda = 3.2$ cm; they were able to set an upper limit of 300 km/sec to the component of velocity of the earth which lies in the plane of the earth's equator. The purpose of our Letter is to show that such measurements can best be made with wideband detectors that are sensitive at millimeter infrared wavelengths where a number of laboratories, including ours, are about to make rocket observations.

Suppose that an extended source of radiation emits $N(\nu)$ photons per unit area per unit time per unit bandwidth so that its brightness is $B(\nu) = h\nu N(\nu)$. If an observer approaches the source with velocity $v = \beta c$, $v \ll c$, he will see the photons emitted at frequency ν Doppler shifted up to frequency $\nu' = \nu(1+\beta)$. Furthermore, in a time interval Δt his detector will sweep up the photons emitted by the source during the time interval $(1+\beta)\Delta t$. Thus, by photon conservation,

$$N'(\nu')d\nu' = N(\nu)d\nu(1+\beta)$$

so

$$\frac{B'(\nu')d\nu'}{h\nu'} = \frac{B(\nu)d\nu}{h\nu}(1+\beta),$$

$$B'(\nu') = \left[B(\nu') \mp \beta\nu' \frac{dB(\nu')}{d\nu}\right](1 \pm \beta),$$

where the top signs refer to an observer approaching the radiation source and the bottom signs refer to a receding observer. If the source emits blackbody radiation with temperature T, then

$$B(\nu') = (2h\nu'^3/c^2)(e^{h\nu'/kT} - 1)^{-1}$$

and

$$B'(\nu') = B(\nu')\left[1 \pm \beta\left(\frac{(h\nu'/kT)e^{h\nu'/kT}}{e^{h\nu'/kT} - 1} - 2\right)\right].$$

If the anisotropy of the received signal is small, then the ratio R of the signal powers seen by an observer looking alternately parallel and antiparallel to the direction of his motion with respect to the radiation source is

$$R = 1 + 2\beta x e^x/(e^x - 1) - 4\beta$$

where $x = h\nu'/kT$. For a detector of unit throughput (1 cm² sr), the received power of the useful anisotropy signal, which is the difference between the signals seen in the parallel and antiparallel directions, in the range $\nu_1' = kTx_1/h$ to $\nu_2' = kTx_2/h$, is

$$P = \beta \frac{4k^4T^4}{c^2h^3} \int_{x_1}^{x_2} \frac{x^3}{e^x - 1}\left(\frac{xe^x}{e^x - 1} - 2\right) dx.$$

The dimensionless function

$$f(x) = \frac{x^3}{e^x - 1}\left(\frac{xe^x}{e^x - 1} - 2\right)$$

is plotted in Fig. 1. At low frequencies it is negative, indicating that the detector receives less power from an approaching source than from a receding one. $f(x)$ becomes positive for $x > 1.60$ and has a strong maximum near $x = 4.5$.

In the case of the 3°K radiation field, almost all of the anisotropy signal power is found in the region around $\lambda = 1$ mm. The total anisotropy signal power that would be received by a bolometric detector of unit throughput because of the earth's motion with respect to the radiation is

$$P = \beta\left(\frac{4k^4T^4}{c^2h^3}\right)\int_0^\infty f(x)dx,$$

$$P = \beta \times 6 \times 10^{-3} \text{ erg/sec}.$$

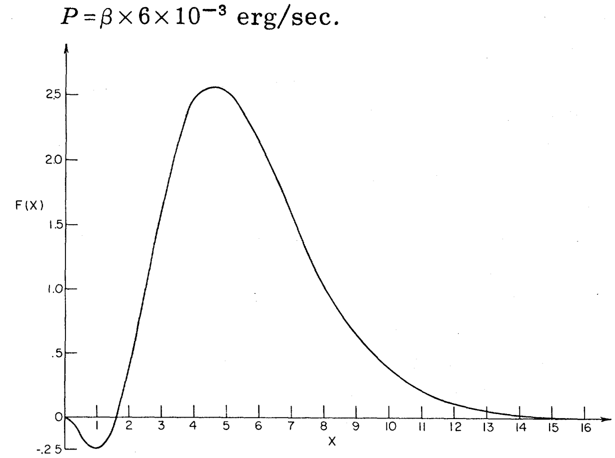

FIG. 1. Relative anisotropy signal strength $f(x)$ (dimensionless) as a function of $x = h\nu'/kT$.

This signal is much greater than the bolometric quantum equivalent noise

$$P_{QN}=\sqrt{2}\,\frac{h}{c}\left[\int_0^\infty \frac{\nu^4 d\nu}{e^{h\nu/kT}-1}\right]^{1/2},$$

which is only 3×10^{-10} erg/sec for $T=3°$.

State-of-the-art bolometric detectors have light gathering power of 1.0 cm^2-sr and noise equivalent powers $\sim10^{-5}$ erg/sec, so that a velocity of $v=300$ km/sec of the earth with respect to the cosmic background radiation could be measured with unit signal-to-noise ratio in about 3 sec. This is over 10^6 times as fast as the radio measurements at 3.2 cm.

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[1]R. B. Partridge and David T. Wilkinson, Phys. Rev. Letters 18, 557 (1967).

OBSERVATIONAL PROPERTIES OF THE HOMOGENEOUS AND ISOTROPIC EXPANDING UNIVERSE

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The purpose of this Letter is to establish a common language between theoretical and observational cosmologists. The first completely satisfactory derivation of the line element for a homogeneous and isotropic expanding universe was given by Robertson[1] and led to the formula

$$ds^2=-\frac{L^2(t)}{[1+r^2/4\mathcal{R}_0{}^2]^2}(dr^2+r^2d\theta^2+\gamma^2\sin^2\theta d\varphi^2)+d(ict)^2, \tag{1}$$

where $\mathcal{R}_0$ is a constant describing the curvature of space (the time axis is always straight) and $L(t)$ is a time-dependent proper distance for the present value of which we assume $L_p=L(t_p)=1$. The curvature radius $\mathcal{R}$ can be real, zero, or imaginary. In the first case, we have a closed spheroidal space with a finite volume; in the second, Euclidean space; and in the third, an open hyperbolic space with an infinite volume. We have $L/L_p=\mathcal{R}/\mathcal{R}_p$ or $=l/l_p$, where l is a distance between any two points (atoms or galaxies) in expanding space.

Writes Tolman,[2] presenting the derivation of that element of length,

"The assumption of spacial isotropy for the large-scale physical findings obtained by observers at rest with respect to the matter in their neighborhood, combined with the principles of relativistic mechanics, does inevitably lead to the proposed line element. Hence, if we should later be dissatisfied on observational or philosophical grounds with the results to be obtained from the proposed models, we must modify either the principles of relativistic mechanics, or the assumption that all the observers in the universe must be expected to obtain large-scale results which are independent of the direction of observation."

The function $L(t)$ is determined by a differential equation:

$$\frac{1}{L}\frac{dT}{dt}=\pm\left(\frac{8\pi G}{3}\rho+\frac{c^2}{\mathcal{R}_0{}^2L^2}\right)^{1/2}, \tag{2}$$

where G is Newton's gravitational constant, c the speed of light, and ρ the combined mass density of matter (particles) and radiation. Thus we can write

$$\rho=\rho_m+\rho_r=\frac{b}{L^3}+\frac{aT^4}{c^2}=\frac{b}{L^3}+\frac{d}{L^4}, \tag{3}$$

where a and d are constants.

The best value for ρ_m today seems to be 0.7 $\times10^{-30}$ g/cm^3 (estimated from the masses and mean distances of the galaxies), while the value of ρ_r is only 6.8×10^{-34} g/cm^3 (which corresponds to the temperature of 3°K). Substituting into (2) the values

$$\delta_p=0.7\times10^{-30}$$

and

$$(1/L_p)(dL_p/dt)=100\text{ km/sec Mpc}$$