photons, namely those corresponding to total isospin $I=\frac{1}{2}$ and $\frac{3}{2}$. It is possible with four measured cross sections to eliminate the three parameters associated with a single isosinglet amplitude, its magnitude and phase. However, one cannot eliminate both isovector amplitudes.

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FOURTH TEST OF GENERAL RELATIVITY: PRELIMINARY RESULTS

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Several years ago it became evident that a new test of general relativity was technically feasible.[1] The proposed experiment was designed to verify the prediction that the speed of propagation of a light ray decreases as it passes through a region of increasing gravitational potential. For a radar pulse transmitted from the earth and reflected by another planet, the increase in the round-trip time delay, attributable to the predicted gravitational influence of the sun on the propagation, would be ≈200 μsec if the path of the pulse were to graze the solar limb.

An intensive program was therefore undertaken early in 1965 to build a new transmitter and receiver system to provide the Lincoln Laboratory Haystack radar with the capability to measure to within 10 μsec the time delays of pulses traveling between the earth and Mercury or Venus when either planet was on the other side of the sun from the earth—the superior-conjunction alignment. The improved radar was put into operation shortly before the last such conjunction of Venus, which occurred on 9 November 1966. Time-delay measurements were made then and during the subsequent superior conjunctions of Mercury on 18 January, 11 May, and 24 August 1967. The most reliable of these data agree, on average, with the excess-delay predictions of general relativity to well within the experimental uncertainity of ±20%. The remainder of this Letter is devoted to a more detailed discussion of the data analysis and the novel experimental techniques required by the echo signal being sometimes as small as 10^{-21} W, i.e., about 10^{27} times weaker than the transmitted signal power of ≈300 kW.

The techniques are most easily explained by describing the experimental procedure. The radar transmitter is accurately controlled by a hydrogen-maser standard and produces a continuous signal at 7.84 GHz whose phase is coherent over intervals in excess of 20 sec. The transmission is actually phase coded; that is, every 60 μsec ($\equiv$ band length) the phase of the wave is either changed by 180° or unaltered, depending on a preselected code. The timing error of the 180° phase-transition points never exceeds 1 μsec. The shift-register code used has 63 elements and hence is repeated every 3.78 msec. The autocorrelation function of this signal exhibits a strong peak with weak sidelobes (none greater than 35 dB below the peak) and enables the round-trip time, i.e., group delay, of the echo to be determined accurately. Theoretical calculations[2] indicated that for this band length and for the radar system employed, delay measurement errors would be only about 10 μsec near superior conjunction. On the other hand, the 3.78-msec codelength implies a corresponding ambiguity in the interpretation of the measured delay. Since the a priori uncertainty in the planetary positions resulting from prior radar determinations[3] was less than 100 μsec, an unambiguous delay interval of 3.78 msec seems a very conservative choice and, in fact, only a small part of the interval is searched.

The phase-coded signal is directed toward the planet, the transmission continuing until the first echo returns to the radar—about 30 min later for Venus near superior conjunction. The receiver is then connected to the antenna and the echo signal has the Doppler shift (introduced by the relative motion of the radar site and the target's center) removed, while the signal is being translated (heterodyned) to near zero frequency. The signal, plus noise, is then passed through a pair of low-pass filters in phase quadrature, each matched to the 60-μsec band length; the two quadrature channels are sampled simultaneously every 30 μsec on a time base modified to account for the continuously changing delay to the target. The samples will, therefore, remain synchronous with a fixed region in delay and can be summed with negligible "smearing" of the echo, because the a priori uncertainty in the delay change to the target over the course of a day is very small. Since there are two samples per band length, the quadrature channels are each separated into even and odd sample numbers, forming four subsets which are each decoded for 16 trial delays. Each second's data are then Fourier-analyzed and the results from each such set are squared and added, as appropriate, to each element of a 32×64 delay-Doppler matrix. The span of the matrix—930 μsec in delay and approximately 64 Hz in frequency—is sufficient for our purposes both because of the rapid falloff with delay of the backscattered power from the target and because of the relatively small a priori uncertainty of the Doppler shift and the round-trip delay to the target.

By the use of very efficient algorithms, the processing leading to the data matrix is completed before the first echo from the next 30-min transmission is received. These alternations of transmitting and receiving cycles, a pair of which constitute a "run," are usually continued throughout a given observing day. After interpolation to provide values at delay intervals of 5 μsec and at frequency intervals of 0.25 Hz, the summation of the matrices obtained for a given day is cross correlated with a two-dimensional array of numbers ("template") that represents in delay and frequency the expected form of the echo from the planet. Because of the large size of the template used, the cross-correlation coefficients are calculated for only a comparatively small number of relative positions ("offsets") of the two arrays. These offsets nevertheless span a region—330 μsec in delay and 40 Hz in frequency—far larger than that corresponding to the a priori uncertainty.

How is the template determined? Since the planetary radius ρ is about a factor of 10^8 larger than the wavelength λ of the radar signal, we can consider the planet to be composed of a great number of independent elements, each large compared with λ and small compared with ρ. When viewed from any given aspect, an element will have a particular backscattering cross section and will impart to the echo signal a particular time delay and Doppler shift which depend on the distance and motion of the element with respect to the radar ("delay-Doppler" mapping[4]). By studying the backscattering properties of a planet at inferior conjunctions, when the signal-to-noise ratio is highest, an "average" template is determined for use in analyzing the echoes near superior conjunction. Because the rotational motion or spin of each of the target planets is now well known,[5] essentially the only unknowns are the two numbers giving the round-trip delay and Doppler shift corresponding to reflections from the point on the surface of the target that lies closest to the earth, the so-called subradar point. The cross correlation of the template with the summed echo signals plus noise is carried out first with a larger, coarse grid (15 μsec in delay, 2 Hz in frequency) and then with a smaller, fine (5 μsec in delay, 0.5 Hz in frequency) grid of relative array offsets. That trial offset which yields the highest cross correlation gives the "best" estimate[2] of the time delay and Doppler shift undergone by the radar signal traversing the round trip from site to subradar point at a given epoch, usually chosen to coincide closely with the midpoint of the observing interval. The associated errors are estimated from the behavior of the cross correlation in the vicinity of the maximum. For illustration we show in Fig. 1 the coarse-grid cross correlation as a function of delay for several values of the frequency for Earth-Mercury measurements near both superior and inferior conjunction—relatively weak- and strong-signal cases, respectively. The difference in signal strength is shown by the relative magnitudes of the noise fluctuations in the two cases. The corresponding comparison for Earth-Venus measurements is far more extreme, the change in signal-to-noise ratio being about 100 times greater in virtue of the much larger variation in Earth-Venus distances.

How may these radar observations be used to test the predicted general relativistic retardation effect on the time delay? One approach consists of separating the data into two sets. The first, composed of all measurements not expected to be influenced substantially by solar gravity, can be used to estimate the orbits and the other unknown but relevant parameters of the dynamical system. The second set, composed only of observations made near superior conjunction, can then be compared directly with predictions based on general relativity and the orbital parameters estimated solely from the first set of data. For the actual analysis, the Haystack contributions to the first set, plus similar data from Lincoln Laboratory's Millstone radar ($\lambda = 23$ cm), were combined with the U. S. Naval Observatory's meridian-circle (optical) observations of the sun, Mercury, and Venus to determine the maximum likelihood estimates of 23 parameters: 18 initial conditions for the orbits of Mercury, Venus, and the center of mass of the earth-moon system; the masses of Mercury and the moon; the equatorial radii of Mercury and Venus (topographical variations were considered separately[6]); and the parameter giving the light-second equivalent of the astronomical unit of

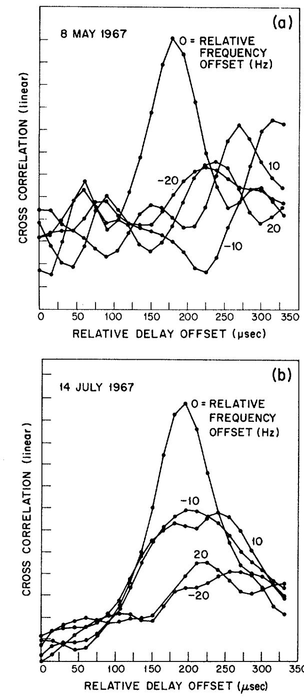

FIG. 1. Coarse-grid cross correlation of delay-Doppler template with partially processed radar echoes from Mercury near (a) superior and (b) inferior conjunction. Small positive slope of mean correlation versus delay offset is as yet unexplained. [The large peaks in (b) at frequency offsets other than zero are manifestations of the strong signal.]

length.[3] Over 400 radar and 6000 optical observations were used; these redundantly span the parameter space. In the computations we assumed that general relativity is correct and that the sun's gravitational quadrupole moment is negligible. Harmonic coordinates were used throughout. The solar corona was ignored since independent evidence[1] indicated that the group delay associated with the plasma along the ray paths of this experiment would be negligible—always less than 1 μsec.[7] The assumed motion of the earth about its center of mass and the orbits used for the moon and other planets were based on standard astronomical sources, as were the masses of the outer planets. The masses of Venus and Mars were taken from the Mariner II and Mariner IV results,[8] respectively, and the mass of the earth plus moon from geopotential theory and earlier radar values of the astronomical unit and the earth-moon mass ratio. The theoretical values of the observations were calculated with a precision of about 1 part in 10^{10} or better, whereas the most accurate measurements had an estimated uncertainty of about 5 parts in

10^9. On the other hand, systematic errors in the orbits of Mars and the outer planets affect the interpretation of the measurements by as much as several parts in 10^9; we are therefore reducing all observations (old and new) of these planets to improve their orbits as well, but the reduction will not be completed for some months.

The residuals (observed minus computed values) obtained when using the maximum likelihood estimates of the parameters are illustrated in Fig. 2, where the time-delay measurements are compared with theoretical values for Earth-Mercury and Earth-Venus observations made between late 1966 and mid 1967. In Fig. 3 we show a comparison, for the last two observed superior conjunctions of Mercury, between the measurements in the second data set and the corresponding predictions based on the first set. Because of the inability to separate completely the orbit determinations from the test of the predicted gravitational increase in time delays, the results shown in Fig. 3 can be somewhat misleading. Nonetheless, it seems safe to conclude that the sun's gravity does slow the speed of propagation of light by about the amount predicted by Einstein's theory.[9] A more quantitative measure of the agreement is provided, for example, by estimating 24 parameters: the 23 described above plus one defined as the coefficient of the "extra" delay in the formula expressing the coordinate time delay as a function of the orbital positions of the planets. A value of zero for this parameter r_l would imply that light travels rectilinearly at a constant speed, whereas a value of unity implies that general relativity is correct. We obtained $r_l = 0.9$, with the post-fit residuals being very similar to the values shown in Fig. 2. The purely formal standard error, resulting from the statistical analysis in which all estimated measurement errors are assumed to be independent and Gaussianly distributed with zero means, is 10%. The actual uncertainty is probably higher, but is very difficult to estimate reliably. We feel that $r_l = 0.9 \pm 0.2$ represents a realistic estimate of the uncertainty. Our present result is therefore incapable of distinguishing between the predictions of general relativity and, for example, those of the Brans-Dicke theory[10] with the free parameter s having the value 0.06[11] which corresponds to $r_l = 0.94$. In terms of the usual notation for the generalized metric, our result implies that $\gamma = 0.8 \pm 0.4$, since $r_l \approx \frac{1}{2}(1+\gamma)$.[12] (The equivalence is not precise because, in our treatment, γ was set equal to unity in computing the orbital motions of the planets.)

In addition to the possible sources of systematic errors already mentioned, two others require separate discussion. First, a subtle manufacturer's design error in the digital computer used at the Haystack site to process the incoming radar signals was not discovered and fixed until after the May 1967 superior conjunction of Mercury. (Only the weak signals were affected significantly and since few observations were

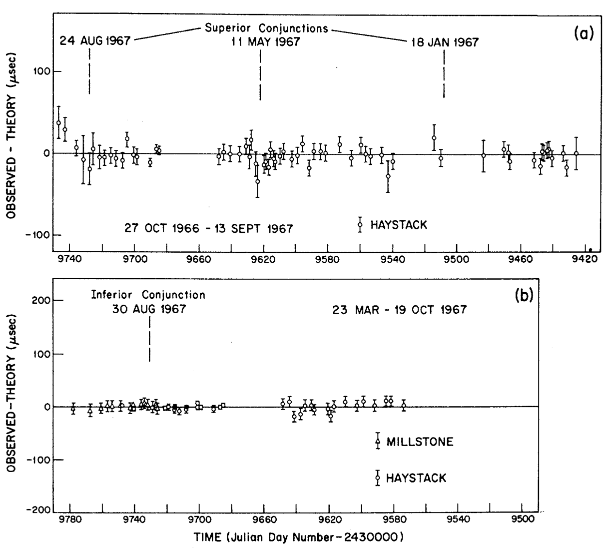

FIG. 2. Post-fit residuals of (a) Earth-Mercury and (b) Earth-Venus time-delay data obtained at Haystack and Millstone. Measurements made near superior conjunction (see Fig 3) were not inculed in the solution for the unknown parameters. (Note that time increases from right to left).

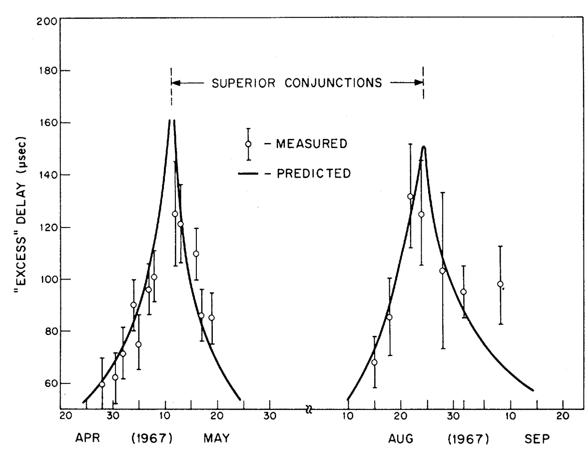

FIG. 3. Comparison of measured and predicted effects of general relativity on Earth-Mercury time delays. Predictions are based on orbits determined from other data.

made at earlier superior conjunctions, our suspicions were not fully aroused before May.) The apparent effect of the error was to introduce a small regular distortion, in frequency alone, into the delay-Doppler data matrix. An empirical correction was then made to the earlier data to undo this effect. The measurements could not be reprocessed ab initio since economic constraints prevented the storing of raw data; only partially processed results were retained. The corrected data presented here, which include all but a few observations, have passed each of our tests for both reliability and consistency.

Second, a program undertaken in the spring and summer of 1967 to make simultaneous interplanetary time-delay measurements at Cornell's Arecibo Ionospheric Observatory[13] ($\lambda = 70$ cm) and at Haystack disclosed slowly varying, systematic differences in the delays—about 20 μsec on average. These differences disappeared at the close approaches between the earth and the target planet but increased gradually as the interplanetary distance increased. No satisfactory explanation for these discrepancies has yet been found: Neither plausible errors in timing nor effects of interplanetary plasma can account for them. Regardless of which results are correct, the effect on our interpretation of the superior conjunction data will not be serious because of the rapidly varying characteristic of the "excess" delays. On the other hand, these discrepancies do prevent us from drawing conclusions at this time on the predicted relativistic advances of the planetary perihelia, on the solar gravitational quadrupole moment, and on possible variations of the gravitational constant. The influences of these latter effects on the time-delay accumulate gradually and are more sensitive to systematic errors. Therefore no important results can be reported in these areas until either the compatibility questions are resolved or a sufficiently long series of observations accumulates.

Repetitions of the superior-conjunction measurements at Haystack and at other installations, as well as the use of space probes, will likely enable a substantial reduction to be made in the systematic and random errors that affect this fourth test of general relativity. The present results are but a start towards full exploitation of this technique for testing gravitational theories.

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