[1]P. H. Barrett, L. M. Bollinger, G. Cocconi, Y. Eisenberg, and K. Greisen, Rev. Mod. Phys. 24, 133 (1952).

[2]W. F. Nash and A. W. Wolfendale, Phys. Rev. Letters 20, 698 (1968).

[3]R. J. Stefanski, R. K. Adair, and H. Kasha, Phys. Rev. Letters 20, 950 (1968).

[4]H. E. Bergeson, J. W. Keuffel, M. O. Larson, E. R. Martin, and G. W. Mason, Phys. Rev. Letters 19, 1487 (1967).

[5]C. G. Callan and S. L. Glashow, Phys. Rev. Letters 20, 779 (1968).

[6]D. E. Dorfan, J. Eades, L. M. Lederman, W. Lee, and S. C. C. Ting, Phys. Rev. Letters 14, 999 (1965).

[7]H. Kasha and R. J. Stefanski, Tenth International Conference on Cosmic Rays, Calgary, Canada (to be published), paper HE35.

[8]D. M. Ritson, Techniques of High Energy Physics, edited by R. E. Marshak (Interscience Publishers, Inc. New York, 1961), Vol. V, p. 526.

[9]The data for Fig. 1, curve A, are taken from G. Lemaître and M. S. Vallarta, Phys. Rev. 43, 87 (1933).

[10]The proton energy spectrum is given in W. R. Weber, Handbuch der Physik 46, 181 (1967), Part II.

PION FORM FACTOR AND $\pi\pi$ DIFFRACTION CROSS SECTION FROM CONSISTENCY RELATIONS AMONG pp, $p\bar{p}$, πp, AND $\pi\pi$ DIFFRACTION CROSS SECTIONS*

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(Received 25 March 1968)

Consistency relations among pp, $p\bar{p}$, πp, and $\pi\pi$ diffraction scattering cross sections and the pion and nucleon form factors are derived on the basis of the multiple-quark-scattering picture of elastic scattering of hadrons. The pion form factor is found to be similar to the nucleon form factor and the charge radius of the pion is $r_\pi \cong 0.65$ F in agreement with experimental evidence. The differential cross section for $\pi\pi$ diffraction scattering is found to be 4/9 times that of πp.

Interesting consistency relations among the leading diffraction peaks for pp, $p\bar{p}$, πp, and $\pi\pi$ scattering and the form factors of p and π are suggested from the systematics of the multiple-quark-scattering picture of high-energy elastic scattering of hadrons.[1] These consistency conditions are of greater generality than the multiple-quark-scattering picture from which they are drawn.[2] By means of these consistency relations, the form factor of π and the differential cross section for $\pi\pi$ diffraction scattering may be determined from the pp, $p\bar{p}$, and πp diffraction cross sections and the form factor of p. The results are that the pion form factor is found to be very similar to the nucleon form factor, and that the differential cross section for $\pi\pi$ diffraction scattering is 4/9 times the πp diffraction cross section. The charge radius of the pion is calculated to be $r_\pi \cong 0.65$ F, in good agreement with the best experimental values.[3,4]

In the self-consistent multiple-quark-scattering analysis of pp scattering, the amplitude for single (charge-independent) quark-quark diffraction scattering f_{QQ} is self-consistently determined from $d\sigma_{pp}^{1}/dt$, the pp cross section in the region of the first diffraction-scattering slope.[1] We have

$$f_{pp}^{\ 1}(t) \simeq [3G_p(t)]^2 f_{QQ}(t), \tag{1}$$

and

$$d\sigma_{pp}^{\ 1}/dt \simeq |f_{pp}^{\ 1}(t)|^2. \tag{2}$$

The superscript 1 indicates that a quantity refers to the first diffraction-scattering slope for the process that is indicated by the subscripts, e.g., $d\sigma_{pp}^{1}$. $G_p(t)$ is the single-quark form factor of p.

Similarly, the single (charge-independent) quark-antiquark ($Q\bar{Q}$) scattering amplitude is self-consistently determined by fitting the first diffraction slope of the elastic $p\bar{p}$ cross section,

$$f_{p\bar{p}}^{\ 1}(t) \simeq [3G_{\bar{p}}(t)]^2 f_{QQ}(t). \tag{3}$$

The single-$\bar{Q}$ distribution in $\bar{p}$ is the same as the single-Q distribution in p; so $G_{\bar{p}}(t) = G_p(t)$. The amplitude for the first diffraction slope for πp scattering involves a combination of QQ and $Q\bar{Q}$ single-scattering amplitudes,

$$f_{\pi p}^{\ 1}(t) \simeq 3G_\pi(t)G_p(t)[f_{QQ}(t) + f_{Q\bar{Q}}(t)]. \tag{4}$$

$G_\pi(t)$ is the single-Q form factor of π, which is the same as the single-$\bar{Q}$ form factor of π. Similarly, the leading diffraction slope in $\pi\pi$ scattering involves a combination QQ, $Q\bar{Q}$, and $\bar{Q}\bar{Q}$ single-scattering amplitudes,

$$f_{\pi\pi}^{1}(t)\simeq[G_\pi(t)]^2[f_{QQ}(t)+2f_{Q\overline{Q}}(t)+f_{\overline{Q}\overline{Q}}(t)]. \quad (5)$$

But charge symmetry gives $f_{QQ}=f_{\overline{Q}\overline{Q}}$, which leads directly to the relation

$$f_{\pi\pi}^{1}(t)\simeq\frac{2}{3}\frac{G_\pi(t)}{G_p(t)}f_{\pi p}^{1}(t), \quad (6)$$

or

$$\frac{d\sigma_{\pi\pi}^{1}}{dt}\simeq\left[\frac{2}{3}\frac{G_\pi(t)}{G_p(t)}\right]^2\frac{d\sigma_{\pi p}^{1}}{dt}. \quad (7)$$

Both $G_\pi(t)$ and $d\sigma_{\pi\pi}/dt$ are not directly observable, but using (1) and (3) to determine f_{QQ} and $f_{Q\overline{Q}}$, respectively, (4) then gives a consistency relation by which $G_\pi(t)$ may be determined from pp, $p\overline{p}$, and πp diffraction cross sections and $G_p(t)$:

$$G_\pi(t)\simeq 3G_p(t)\left[\frac{d\sigma_{\pi p}^{1}}{dt}\right]^{1/2}\times\left[\left(\frac{d\sigma_{pp}^{1}}{dt}\right)^{1/2}+\left(\frac{d\sigma_{p\overline{p}}^{1}}{dt}\right)^{1/2}\right]; \quad (8)$$

and (7) gives a consistency relation by which $d\sigma_{\pi\pi}^{1}/dt$ may be determined also in terms of the nucleon observables:

$$\frac{d\sigma_{\pi\pi}^{1}}{dt}\simeq 4\left[\frac{d\sigma_{\pi p}^{1}}{dt}\right]^2\times\left[\left(\frac{d\sigma_{pp}^{1}}{dt}\right)^{1/2}+\left(\frac{d\sigma_{p\overline{p}}^{1}}{dt}\right)^{1/2}\right]^{-2}. \quad (9)$$

We assume here that all these scattering processes are diffractive enough so that the leading diffraction amplitudes for all these processes have roughly the same phases over the regions of the first diffraction slopes, starting from almost pure imaginary at $t=0$.

G_π and G_p that occur in our multiple-quark-scattering analysis are the one-Q form factors of π and p, respectively, and are related to the observable electric-charge form factors of π and p as

$$G_\pi^{E}(t)=G_Q^{E}(t)G_\pi(t)$$

and

$$G_p^{E}(t)=G_Q^{E}(t)G_p(t), \quad (10)$$

where G_Q^E is the electric-charge form factor of Q, which we assume to be charge independent. Thus, our expressions (6)-(8) are all valid for either the single-Q form factors G_π and G_p or the observable electric form factors G_π^E and G_p^E, and knowledge of the electric form factor of the quark G_Q^E is not separately required.

Assuming leading diffraction peaks of the (energy-independent) form

$$\frac{d\sigma_{pp}^{1}}{dt}\simeq\left.\frac{d\sigma_{pp}^{1}}{dt}\right|_{t=0}\exp(\xi_{pp}t), \quad (11)$$

where $\xi_{pp}\cong 9\text{-}10$, $\xi_{\pi p}\cong 8\text{-}9$, and $\xi_{p\overline{p}}\cong 11\text{-}12$ $(\mathrm{BeV}/c)^{-2}$, and assuming that the magnitudes at the forward angle are given roughly by the optical theorem as proportional to the total cross sections[5] $\sigma_{pp}\cong 38$ mb, $\sigma_{p\overline{p}}\cong 48$ mb, and $\sigma_{\pi p}\cong 25$ mb, Eq. (8) becomes, for either G_π and G_p or for G_π^E and G_p^E at $|t|\lesssim 1$ $(\mathrm{BeV}/c)^2$,

$$G_\pi(t)\simeq 3G_p(t)\left\{\frac{\sigma_{pp}}{\sigma_{\pi p}}\exp[\tfrac{1}{2}(\xi_{pp}-\xi_{\pi p})t]+\frac{\sigma_{p\overline{p}}}{\sigma_{\pi p}}\exp[\tfrac{1}{2}(\xi_{p\overline{p}}-\xi_{\pi p})t]\right\}^{-1}, \quad (12)$$

and (9) becomes

$$\frac{d\sigma_{\pi\pi}^{1}}{dt}\simeq\frac{d\sigma_{\pi p}^{1}}{dt}4\left\{\frac{\sigma_{pp}}{\sigma_{\pi p}}\exp[\tfrac{1}{2}(\xi_{pp}-\xi_{\pi p})t]+\frac{\sigma_{p\overline{p}}}{\sigma_{\pi p}}\exp[\tfrac{1}{2}(\xi_{p\overline{p}}-\xi_{\pi p})t]\right\}^{-2}. \quad (13)$$

If the Pomeranchuk theorem were exact for pp, $p\overline{p}$, and also for QQ and $Q\overline{Q}$ cross sections such that $\sigma_{\pi p}/\sigma_{pp}=\sigma_{\pi p}/\sigma_{p\overline{p}}=\frac{2}{3}$ and $\xi_{pp}=\xi_{p\overline{p}}$, and we take $\xi_{\pi p}\cong\xi_{pp}$, then

$$G_\pi(t)\simeq G_p(t), \quad \text{(Pomeranchuk limit)}$$

$$G_\pi^{E}(t)\simeq G_p^{E}(t), \quad (14)$$

and

$$\frac{d\sigma_{\pi\pi}^{1}}{dt}\simeq\frac{4}{9}\frac{d\sigma_{\pi p}^{1}}{dt}. \quad \text{(Pomeranchuk limit)}. \quad (15)$$

Ignoring the Pomeranchuk theorem and treating the present (approximately energy-independent) high-energy cross sections as the asymptotic values, we obtain[6]

$$G_\pi^E(t) \simeq \frac{0.87}{1-t/M^2}, \tag{16}$$

for $|t| < M^2 = 0.57$ (BeV/c)^2. The normalization is off here as $G_\pi^E(0) \cong 3\sigma_{\pi p}(\sigma_{pp}+\sigma_{p\bar{p}})^{-1} \cong 0.87$, which is probably indicative of the order of accuracy of the whole calculation. However, the mean-square charge radius of the pion does not depend on this normalization and is given directly from (8) as

$$\langle r_\pi^2 \rangle = 6(dG_\pi^E/dt)_{t=0}[G_\pi^E(0)]^{-1}, \tag{17}$$

or

$$r_\pi \cong 0.65 \text{ F}.$$

This value is in good agreement with the best experimental determinations of $r_\pi = 0.8 \pm 0.1$ F from pion electroproduction[3] and $r_\pi \le 0.9$ F from $\pi\alpha$ Coulomb scattering,[4] and with the value $r_\pi \cong 0.6$ F calculated from current algebra and pole dominance.[7]

Similarly, the inverse diffraction width in $\pi\pi$ scattering can be determined directly from (13) as

$$\xi_{\pi\pi} \simeq 2\xi_{\pi p} - (\xi_{pp} + \xi_{p\bar{p}}\sigma_{p\bar{p}}/\sigma_{pp}) \times (1+\sigma_{p\bar{p}}/\sigma_{pp})^{-1} \simeq 5.8 \text{ (BeV/}c)^{-2}, \tag{18}$$

and the total cross section as

$$\sigma_{\pi\pi} \cong 2(\sigma_{\pi p})^2(\sigma_{pp}+\sigma_{p\bar{p}})^{-1} \cong 14 \text{ mb}. \tag{19}$$

These values agree closely with those obtained from Regge theory of $\xi_{\pi\pi} = 2\xi_{\pi p} - \xi_{pp} \cong 7$ (BeV/c)$^{-2}$ and $\sigma_{\pi\pi} \cong (\sigma_{\pi p})^2/\sigma_{pp} \cong 16$ mb.[8]

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*Work supported in part by the National Science Foundation.

[1]E. Shrauner, "Self-Consistent Multiple Quark Scattering Analysis of Elastic pp Scattering," to be published. The interpretation of higher-energy (elastic) collisions of hadrons in terms of a picture in which multiple internal diffraction scattering of constituent quarks is the dominant process extends to momentum-transfer–dependent quantities, differential cross sections and form factors, the same type of systematic analysis for which previously the quark model has been so surprisingly successful mainly for momentum-transfer–independent quantities, total cross sections and branching ratios. The details of the broken-slope structure of the differential cross sections for elastic pp, $p\bar{p}$, and πp scattering at higher energies have been analyzed in terms of this picture where large-momentum-transfer scatterings are viewed as the cumulative effects of multiple internal diffraction scatterings of constituent quarks. The first, second, and third slopes of $d\sigma/dt$ are interpreted as due, respectively, to single, double, and triple internal quark-quark scatterings. The higher-order multiple-scattering amplitudes determined from the first slope yield striking agreement with experiment. The broken-slope structure of the pp differential cross section has been emphasized by A. D. Krisch, Phys. Rev. Letters 19, 1149 (1967), and earlier references therein. The data for $p\bar{p}$ and πp differential cross sections also tend toward energy-independent broken-slope structures similar to that of pp. These data are all summarized in Y. Sumi, Progr. Theoret. Phys. (Kyoto) Suppl. Extra No., 3 (1967), and by Y. Sumi and T. Yoshida, Progr. Theoret. Phys. (Kyoto) Suppl. Extra No., 53 (1967), and they have been analyzed according to the above picture by L. Benofy, D. W. Cho, and E. Shrauner (to be published).

[2]This determination of $G_\pi(t)$ from analysis of diffraction scattering peaks could be applied also in the case of continuous distributions of hadronic matter, but then a different argument would have to be proposed in order to extend the relevance of the consistency relations to momentum transfers as large as, or beyond, the first breaks and to interpret the relations of the hadronic and electric form factors. Our semiclassical approach corresponds to a generalization of the factorized Regge-pole picture. See W. J. Abbe, Phys. Rev. 160, 1519 (1967).

[3]C. W. Akerlof, W. W. Ash, K. Berkelman, C. A. Lichtenstein, A. Ramanauskas, and R. H. Sieman, Phys. Rev. 163, 1482 (1967).

[4]M. M. Block, Bull. Am. Phys. Soc. 13, 112(T) (1968), and private communication.

[5]Sumi, Ref. 1, and Sumi and Yoshida, Ref. 1.

[6]We use the Wilson-Hofstadter dipole fit to the nucleon form factors. See W. Albrecht, H. J. Behrend, R. W. Brasse, W. Flarger, H. Hultschig, and K. G. Steffen, Phys. Rev. Letters 17, 1192 (1966).

[7]R. Arnowitt, M. H. Friedman, and P. Nath, Phys. Rev. Letters 19, 1085 (1967).

[8]See Abbe, Ref. 2.