PARAMETER ξ IN K_{l3} DECAY

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Implications of discrepancies between different experimental determinations of the parameter ξ in K_{l3} decay are discussed. In particular, it is pointed out that (a) the presence of a scalar interaction less than the present experimental upper limit will have a large effect on ξ, and (b) different experimental determination of ξ could be used to test electron-muon universality.

In order to relate experimental data on K_{e3} and $K_{\mu3}$ decays, it is necessary to assume electron-muon universality; and conversely, a comparison of such data provides a test of electron-muon universality. It has been noted by Willis[1] that this universality principle is not well tested in strangeness-changing decays. In addition, although the presence of a local pseudoscalar interaction seems ruled out by the $\pi_{e2}/\pi_{\mu2}$ branching ratio and more recently by the $K_{e2}/K_{\mu2}$ branching ratio[2] (which is a strangeness-changing decay and hence more relevant to the present discussion), the presence of a local scalar interaction has not been well tested.[3]

Let us suppose that only scalar and vector form factors are involved in K_{l3} decay and that time-reversal invariance holds. Then the matrix element for K_{l3} decay is proportional to

$$m_K f_s^l \bar{u}_l(1+\gamma_5)u_{\nu l} + \tfrac{1}{2}if_+^l(P_K+P_\pi)_\alpha \bar{u}_l\gamma_\alpha(1+\gamma_5)u_{\nu l} + \tfrac{1}{2}if_-^l(P_K-P_\pi)_\alpha \bar{u}_l\gamma_\alpha(1+\gamma_5)u_{\nu l}, \tag{1}$$

where we have used a standard notation for the form factors. It is well known that by use of the Dirac equation, the three form factors in expression (1) reduce to two. The form factor f_-^l leads to an induced scalar interaction. We may parametrize the system by the quantities f_+^l and ξ_l where

$$\xi_l = \frac{f_-^l}{f_+^l} - 2\frac{m_K f_s^l}{m_l f_+^l}. \tag{2}$$

It is impossible to tell by measuring the lepton polarization or the pion-lepton angular correlation or the energy spectrum of the pion or lepton whether or not a scalar interaction exists in either of the decays K_{e3} or $K_{\mu3}$. An analysis of the data would lead to a knowledge of f_+^e, f_+^μ, ξ_e, and ξ_μ, and one could not in principle separate out the contributions to ξ_l which are shown in Eq. (2). If we allow for the possibility of a tensor interaction, then our conclusions are unchanged. It would be possible to parametrize the system by three quantities, the new quantity being the tensor form factor, but, as before, one cannot separate the vector and scalar form-factor contributions to ξ. We shall assume that there is no tensor interaction present.

If one assumes electron-muon universality, then

$$f^e = f^\mu \tag{3}$$

for all form factors involved. However, we would still have

$$\xi_e \neq \xi_\mu, \tag{4}$$

unless $f_s = 0$.

Once we assume electron-muon universality, we have one more piece of experimental information that we can use, i.e., the $K_{\mu3}/K_{e3}$ branching ratio. It is the purpose of this note to point out that, if a scalar interaction is present, one must use care in comparing the values of ξ_μ obtained from experiments which measure the $K_{\mu3}/K_{e3}$ branching ratio and those which determine ξ_μ by a study of the final state in $K_{\mu3}$ decay. The latter type of experiment is those which measure muon polarization or pion-muon angular correlation or shape of the pion or muon spectrum. These experiments yield "direct" information on the parameter ξ_μ and should be internally consistent. For the sake of brevity, we shall henceforth refer to such experiments as muon polarization experiments.

The most recent experiment,[3] on the basis of a study of the positron spectrum in K_{e3}^+ decay, could only conclude that

$$|f_S/f_+|<0.23, \tag{5}$$

at a 90% confidence level and assuming f_+ to be a constant.

Assuming that all form factors are constant, we may write

$$\frac{\Gamma(K_{\mu 3})}{\Gamma(K_{e3})}=\frac{\frac{1}{4}X_\mu(m_\mu/m_K)^2(\xi_\mu-1)^2+Y_\mu(m_\mu/m_K)^2(\xi_\mu-1)+Z_\mu}{\frac{1}{4}X_e(m_e/m_K)^2(\xi_e-1)^2+Y_e(m_e/m_K)^2(\xi_e-1)+Z_e}, \tag{6}$$

where X_l, Y_l, and Z_l are given in an analytic form by Fujii and Kawaguchi.[4] The coefficients X_e, Y_e, and Z_e may be written down in a simple form accurate to order $(m_e/m_K)^2$:

$$X_e=\tfrac{1}{3}+3\mu^2-3\mu^4-\tfrac{1}{3}\mu^6+4\mu^2(1+\mu^2)\ln\mu,$$

$$Y_e=\tfrac{1}{3}+\tfrac{1}{2}\mu^2-\mu^4+\tfrac{1}{6}\mu^6+2\mu^2\ln\mu,$$

$$Z_e=\tfrac{1}{12}-\tfrac{2}{3}\mu^2+\tfrac{2}{3}\mu^6-\tfrac{1}{12}\mu^8-2\mu^2\ln\mu, \tag{7}$$

where

$$\mu=m_\pi/m_K.$$

In Table I we give the numerical values of the various X_l, Y_l, and Z_l for both K_{l3}^+ and K_{l3}^0 decays.[5]

If we neglect the possible scalar admixture, we obtain the branching ratios

$$\Gamma(K_{\mu 3}^+)/\Gamma(K_{e3}^+)=0.6456+0.1264\xi_\mu+0.019\,20\xi_\mu^2, \tag{8}$$

$$\Gamma(K_{\mu 3}^0)/\Gamma(K_{e3}^0)=0.6452+0.1245\xi_\mu+0.018\,65\xi_\mu^2. \tag{8'}$$

We pause at this point to note that, if one parametrizes the K_{l3} decay in the usual way (see, e.g., Jackson[6]), then apart from the phase-space factors with which we have been concerned, a factor m_K^5 appears in the decay rate. If we take this into account, then assuming there is no scalar interaction[7]

$$\frac{\Gamma(K_{e3}^0)}{\Gamma(K_{e3}^+)}=\left(\frac{m_{K^0}}{m_{K^+}}\right)^5\frac{Z_e^0}{Z_e^+}=1.0121, \tag{9}$$

and

$$\frac{\Gamma(K_{\mu 3}^0)}{\Gamma(K_{\mu 3}^+)}=1.0121F(\xi), \tag{9'}$$

where $F(\xi)$ is a slowly varying function of ξ given by the ratio of Eq. (8′) to Eq. (8). For example, $F(1)=0.9963$, $F(0)=0.9993$, and $F(-1)=1.0016$.

The coefficients of X_e and Y_e in Eq. (6) are usually neglected since they are assumed to be of order $(m_e/m_K)^2$. However, from Eq. (2),

$$(m_e/m_K)[\tfrac{1}{2}(\xi_e-1)]=-f_S/f_+, \tag{10}$$

to a good approximation, assuming f_+ and f_- are of the same order and $|f_S/f_+|$ to be of order 0.1. In this case the branching ratios given in Eqs. (8) and (8′) must be replaced by

$$\frac{\Gamma(K_{\mu 3}^+)}{\Gamma(K_{e3}^+)}=\frac{0.6456+0.1264\xi_\mu+0.019\,20\xi_\mu^2}{1+2.57(f_S/f_+)^2}, \tag{11}$$

and

$$\frac{\Gamma(K_{\mu 3}^0)}{\Gamma(K_{e3}^0)}=\frac{0.6452+0.1245\xi_\mu+0.018\,65\xi_\mu^2}{1+2.54(f_S/f_+)^2}. \tag{11'}$$

To obtain this, we have simplified the denomina-

Table I. Numerical values of the quantities X_l, Y_l, and Z_l introduced in Eq. (6).

Decay	X_l	Y_l	Z_l
$K^+\to\pi^0\mu^+\nu$	0.081 00	0.086 91	0.038 20
$K^+\to\pi^0e^+\nu$	0.1240	0.1714	0.048 28
$K^0\to\pi^-\mu^+\nu$	0.077 74	0.084 33	0.037 02
$K^0\to\pi^-e^+\nu$	0.1191	0.1666	0.046 96

tor in Eq. (6) to

$$X_e\left(\frac{f_s}{f_+}\right)^2-\frac{m_e}{m_K}\left(\frac{f_s}{f_+}\right)\left[X_e\left(\frac{f_--f_+}{f_+}\right)+4Y_e\right]+Z_e. \quad (12)$$

For $|f_S/f_+|\approx 0.1$, the contribution of the term of order m_e/m_K to this expression is about $\frac{1}{10}$ relative to the first term, which is itself a small correction term. For larger values of $|f_S/f_+|$, the term of order m_e/m_K is even less important. Thus we simplify Eq. (12) to

$$X_e(f_s/f_+)^2+Z_e, \quad (12')$$

which leads to Eqs. (11) and (11′).

To illustrate the effect of a scalar admixture, we show in Table II the values of ξ_μ corresponding to branching ratios in the region 0.60-0.70. Since we are now dealing with the parameter ξ_l only for $K_{\mu 3}$ decay, we shall drop the subscript μ. One set of solutions (ξ_+, ξ_0) corresponds to zero scalar admixture and hence to the smaller solutions of the quadratic branching-ratio equations (8) and (8′). The other set (ξ_+^S, ξ_0^S) corresponds to a scalar admixture $|f_S/f_+|=0.1$. In this case we have taken the smaller solutions of the quadratic branching-ratio equations (11) and (11′). (The larger solutions for ξ are ruled out by the muon energy spectrum in $K_{\mu 3}$ decay.) Since we only wish to indicate the effect heuristically, and because of the uncertainty of the effect of the contribution of order m_e/m_K to the expression given in Eq. (12), we have assumed the denominators of Eqs. (11) and (11′) are replaced by 1.025.

The third decimal place should not be taken too seriously. It is included only to emphasize that, for the same branching ratio $\Gamma(K_{\mu 3})/\Gamma(K_{e3})$, slightly different values of ξ (i.e., ξ_+ and ξ_0) are obtained from K^+ and K^0 semileptonic decays. Alternatively, if the $|\Delta I|=\frac{1}{2}$ rule is assumed so that ξ would be the same for both K^+ and K^0 decays, then slightly different branching ratios would be obtained in each case. This is closely related to the fact that the quantity $F(\xi)$ introduced in Eq. (9′) is a slowly varying function of ξ. We observe from Table II that, if a scalar interaction is present, then one deduces from the correct branching-ratio equations (11) and (11′) a value ξ^S which is larger than the value ξ which one would deduce from the incorrect branching-ratio equations (8) and (8′) for a fixed value of $\Gamma(K_{\mu 3})/\Gamma(K_{e3})$. If a scalar interaction is present, one must compare the solution ξ^S with the parameter ξ_μ deduced from observation of the

Table II. Solution of branching-ratio equations (8), (8′), (11), and (11′) for branching ratios in the range 0.60-0.70. ξ_+ and ξ_0 are the solutions for zero scalar admixture, ξ_+^S and ξ_0^S are those for a scalar admixture of $|f_S/f_+|=0.1$.

Branching ratio	ξ_+	ξ_+^S	ξ_0	ξ_0^S
0.70	0.405	0.527	0.414	0.537
0.68	0.261	0.384	0.269	0.393
0.66	0.112	0.236	0.117	0.243
0.64	−0.045	0.081	−0.042	0.086
0.62	−0.209	−0.081	−0.209	−0.079
0.60	−0.383	−0.252	−0.385	−0.252

muon polarization in $K_{\mu 3}$ decay. Thus assuming electron-muon universality and $f_S\neq 0$, we have

$$\xi^S(\text{branching ratio})=\xi_\mu(\text{polarization}),$$

but

$$\xi(\text{branching ratio})<\xi_\mu(\text{polarization}).$$

This effect presumably persists when allowance is made for variation of the form factors with energy.

Since we do not know a priori what the value of f_S is, it is impossible to solve the correct branching-ratio equations (11) and (11′) and obtain ξ^S. All we can say is that if the branching-ratio equations (8) and (8′) are solved for ξ and $f_S\neq 0$, then

$$\xi(\text{branching ratio})<\xi_\mu(\text{polarization}),$$

and the discrepancy between the two is a measure of f_S.

It is clear that this will do nothing to explain the present experimental discrepancies. The reverse is, in fact, the case since the present situation suggests[1]

$$\xi(\text{polarization})<\xi(\text{branching ratio}).$$

However, it is expected that this discrepancy will be reduced or removed by (a) taking into account the energy dependence of the form factors f_+ and f_-,[8] and (b) reduction of errors in both kinds of experiment.

In fact the most recent determinations[9] of the $K_{\mu 3}/K_{e3}$ branching ratio are consistent with a negative ξ, which is in agreement with the negative sign of ξ_μ obtained from muon-polarization experiments in $K_{\mu 3}$ decay. There is still a discrepancy in the magnitude of ξ as determined by the different methods. Hopefully, however, this further discrepancy will be reduced or removed as suggested above.

Only then can one look for discrepancies of the sign and order that we are talking about here. Of course, the magnitude of the difference between ξ and ξ^S could be increased by very much more than what is shown in Table II if we allow a larger admixture of $|f_S/f_+|$, say 0.2. Such a comparison could be regarded as an independent check for the presence of a scalar interaction, although it is certainly less direct than an analysis of the positron spectrum in K_{e3}^+ or K_{e3}^0 decay.[10]

Finally we note that, simply by allowing $|f_S/f_+|$ to be 0.1, then from Eq. (2) we have approximately

$$\xi_\mu \approx f_-/f_+ \pm 1. \qquad (13)$$

Thus the parameter ξ_μ measured by muon-polarization experiments (or more approximately by branching-ratio comparisons) could be appreciably different from the vector form-factor ratio f_-/f_+. For example, a simple K^*-dominance model of the K_{l3} decay process leads to a constant $f_-/f_+ \approx -0.3$ and a branching ratio $\Gamma(K_{\mu3})/\Gamma(K_{e3}) \approx 0.65$.[11] We would not expect a small admixture of scalar interaction to greatly influence the branching ratio of 0.65 as suggested by the results shown in Table II. But an admixture of $|f_S/f_+|$ rather less than 0.1 would have a significant effect on the parameter ξ_μ because of the enhancement factor $2m_K/m_\mu \approx 10$ which appears in Eq. (2), so that muon-polarization experiments could yield a value of ξ_μ rather different from the value ≈ -0.3 which the K^*-dominance model predicts.

Likewise, accurate measurements of the $K_{\mu3}/K_{e3}$ branching ratio would yield a value ξ [obtained from the incorrect branching-ratio equations (8) and (8′)] rather different from -0.3; but these values of ξ, although both quite different from -0.3, would differ from each other by an amount dependent on the scalar admixture. Thus we conclude that in order to seriously test models of K_{l3} decay which assume dominance of the decay process by the vector form factors f_+ and f_-, it is important to place more stringent experimental limits on the admixture of scalar interaction present.

We conclude by returning to the possibility of violation of e-μ universality in the weak interactions. Although it is theoretically an unattractive notion, it should be tested for strangeness-changing decays, especially those involving vector currents, e.g., K_{l3} decay.[12] We suppose for simplicity that the only form vectors involved are constant vector form factors. Then the branching-ratio equation (8) would be replaced by

$$\Gamma(K_{\mu3}^+)/\Gamma(K_{e3}^+) = (f_+^{\mu}/f_+^{e})^2(0.6456 + 0.1264\xi_\mu + 0.019\,20\xi_\mu^2), \qquad (14)$$

so that if $f_+^e \neq f_+^\mu$, then a discrepancy would again exist between ξ as measured from the usual branching-ratio equation (8) and from muon-polarization experiments. Now, however, the discrepancy may be of either sign. As may be inferred from the above discussion, $|f_+^e/f_+^\mu| > 1$ implies

$$\xi(\text{branching ratio}) < \xi(\text{polarization}),$$

and $|f_+^e/f_+^\mu| < 1$ implies

$$\xi(\text{polarization}) < \xi(\text{branching ratio}).$$

Since ξ_μ may be determined independently by means of measurement of the muon polarization in $K_{\mu3}$ decay, it is clear that a knowledge of the $K_{\mu3}/K_{e3}$ branching ratio enables one to determine the ratio $(f_+^e/f_+^\mu)^2$. The same argument will carry over with only minor details changed if the form factors are energy dependent.

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[1]W. J. Willis, in Proceedings of the International Conference on Elementary Particles, Heidelberg, Germany, 1967 (North-Holland Publishing Company, Amsterdam, The Netherlands, to be published).

[2]D. R. Botterill et al., Phys. Rev. Letters 19, 982 (1967).

[3]D. R. Botterill et al., to be published.

[4]A. Fujii and M. Kawaguchi, Phys. Rev. 113, 1156 (1959).

[5]We have used masses for all particles involved as given by the data tables of A. H. Rosenfeld et al., Rev. Mod. Phys. 40, 77 (1968). Slight uncertainties (e.g., ±0.1 MeV in the K^+ or K^0 mass) only affect the branching-ratio equations by one or two parts in the fourth significant figure. These equations differ slightly from many previous versions for reasons which are not clear to us. In addition, we distinguish between the K^0 and K^+ branching ratios.

[6]J. D. Jackson, 1962 Brandeis Lectures (W. A. Benjamin, Inc., New York, 1963), p. 263.

[7]The fact that the overall phase-space difference

between K_{e3}^{+} and K_{e3}^{0} decays is 1% rather than 4% as quoted by Willis in Ref. 1 seems to have been first noted by Dr. T. W. Quirk (private communication).

[8]N. Cabibbo, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, 1966 (University of California Press, Berkeley, Calif., 1967), p. 29.

[9]Aachen-Bari-CERN-Padova-Madrid-Valencia Collaboration, in Proceedings of the International Conference on Elementary Particles, Heidelberg, Germany, 1967 (North-Holland Publishing Company, Amsterdam, The Netherlands, to be published); private communication from Dr. R. C. Field (to be published).

[10]The most direct test is to examine the π-e angular correlation in the dilepton center of mass. See S. W. MacDowell, Ann. Phys. (N.Y.) 18, 171 (1962).

[11]J. L. Acioli and S. W. MacDowell, Nuovo Cimento 24, 606 (1962).

[12]The branching ratio $K_{e2}/K_{\mu2}$ (see Ref. 2), as well as being a test of the presence of a pseudoscalar interaction, is also a test of e-μ universality. This process, however, involves the axial-vector current.

ERRATA

SOME EFFECTS OF QUANTIZATION OF INTERNAL ROTATION ON SPIN-LATTICE RELAXATION AND HYPERFINE STRUCTURE. W. L. Gamble, I. Miyagawa, and R. L. Hartman [Phys. Rev. Letters 20, 415 (1968)].

Line 4 of the first column of p. 416 should read "2, 4, and 6 at low temperatures."

The sentence beginning on line 15 of paragraph 3, column 2, on p. 416 should read, "The levels marked A are nondegenerate and totally symmetric with respect to the elements of the symmetry group C_3."

The chemical formula in line 14, paragraph 2, column 1, on p. 418 should be "H_3C-$\dot{C}HR$."

Since there has been some confusion, the authors would like to point out explicitly that the quantum effect referred to in the last sentence of the abstract is that of the failure of the modified Bloch equation. Additional evidence that a quantized rotational model for methyl groups is necessary at low temperatures has been obtained by ENDOR experiments [James W. Wells and Harold C. Box, J. Chem. Phys. 46, 2935 (1967); S. Clough and F. Poldy, Phys. Letters 24A, 545 (1967), and 25A, 186 (1967)].

ORIGIN OF SOLVENT KNIGHT SHIFTS IN ALLOYS. R. E. Watson, L. H. Bennett, and A. J. Freeman [Phys. Rev. Letters 20, 653 (1968)].

The curves of the figure labeled "theory" (Fig. 1) include the effects of lattice volume changes on P_F (contrary to the statement in the caption) although the P_F curves, as shown, do not include these volume effects. In any case, as noted in footnote 10, the placement of the "theory" curves is uncertain (whether above or below the experimental curves) because of the difficulties in obtaining any estimate of χ_p. Details have been given elsewhere [R. E. Watson, L. H. Bennett, and A. J. Freeman, Bull. Am. Phys. Soc. 12, 689 (1968), and L. H. Bennett, R. W. Mebs, and R. E. Watson, Phys. Rev. (to be published)]. We thank A. J. McAlister for pointing out this error in the figure.

CORRECTIONS TO THE EXPERIMENTAL VALUE FOR THE ELECTRON g-FACTOR ANOMALY. Arthur Rich [Phys. Rev. Letters 20, 967 (1968)].

The following typographical errors were made: Page 967, read

$$a_{\text{theory}}^{B} = 0.001\,159\,641(3)$$

instead of 0.001 159 614(3).

Page 968, Eq. (1),

$$a\left(\frac{\gamma}{\gamma+1}\right)\frac{[v_z^{\,2}]}{c^2}$$

instead of

$$a\left(\frac{\gamma}{\gamma+1}\right)\frac{[v_z^{\,2}]}{C^2};$$