EFFECTIVE OPERATORS OF ELECTROMAGNETIC INTERACTIONS IN NUCLEI AND REALISTIC NUCLEON-NUCLEON POTENTIALS*

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Electromagnetic interactions in nuclei are described in terms of effective operators for valence (open-shell) nucleons only. The effective operators are calculated theoretically from a realistic nucleon-nucleon potential (Yale-Shakin) in terms of excitations and de-excitations of particle-hole pairs of the core nucleons. The theory is successfully applied to Sn^{116} whose states are described in terms of the two- and four-quasiparticle Tamm-Dancoff theories involving valence neutrons only.

In describing electromagnetic properties of nuclei, the contributions of the core nucleons are often essential (e.g., in the case of electric interaction with nuclei which have closed proton shells and only neutrons in the valence subshells) in spite of the fact that the core is usually supposed to be inert in shell-model calculations. The only way out in similar situations has been to sacrifice the microscopic point of view and to introduce the notion of an effective charge, a constant supposed to simulate the cumulative effect of all the core nucleons. The numerical values of such effective charges of neutrons and/or protons depend on the multipole in question, and are almost adjustable parameters. A "derivation" of these quantities from a hydrodynamical picture of the nucleus leads to mixing a phenomenological description with a microscopic theory. To sum up, an effective charge is in fact just the measure of the extent of our ignorance of a given electrodynamic process inside the nucleus. It is clear that, particularly when one works with realistic nucleon-nucleon potentials, a fully microscopic theory of nuclear structure should be free of the concept of an adjustable effective charge.

Several authors[1,2] have tried to estimate the effective charges using the picture of virtual excitations of core nucleons. In particular, a neutron effective charge could originate from second-order processes in which a virtual or a real photon is absorbed by a core proton creating a particle-hole pair which is subsequently de-excited in a collision with a valence neutron. The analysis of Refs. 1 and 2 has, however, been only qualitative, involving only schematic approximations and purely phenomenological nuclear forces.

It is our aim now to study the problem in a quantitative way in relation to a realistic nucleon-nucleon potential. We choose the example of the even tin isotopes which are representative of an important region of the periodic table: that of the so-called vibrational nuclei. We derive formulas for the effective electric (or magnetic) 2^λ-pole operator, $\hat{O}_{\mathrm{eff}}^{\lambda}$, in the representation of shell-model single-particle (Hartree-Fock) states. In particular, we examine the question to what extent can $\hat{O}_{\mathrm{eff}}^{\lambda}$ be replaced by $e_{\mathrm{eff}}^{(\lambda)}\hat{O}^{\lambda}$ where $e_{\mathrm{eff}}^{(\lambda)}$ is a constant "effective charge" (independent of the configurations of the transitions). In the present Letter we limit ourselves to giving numerical results on only the effective operator, $\hat{O}_{\mathrm{eff}}^{E2}$, on the most important reduced transition probability $B(E2, 2_1^+ \to 0_1^+)$, and on the quadrupole moment of the first excited 2_1^+ state, $Q(2_1^+)$.

For realistic two-nucleon potential we have chosen the Yale potential.[3] The effective interac-

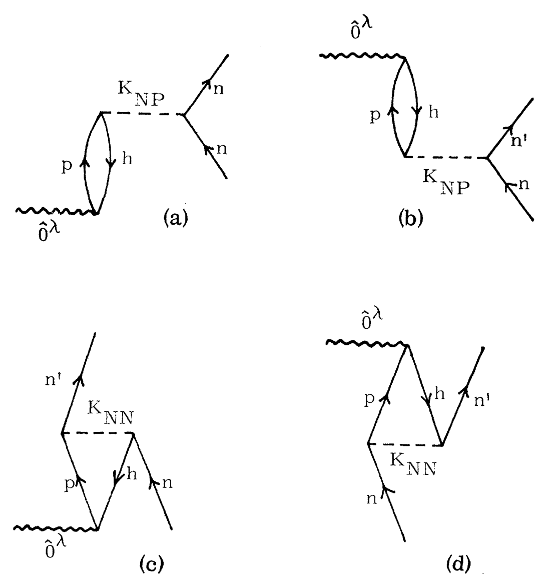

FIG. 1. Lowest-order diagram for processes contributing to matrix elements $\langle n' \| \hat{O}_{\mathrm{eff}}^{\lambda} \| n\rangle$ of the effective electromagnetic interaction $\hat{O}^{\lambda}$.

tion operator is the Brueckner reaction matrix K taken in the approximation given by Shakin et al.[4]

For the Sn nuclei we have a 50-50 doubly magic core. The ground states and the low-lying excited states of the even tin isotopes have been successfully described in the quasiparticle Tamm-Dancoff (QTD) and quasiparticle second Tamm-Dancoff (QSTD) approximations.[5-7] In these microscopic theories the eigenvectors of the nuclear states are described in terms of zero and two (and four) valence-neutron quasiparticle (qp) excitations.

Suppose now we have to calculate the effective operator of an electric 2^λ pole $\hat{O}_{\rm eff}{}^\lambda$ for the valance neutrons. It is the neutron-proton two-body force K_{np} which is responsible for the transmission of the $\hat{O}^\lambda$ interaction from the core protons to the valence neutrons. To lowest order, $\hat{O}_{\rm eff}{}^\lambda$ is represented by Figs. 1(a) and 1(b). A reduced single-particle (s.p.) matrix element corresponding to the two processes of Fig. 1 can be put in the form

$$\langle n'\|\hat{O}_{\rm eff}{}^\lambda\|n\rangle = 2\sum_{PH}^{\rm (protons)}[F_{np}(n'nPH,\lambda)e_1^{-1}\langle P\|e\hat{O}^\lambda\|H\rangle + \langle H\|e\hat{O}^\lambda\|P\rangle e_2^{-1}F_{np}(n'nHP,\lambda)], \tag{1}$$

where P and H are the proton particle and hole states, respectively, $e_1 \equiv E_P{}^0 - E_H{}^0 + (E_{n'}{}^0 - E_n{}^0)$, $e_2 \equiv E_P{}^0 - E_H{}^0 - (E_{n'}{}^0 - E_n{}^0)$, and $F_{np}(abcd, J)$ is a particle-hole coupled reduced matrix element of the operator K_{np} defined in a complete analogy to Eq. (11) of Baranger.[8] Equation (1) is obviously valid for any single-particle tensor operator with vanishing neutron matrix elements, e.g., for any 2^λ-pole operator of the Coulomb potential in the problem of electron scattering from nuclei.[9] In the case of an effective operator $\hat{O}_{\rm eff}{}^\lambda$ of a magnetic interaction, the expression on the right-hand side of Eq. (1) has to be supplemented by the first-order term $\langle n'\|e\hat{O}^\lambda\|n\rangle$ and by an extra sum over the neutron-core (PH) pairs identical in form to the sum of Eq. (1) except that the elements F_{np} are replaced by F_{nn} involving antisymmetrization of the elements of K_{nn} and, therefore, Figs. 1(c) and 1(d).

We have also examined the corrections of higher order processes corresponding to iterating the diagrams of Fig. 1, i.e., containing the chains of all the random-phase-approximation bubble and related exchange diagrams. Such corrections are found to be quite small, in fact, practically negligible. This is consistent with the smallness of the higher iteration corrections[7,10] of the core polarization bubbles in renormalizing "bare" matrix elements of the two-nucleon interaction K in the theory of the effective nuclear forces.[11,7,10] It is then sufficient to use in Eq. (1) F elements appropriate to the "bare" force K.

For our example of the $E2$ operator in the Sn^{116} nucleus we have considered the s.p. binding energies computed by the Bonn group[12] with a most reasonable Woods-Saxon potential. The energies (in MeV) for the five valence (nlj) subshells are as follows: -10.52 $(2d_{5/2})$, -9.36 $(1g_{7/2})$, -8.45 $(3s_{1/2})$, -7.78 $(2d_{3/2})$, and -7.16 $(1h_{11/2})$; for the eight important proton core (hole) subshells we have -30.09 $(1d_{5/2})$, -27.93 $(1d_{3/2})$, -27.07 $(2s_{1/2})$, -22.91 $(1f_{7/2})$, -19.07 $(1f_{5/2})$, -18.82 $(2p_{3/2})$, -17.28 $(2p_{1/2})$, and -15.24 $(1g_{9/2})$. In addition to the five valence subshells we consider six higher proton particle subshells: -2.56 $(2f_{7/2})$, -1.14 $(3p_{3/2})$, -0.23 $(3p_{1/2})$, $+1.01$ $(2f_{5/2})$, $+1.04$ $(1i_{13/2})$, and $+1.07$ $(1h_{9/2})$. Any other particle or hole subshells give only negligible contributions. The Woods-Saxon radial wave functions are reasonably approximated with those of the harmonic oscillator (h.o.) with $\nu = 0.46\ {\rm F}^{-1}$. With the above states there are in all 29 nonvanishing ($E2$ allowed transitions) proton matrix elements $\langle p\|e\hat{O}^{\lambda=2}\|H\rangle$ giving contributions to the nine distinct matrix elements $\langle n'\|\hat{O}_{\rm eff}{}^{\lambda=2}\|n\rangle$ $(n \le n')$ for the valence neutrons.

We define the "effective charge matrix" (ECM) as

$$e_\lambda(n,n') \equiv \langle n'\|\hat{O}_{\rm eff}{}^\lambda\|n\rangle / \langle n'\|e\hat{O}^\lambda\|n\rangle_{\rm ref}, \tag{2}$$

where $\langle n'\|e\hat{O}^\lambda\|n\rangle_{\rm ref}$ is the "reference matrix" defined in the usual way for "direct" $n\to n'$ transitions and $e_{\rm eff}{}^{(\lambda)} = 1$. ECM gives the actual theoretical effective charge for each individual $n \to n'$ transition.

In Table I we give $e_2(n,n')$ for Sn^{116} computed from Eq. (1) with the elements F_{np} of the bare Yale-Shakin K_{np} force and the single-particle parameter as defined above. Although of the same sign and of the same order of magnitude, the $e_2(n,n')$ are actually grouped in two clusters: those somewhat higher than unity and those somewhat smaller than 0.7. The entire $1f2p$ ma-

Table I. Matrix of effective charge of Eq. (2) for $E2$ transitions for the five valence-neutron subshells in Sn.

	$3s_{1/2}$	$2d_{3/2}$	$2d_{5/2}$	$1g_{7/2}$	$1h_{11/2}$
$3s_{1/2}$	···	0.6143	0.6757	···	···
$2d_{3/2}$		0.6459	0.6989	1.1636	···
$2d_{5/2}$			0.6521	1.1132	···
$1g_{7/2}$				1.0844	···
$1h_{11/2}$					0.6535

jor shells, plus $1g_{9/2}$ of the core with all their transitions to the five lowest lying particle subshells, contribute on the average slightly more than about 50% of all the $e_2(n,n')$. Transitions from the same to the six s.p. levels of the upper major shell contribute the surprisingly large amount of 30-40% of all the $e_2(n,n')$. The $1d2s$ major shell of the core is of little importance. Both terms on the right-hand side of Eq. (1) give contributions of the same order of magnitude.

In order to examine the relative importance of our individual $e_2(n,n')$ we compute with the numbers of Table I the observables $B(E2, 2_1^+ \rightarrow 0_1^+)$ and $Q(2_1^+)$. We first solve the appropriate QTD and QSTD secular problems,[5,6] and find the desired eigenvalues and the eigenvectors $|0_1^+\rangle$ and $|2_1^+\rangle$, both strictly compatible with the model parameters involved in $e_2(n,n')$ of Table I. The effective nuclear force in mixing the QTD and QSTD configurations of the valence neutrons contains the second-order renormalizations of "core polarization" of all the proton and neutron subshells mentioned above; the valence-neutron subshells are assumed to be, on the average, exactly half occupied; no other approximation of the propagators of the core-polarization terms is made. The s.p. energies and wave functions are exactly those of our $e_2(n,n')$ calculation. While $|0_1^+\rangle$ is, in QTD, the qp (quasiparticle) vacuum itself and $|2_1^+\rangle$ has nine two-qp components, the corresponding vectors in our QSTD theory[6,7] have 56 ($|0_1^+\rangle$) and 94 components; these are free of all the basic spurious kets due to the nucleon-number nonconservation (such kets are projected out). The QSTD 0_1^+ eigenvalue lies lower by -0.363 MeV than the qp vacuum and the QSTD 2_1^+ eigenvalue is 1.153 MeV; the QTD 2_1^+ energy lies at 1.259 MeV; the observed 2_1^+ energy is 1.291 MeV.

In Table II we give the QTD and QSTD values of the $B(E2, 2_1^+ \rightarrow 0_1^+)$ and $Q(2_1^+)$ both "theoretical" [computed with the $e_2(n,n')$ of Table I] and those calculated with the neutron effective charge, $e_{\mathrm{eff}}^{(2)} = 1$. The reported observed value of $B(E2,$

Table II. $B(E2, 2_1^+ \rightarrow 0_1^+)$ (in e^2 F^4) in Sn^{116} computed with QTD and QSTD eigenvectors for the Yale-Shakin force with core polarization; $e_2(n,n')$ is computed from Eqs. (1) and (2); the quadrupole moment of the 2_1^+ state, $Q(2_1^+)$ (in barns), is given for QSTD in the same theories.

	$B(E2, 2_1^+ \rightarrow 0_1^+)$ (e^2 F^4) QTD	QSTD	$Q(2_1^+)$ QSTD (b)
$e_2(n,n')$ (theoretical)	232.3	202.2	+0.094
$e_{\mathrm{eff}}^2 = 1$	317.0	273.7	+0.125

$2_1^+ \rightarrow 0_1^+)$ varies between about 200 and about 500 e^2 F^4. One must keep in mind that our predicted result of 202.2 e^2 F^4 was obtained with no adjustable parameter involved. Clearly, a better agreement with experiment could be obtained if, e.g., we were to vary the Woods-Saxon s.p. pa rameters.

The observed value of $Q(2_1^+)$ of Sn^{116} is[13] +0.4 ± 0.3 b. Our predicted values lie around the lower limit of the experimental error. One has to keep in mind the fact that $Q(2_1^+)$ is a very "delicate" quantity sensitive to the detailed structure of the $|2_1^+\rangle$ vector. The QSTD predictions are much better than those of QTD because of the most important enhancement due to the large two-qp–four-qp interference terms even in the case of quite small four-qp components. It should be noted that our theory is based on the purely spherical shell model; we feel that the assumption of a stable deformation in the 2_1^+ state in Sn is probably premature.

The results of Tables I and II obtained with no ad hoc adjustable parameter seem to support strongly our present theory of the effective electromagnetic interaction operators based on realistic nucleon-nucleon potentials. Detailed analysis of other transition probabilities, static moments, and inelastic electron-scattering form factors by the present methods will be published elsewhere.

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