

## DISTORTION OF NUCLEAR SPECTRA BY FINAL-STATE COULOMB INTERACTIONS\*

Edwin Norbeck and F. Duane Ingram

Department of Physics and Astronomy, University of Iowa, Iowa City, Iowa

(Received 18 April 1968)

When charged-particle nuclear spectra are used to determine the location and width of an energy level in a nucleus with a moderately short lifetime for charged particle decay, corrections for final-state Coulomb interactions can sometimes be very important. The effect is illustrated by application to the 16.62- and 16.92-MeV levels in  $\text{Be}^8$  produced by the reaction  $\text{Li}^6(\text{Li}^6, \alpha)\text{Be}^8$ .

With the help of the magnetic spectrograph, charged-particle energy spectra are now being measured with great precision. Subtle interference effects are studied by measuring the difference between the actual shape of a peak and that predicted by the Breit-Wigner formula.<sup>1,2</sup> It is therefore important to recognize all potential sources of distortion of an energy spectrum. The effect described below can change the apparent width and location of an energy level, and in some cases cause a noticeable distortion in the shape of a peak in the energy spectrum. The effect may be different for different reactions leading to the same nuclear level and may be a function of bombarding energy and sometimes even of angle.

One can see how a final-state Coulomb interaction changes the energy spectrum by referring to Fig. 1. The two particles  $P_1$  and  $P_2$  are charged. The average time delay between the emission of  $P_1$  and the later emission of  $P_2$  is related to the width of the state. For the motion of  $P_2$  to affect the motion of  $P_1$ , it is not necessary for the two particles to "collide," although the effect is increased by making the angle  $\theta$  smaller. To get a large effect the velocity  $V_2$  should not be too much smaller than  $V_1$ . The energy of particle  $P_1$  is changed because it experiences a larger Coulomb repulsion when  $P_2$  follows along behind it than it would if  $P_2$  were not emitted at all.

The shift in energy of  $P_1$  was demonstrated with the reaction  $\text{Li}^6(\text{Li}^6, \alpha)\text{Be}^8$ . Particle  $P_1$  was an alpha particle which was detected at a laboratory angle of  $15^\circ$ . The intermediate nucleus was  $\text{Be}^8$  in the 16.62- or 16.92-MeV state, which de-

cayed to produce  $P_2$  and the residual nucleus, both alpha particles. The angle  $\theta$  was determined by detecting the residual nucleus in a position-sensitive detector.<sup>3</sup> The position detector spanned an angular range of  $35^\circ$  which was divided up after the experiment into small segments.<sup>4</sup> Figure 2 shows two of the  $P_1$  spectra which were collected simultaneously, under identical conditions except that the data shown as crosses represent a smaller value of  $\theta$ . The vertical scale has been adjusted so that the area under the 16.62- and 16.92-MeV peaks is the same for both sets of data. The difference in height between the peaks in the two sets reflects the difference in width. The lines drawn through the points were calculated as described below. It can be seen that the calculated curves fit the experimental points quite well. The poor fit to the low-energy side of the dot data was caused by  $\text{Be}^8$  breakup particles getting into the  $P_1$  energy detector. At all angles  $\theta$  between  $10^\circ$  and  $90^\circ$ , the positions of the 16.62- and 16.92-MeV peaks agreed with the calculations within an experimental uncertainty which was never more than 30 keV. For  $\theta$  less than about  $10^\circ$ , the distortion of the peaks was so great that it was difficult to separate the Coulomb shifts from the distortion of the spectrum caused by both  $P_1$  and  $P_2$  hitting the same detector.

The calculation of the final-state Coulomb interaction begins with the assumption that the energy level of the intermediate nucleus has a sharp energy equal to its central value and that it decays exponentially with a mean life of  $\hbar/\Gamma$ . At  $t=0$ , the particle  $P_1$  starts out from a dis-

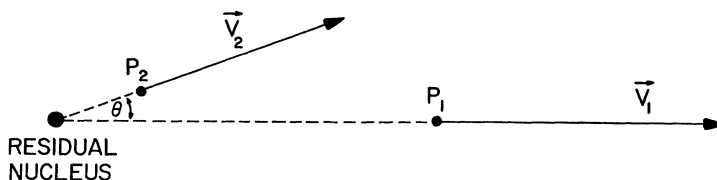


FIG. 1. The final-state Coulomb interaction changes the energy of particles  $P_1$  and  $P_2$ .

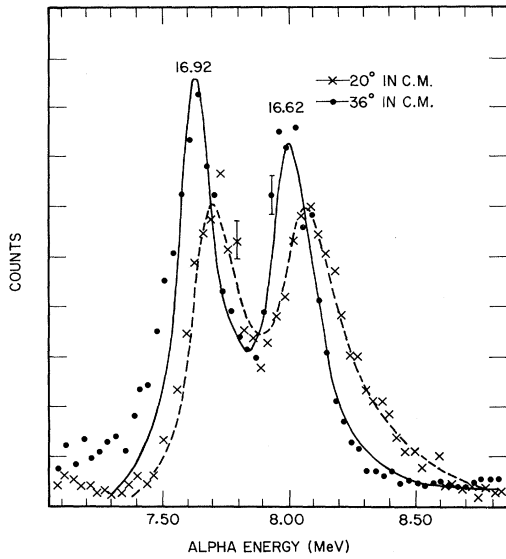


FIG. 2. Alpha-particle spectrum at  $15^\circ$  (lab) from  $\text{Li}^6(\text{Li}^6, \alpha)\text{Be}^8$  with a beam energy of 6.0 MeV and 5.2 mg of Ni over the detector. The dot and the cross data were collected simultaneously in the same detector. They differ only by the location of another detector which looked at  $\text{Be}^8$  breakup particles and which was used to gate the first detector. The angle in the figure is the angle  $\theta$  in Fig. 1. The vertical scale has been adjusted so that the total area under the 16.62- and 16.92-MeV peaks is the same for both the dot and cross data. The lines were calculated from the theory.

tance of 6 F with a kinetic energy  $E - V$ .  $E$  is the energy it would have at infinity in the absence of a breakup of the intermediate nucleus, and  $V$  is the electrical potential energy. At some radius  $R$ , the intermediate nucleus emits  $P_2$  at an angle  $\theta$  with respect to the direction of  $P_1$ . The time of emission of  $P_2$  is calculated<sup>5</sup> and this time used to determine the fraction of the intermediate nuclei that remain. The paths of the three final charged particles,  $P_1$ ,  $P_2$ , and the recoiling final nucleus, are determined by a numerical integration of  $\vec{F} = m\vec{a}$  where  $\vec{F}$  is determined by Coulomb's law. The integration program applies the Nyström<sup>6</sup> scheme to the six coupled equations for the  $x$  and  $y$  components of the position and velocity vectors for the three particles. The integration continues until the potential energy between the three particles is down to about 1 keV. At this distance, electron screening effects are comparable in importance with the effect being calculated. The final kinetic energy of  $P_1$  is the chief object of the calculation. The other velocities are used to check for overall momentum and energy conservation as an indication of the accuracy of the numerical integration.

For small values of  $\theta$ , the shift in the energy of  $P_1$  can be substantial even when the radius  $R$  is large compared with nuclear dimensions. For the reaction we studied,  $V_1$  is about  $1.5 \times 10^9$  cm/sec and  $V_2$  is  $1.2 \times 10^9$  cm/sec for the 16.62-MeV state. For  $\theta = 15^\circ$  and a radius  $R$  of 100 F, at which time about half of the  $\text{Be}^8$  have decayed,  $P_1$  is shifted by 212 keV. The momentum exchanged between  $P_1$  and  $P_2$  is very small. The large energy shift comes from the cross term that arises when the kinetic energy is calculated from the momentum. The change in the direction of  $P_1$  is generally small. In the example just discussed, it is about  $1^\circ$ . Both this small change of angle and the recoil of the residual nucleus introduce additional details into the definition of the angle  $\theta$ . These are readily treated<sup>7</sup> but will not be dealt with here.

The calculation is repeated for selected values of the breakup radius  $R$  ranging out to 1500 F. The energy shifts for intermediate values of  $R$  are found by interpolation. This list of energy shifts is then transformed into a curve that represents the energy spectrum one would observe if the unshifted energy spectrum could be represented by a delta function. This curve is folded into a realistic unshifted spectrum, which includes both the natural width and the experimental broadening, to produce curves such as are shown in Fig. 2.

The calculated curves fitted the width and location of the peaks, but they showed a high-energy tail that was not in agreement with the experiments. We found we could get a better fit to the data by not allowing any  $\text{Be}^8$  breakups until  $P_1$  had reached a radius of 30 F. The first 15% of the events, which were eliminated in this manner, produced large energy shifts for small angles  $\theta$  because  $P_1$  and  $P_2$  were initially close together with similar velocities. The high-energy tail should not appear in a quantum mechanical calculation where the positions and velocities cannot be so well defined.

If  $P_2$  is now observed so that the  $P_1$  energy spectrum reflects all values of  $\theta$ , the effect on the  $P_1$  energy spectrum is quite small, although still significant, if the energy measurements are made with high precision. In this case, it is necessary to average over all values of  $\theta$ , with due regard for the  $\sin\theta$  weighting factor and for angular correlations between  $P_1$  and  $P_2$ . Our experimental measurements were not sufficiently precise to measure these small averaged shifts, so we calculated the shifts to be expected in the

reaction  $B^{10}(d, \alpha)Be^8$  which has been the subject of extensive measurements with a magnetic spectrograph.<sup>1</sup> The calculations were for the 16.62-MeV state in  $Be^8$  with a 12-MeV deuteron beam and the detector at a laboratory angle of  $30^\circ$ . For an isotropic breakup of the  $Be^8$ , the energy of  $P_1$  was shifted down by about 9 keV. With angular correlation patterns such as have been observed in a similar reaction,<sup>8</sup> the negative energy shift varied from about 7 to 10 keV. This is the same order of magnitude as is caused by the interference between the two  $Be^8$  states.<sup>1,2</sup> When final-state Coulomb interaction effects are likely to be important, one should report the detector angle and beam energy along with the energy level parameters, as has been done by Kroepfl and Browne.<sup>9</sup>

The negative energy shift can be understood by consulting Fig. 1 again. If  $P_2$  moves rapidly away with a large angle  $\theta$ , particle  $P_1$  gains less energy from electrostatic repulsion because of the charge carried away by  $P_2$ . When the intermediate nucleus is  $Be^8$ , the symmetry of the nucleus limits the effective  $\theta$  to  $90^\circ$ . Even though the negative shift is small, it overpowers the larger positive shift in the integrated curve because of the  $\sin\theta$  factor.

Our experiments were not sufficiently precise to allow a measurement of the negative energy shift. Because of its importance in determining the location of energy levels, this measurement should be made as soon as possible. One might expect the classical calculation to be better for the negative shift since it depends only on  $P_1$  and

$P_2$  being far apart and does not require precise localization.

A proper quantum mechanical treatment of the final-state Coulomb interaction which included angular-momentum effects should give a quantitative description of the peak shapes to be expected. If this theory were checked once with a high-precision coincidence experiment, it could then be used to calculate corrections for the Coulomb shifts in all other experiments.

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\*Work supported in part by the National Science Foundation.

<sup>1</sup>C. P. Browne, W. D. Callender, and J. R. Erskine, *Phys. Letters* **23**, 371 (1966).

<sup>2</sup>J. B. Marion, P. H. Nettles, C. L. Cocke, and G. J. Stephenson, Jr., *Phys. Rev.* **157**, 847 (1967).

<sup>3</sup>E. Norbeck and R. R. Carlson, *Instrumentation Techniques in Nuclear Pulse Analysis* (National Academy of Sciences-National Research Council, Washington, D. C., 1964), p. 42.

<sup>4</sup>E. Norbeck and M. D. Mancusi, *Nucl. Instr. Methods* **56**, 296 (1967).

<sup>5</sup>H. Goldstein, *Classical Mechanics* (Addison-Wesley Publishing Company, Inc., Reading, Mass., 1950), p. 63.

<sup>6</sup>W. E. Grove, *Brief Numerical Methods* (Prentice-Hall, Inc., Englewood Cliffs, N. J., 1966), p. 107. [Be sure to note the error in the second formula.]

<sup>7</sup>F. D. Ingram, thesis, University of Iowa, 1968 (unpublished).

<sup>8</sup>C. Moazed and H. D. Holmgren, *Phys. Rev.* **166**, 977 (1968).

<sup>9</sup>J. J. Kroepfl and C. P. Browne, *Nucl. Phys.* **A108**, 289 (1968).

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#### NUCLEAR SPIN FILTER\*

Joseph L. McKibben, George P. Lawrence, and Gerald G. Ohlsen  
Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico  
(Received 15 April 1968)

A "spin filter" for selecting metastable hydrogen, deuterium, or tritium atoms with a given nuclear spin magnetic quantum number ( $m_I$ ) has been built and tested. With the device installed in the Los Alamos "Lamb-shift" polarized-ion source, we have obtained a deuterium negative-ion beam with  $\sim 55\%$  spin-state purity for  $m_I = 1, 0, \text{ or } -1$ . An improved magnetic field homogeneity in the apparatus is expected to increase the purity to  $\sim 75\%$ .

In connection with the development of a "Lamb-shift"-type source of polarized negative hydrogen, deuterium, or tritium ions,<sup>1</sup> we have developed and tested a device through which metastable atoms with a particular nuclear spin orientation ( $m_I$ ) may be transmitted while the remain-

ing atoms are quenched to the ground state. The device exploits the "three-level interaction" phenomenon which was discovered by Lamb and Retherford<sup>2</sup>; a rather complete discussion of the relevant theory has been given in a report.<sup>3</sup>

In this method, a longitudinal rf electric field