[4]James S. Ball, William R. Frazer, and M. Jacob, Phys. Rev. Letters 20, 518 (1968).
[5]Akbar Ahmadzadeh, R. J. Jacob, and B. P. Nigam, to be published.
[6]Akbar Ahmadzadeh, Phys. Rev. Letters 16, 952 (1966); A. Ahmadzadeh and C. H. Chan, Phys. Letters 22, 692 (1966).
[7]Farzam Arbab and Jan W. Dash, Phys. Rev. 163, 1603 (1967).
[8]D. Z. Freedman and J. M. Wang, Phys. Rev. 153, 1596 (1967).
[9]A. Sciarino and M. Toller, University of Rome Report No. 108, 1967 (unpublished); D. Z. Freedman and J. M. Wang, Phys. Rev. 160, 1560 (1967).

USE OF SINGLE-PION PRODUCTION TO REMOVE AMBIGUITIES IN PARTIALLY CONSERVED AXIAL-VECTOR CURRENT AND CURRENT-ALGEBRA PREDICTIONS OF π-π SCATTERING*

M. G. Olsson and Leaf Turner
Department of Physics, University of Wisconsin, Madison, Wisconsin 53706
(Received 1 March 1968)

We show that the ambiguities present in the current-algebra and partially conserved axial-vector current model of low-energy π-π scattering are the same as those in the single-pion production amplitude. The experimental data strongly support the Weinberg formulation.

For some time, it has been apparent that there is considerable ambiguity in the prediction of the pion scattering lengths by the hypothesis of partially conserved axial-vector current (PCAC) and current algebra.[1] In Weinberg's original calculation[2] of π-π scattering, he requires that $\partial^\mu A_\mu{}^\alpha$ form a chiral quadruplet with an isoscalar field. In a later paper[3] he derives a phenomenological Lagrangian that reproduces the same π-π scattering with all pions on the mass shell. Schwinger's Lagrangian[4] differs by assuming a chiral-symmetry-breaking term which implies that $\partial^\mu A_\mu{}^\alpha$ is proportional to the physical pion field. Schwinger's Lagrangian yields the exact Adler consistency condition[5] on π-π scattering which the Weinberg Lagrangian fails to do. The Schwinger Lagrangian differs also in its prediction of π-π scattering lengths.

The phenomenological Lagrangian technique allows easy reproduction of the current-algebra and PCAC results for the π-π scattering amplitudes and pion-production amplitudes. A further advantage is the ease with which one can isolate and study ambiguities remaining after the application of PCAC and current algebra. We shall show that the most general Lagrangian derived in accordance with current algebra and PCAC introduces a single parameter ξ into the π-π scattering lengths which can be determined only by additional assumptions.[6] However, we shall further demonstrate that ξ is the only parameter that enters the amplitude for single-pion production with external pions on the mass shell. Experimental data favor the value $\xi=0$. This value of ξ yields the Weinberg π-π amplitude on the mass shell; $\xi=1$ yields the Schwinger mass-shell amplitude.

Several techniques exist for generating chiral-invariant Lagrangians.[3,4,7] Our notation is that used by Bardeen and Lee. We assume PCAC in the form $\partial^\mu A_\mu{}^\alpha = f_\pi \mu^2 \varphi^\alpha(1+h_0\varphi^2+\cdots)$ to break the chiral invariance. Our pion Lagrangian to order φ^4 follows at once[8]:

$$\mathcal{L}(\pi) \equiv \tfrac{1}{2}(\partial_\mu\varphi)^2 - \tfrac{1}{2}\mu^2\varphi^2 + \mathcal{L}_\pi,$$

where

$$\mathcal{L}_\pi = (1/8f_\pi{}^2)[(\partial_\mu\varphi^2)^2 - \mu^2(\varphi^2)^2] - (g_0/f_\pi)[\tfrac{1}{2}\varphi^2(\partial_\mu\varphi)^2 + \tfrac{1}{4}(\partial_\mu\varphi^2)^2 - \tfrac{3}{8}\mu^2(\varphi^2)^2] - \tfrac{1}{4}\mu^2 h_0(\varphi^2)^2.$$

Choosing the two undetermined parameters to be $g_0 = 1/2f_\pi$ and $h_0 = -1/4f_\pi{}^2$ yields the Weinberg Lagrangian.[3] The values $g_0 = 1/2f_\pi$ and $h_0 = 0$ produce the Schwinger Lagrangian.[4]

Defining[8,9]

$$\frac{\delta^{(4)}(q+l-p-k)}{4(2\pi)^2(q_0 l_0 p_0 k_0)^{1/2}} M_{abcd}(s,t,u) \equiv -\int d^4x \langle \pi^c(l)\pi^d(q)|\mathcal{L}_\pi|\pi^a(p)\pi^b(k)\rangle,$$

we obtain

$$M_{abcd}(s,t,u) = \delta_{ab}\delta_{cd}[As + (A+f_\pi{}^{-2})(t+u) + C\mu^2]$$

plus crossing symmetric permutations, where $A \equiv g_0/f_\pi - f_\pi^{-2}$ and $C \equiv 2h_0 + f_\pi^{-2} - 3g_0/f_\pi$. Using the well-known isospin projection operators,[10]

$$M_{abcd}(s,t,u) = P_0[(5A+2/f_\pi^2)s + (5A+4/f_\pi^2)(t+u) + 5C\mu^2] + P_1[f_\pi^{-2}(u-t)] + P_2[(2A+2/f_\pi^2)s + (2A+f_\pi^{-2})(t+u) + 2C\mu^2],$$

which yields

$$2a_0 - 5a_2 = 6L; \quad L = \mu/8\pi f_\pi^2. \tag{1}$$

If we require $\partial^\mu A_\mu^\alpha = f_\pi \mu^2 \Phi^\alpha(\varphi)$, then the chiral-symmetry-breaking part of the Lagrangian is $f_\pi\mu^2\sigma(\varphi^2)$. Expanding $\sigma(\varphi^2)$, we recognize $h_0 = -(g_0/2f_\pi)$. With this choice and with all pions on the mass shell,

$$M_{abcd}(s,t,u) = f_\pi^{-2}[\delta_{ab}\delta_{cd}(\mu^2 - s) + \delta_{ac}\delta_{bd}(\mu^2 - t) + \delta_{ad}\delta_{bc}(\mu^2 - u)],$$

independently of the value of g_0. This is the Weinberg π-π scattering amplitude.[2] The imposition of the Adler consistency condition[5] on M_{abcd} requires merely that $h_0 = 0$. If, in addition, we wish to obtain Weinberg's amplitude, then we must also set $g_0 = 0$.

We now derive the pion scattering lengths. Setting $\xi = 2f_\pi(g_0 + 2h_0 f_\pi)$, we rewrite the pion scattering amplitude with all pions on the mass shell:

$$f_\pi^2 M_{abcd} = [(\tfrac{5}{2}\xi + 1)\mu^2 - 2s]P_0 + (u-t)P_1 + [(\xi - 2)\mu^2 + s]P_2.$$

Hence

$$a_0/a_2 = (\tfrac{5}{2}\xi - 7)/(\xi + 2).$$

Using Eq. (1), a_0 and a_2 are uniquely determined.

We wish to indicate the physical meaning of the parameter ξ. To order φ^2,

$$[Q_5^\alpha, \partial^\mu A_\mu^\beta] = f_\pi\mu^2[Q_5, \varphi^\beta(1 + h_0\varphi^2)] = if_\pi\mu^2[\delta^{\alpha\beta}(f_\pi - \varphi^2/2f_\pi) + (\xi/4f_\pi)(\delta^{\alpha\beta}\varphi^2 + 2\varphi^\alpha\varphi^\beta)].$$

Since the term $f_\pi - \varphi^2/2f_\pi$ is the σ field to order φ^2, we see that ξ measures the amount of departure from the usual assumption

$$[Q_5^\alpha, \partial^\mu A_\mu^\beta] = if_\pi\mu^2\delta^{\alpha\beta}\sigma(\varphi^2).$$

Whereas most of the papers in Ref. 1 effectively choose $\xi = 0$ and make various assumptions about the extrapolation to the physical threshold, we vary ξ and determine the amplitude by the phenomenological Lagrangian prescription. It is interesting to note that the p-wave scattering length a_1 is free from ambiguity:

$$18\mu^2 a_1 = 2a_0 - 5a_2 = 6L.$$

It is also the only π-π scattering length amenable to reasonably direct experimental test. A dispersion-relation calculation based mainly on the ρ-meson parameters provides good agreement with the current-algebra prediction.[11]

Using Weinberg's covariant-derivative formalism[7] for generating chiral-invariant pion-nucleon coupling, we find the Lagrangian for π-N interactions relevant to single-pion production, $\pi N \to \pi\pi N$:

$$\mathcal{L}_{NN\pi} = (G/2M)\bar{\psi}\gamma_\mu\gamma_5\vec{\tau}\psi\cdot\partial^\mu\vec{\varphi},$$

$$\mathcal{L}_{NN\pi\pi\pi} = -(G/2M)^3(g_v/g_A)^2[2g_0 f_\pi\bar{\psi}\gamma_\mu\gamma_5\vec{\tau}\psi\cdot\partial^\mu\vec{\varphi}\varphi^2 + 2(2g_0 f_\pi - 1)\bar{\psi}\gamma_\mu\gamma_5\vec{\tau}\psi\cdot\vec{\varphi}\vec{\varphi}\cdot\partial^\mu\vec{\varphi}],$$

$$\mathcal{L}_{NN\pi\pi} = -(G/2M)^2(g_v/g_A)^2\bar{\psi}\gamma_\mu\vec{\tau}\psi\cdot\vec{\varphi}\times\partial^\mu\vec{\varphi}.$$

For completeness, we rewrite the π-π interaction Lagrangian:

$$\mathfrak{L}_\pi = (G/2M)^2 (g_v/g_A)^2 [2(1-2g_0 f_\pi)(\varphi\partial^\mu\varphi)^2 - 2g_0 f_\pi \varphi^2 (\partial^\mu\varphi)^2 + \tfrac{1}{2}(3g_0 f_\pi - 2h_0 f_\pi{}^2 - 1)\mu^2(\varphi^2)^2]$$

with $(2f)^{-1} = (G/2M)(g_v/g_A)$.

Current-algebra calculations are most straightforward at threshold. Above threshold, other terms which are noncalculable except by more detailed models[12] become important. In this spirit, we calculate the $\pi N \to \pi\pi N$ production amplitude according to prescription. The two diagrams which can be important are shown in Fig. 1. The remaining diagrams, involving a virtual nucleon, are negligible at threshold. We calculate the contributions from all diagrams to two charge-state amplitudes. Defining the amplitude $A(\pi N \to \pi\pi N)$ by[8,13]

$$\langle N(p_f)\pi^\alpha(q_1)\pi^\beta(q_2)|S|N(p_i)\pi^\gamma(Q)\rangle$$
$$\equiv -\left(\frac{G}{2M}\right)^3 \left(\frac{g_v}{g_A}\right)^2 \frac{M\delta^{(4)}(p_f+q_1+q_2-p_i-Q)}{(2\pi)^{7/2}(E_i E_f \omega_1 \omega_2 \omega_Q)^{1/2}} \times A(\pi^\gamma N \to \pi^\alpha \pi^\beta N),$$

we arrive at the threshold amplitudes in a two-component spinor representation:

$$A(\pi^- p \to \pi^+\pi^- n) = \left(\frac{M}{2(E_i+M)}\right)^{1/2} \times \left[\frac{2\omega_Q - \xi\mu}{\omega_Q - \mu} + \frac{2\mu}{M} - 2 - \frac{2\mu}{\mu+2M} + \frac{2\mu}{2E_i - \mu}\right] \times \varphi_f{}^\dagger \vec{Q}\cdot\vec{\sigma}\varphi_i,$$

$$A(\pi^+ p \to \pi^+\pi^+ n) = \left(\frac{M}{2(E_i+M)}\right)^{1/2} \times \left[\frac{4\mu}{\mu+2M} - \frac{4\mu}{2E_i - \mu} - \frac{4\mu}{M} - \frac{(2+\xi)\mu}{\omega_Q - \mu}\right] \times \varphi_f{}^\dagger \vec{Q}\cdot\vec{\sigma}\varphi_i.$$

At threshold, use of charge independence specifies the other charge-state amplitudes in terms of these. The best data available near threshold[14] are for the reaction $\pi^- p \to \pi^+\pi^- n$. In Fig. 2 we plot the available cross-section measurements for this reaction. The curves drawn represent the cross section predicted by multiplying the threshold amplitude by phase space[15] for various values of the parameter ξ. As one sees, as threshold is approached, the data seem to converge to the curve specified[16] by $\xi = 0$ or 4.5. In contrast, the prediction of Schwinger's Lagrangian ($\xi = 1$) seems considerably low. Hence, the data favor $\xi = 0$ or 4.5. Although there are no data available close to threshold for the other charge-state reactions, nevertheless the present data rule out the value $\xi = 4.5$. For example, at pion laboratory kinetic energy $T_\pi = 357$ MeV, $\sigma(\pi^+ p \to \pi^+\pi^+ n) = 0.12 \pm 0.01$ mb. The predictions for this charge state are $\sigma(\pi^+ p \to \pi^+\pi^+ n) = 0.24$, 0.48, and 1.9 mb for $\xi = 0$, 1, and 4.5,

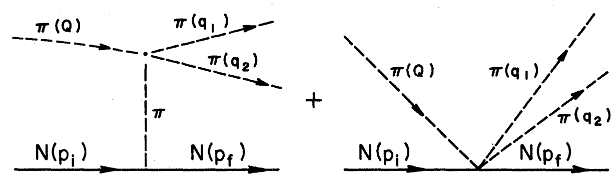

FIG. 1. Diagrams which can be important in the single-pion production process. Other contributions involving a nucleon pole are small at threshold.

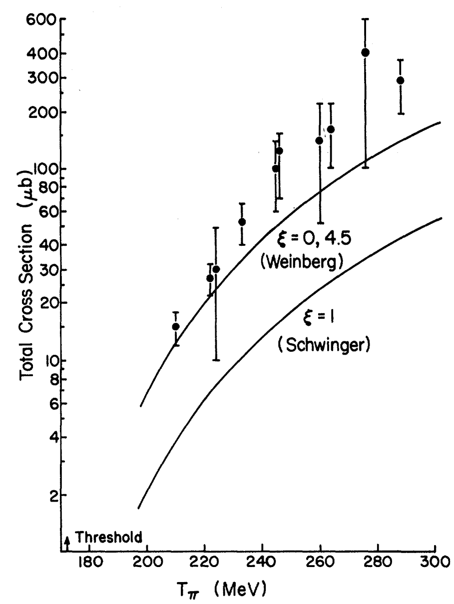

FIG. 2. The available data for the reaction $\pi^- p \to \pi^-\pi^+ n$ near threshold (cf. Ref. 14). The curves drawn represent predictions for different values of the parameter ξ. Above threshold the cross-section prediction is made by multiplying the square of the threshold amplitude by the physical phase space. The data appear to converge to the upper curve as threshold is approached.

respectively. Similarly, for $T_\pi = 400$ MeV, $\sigma(\pi^- p \to \pi^0\pi^0 n) = 1.4 \pm 0.2$ mb. Our predictions are $\sigma(\pi^- p \to \pi^0\pi^0 n) = 0.85$, 0.63, and 0.11 mb for $\xi = 0$, 1, and 4.5. Hence, we see a strong indication that $\xi \simeq 0$ is indeed the correct choice.

We have shown how the pion production process removes ambiguity in the PCAC and current-algebra prediction of π-π scattering. It will be most interesting to see if a consistent picture is maintained when pion production experiments are done near threshold for the other charge states. Such experiments will be important, not only as a test of current algebra, but also as a probe of the π-π interaction which still remains difficult to measure directly.

We would like to thank Professor C. J. Goebel and Professor B. Sakita for several useful discussion

---

*Work supported in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation, and in part by the U. S. Atomic Energy Commission under Contract No. AT (11-1)-881, No. COO-881-161.

[1]N. N. Khuri, Phys. Rev. 153, 1477 (1967); J. Sucher and C.-H. Woo, Phys. Rev. Letters 18, 723 (1967); K. Kang and T. Akiba, Phys. Rev. 164, 1836 (1967); H. R. Rubinstein and G. Veneziano, to be published; J. Iliopoulos, Nuovo Cimento 53A, 552 (1968); A. Donnachie, CERN Report No. TH. 804, 1967 (unpublished); L. S. Kisslinger, Phys. Rev. Letters 18, 861 (1967); J. R. Fulco and D. Y. Wong, Phys. Rev. Letters 19, 1399 (1967); S. Matsuda and S. Oneda, Phys. Rev. 165, 1749 (1968).

[2]S. Weinberg, Phys. Rev. Letters 17, 616 (1966).

[3]S. Weinberg, Phys. Rev. Letters 18, 188 (1967).

[4]J. Schwinger, Phys. Letters 24B, 473 (1967).

[5]S. L. Adler, Phys. Rev. 137, B1022 (1965).

[6]We have recently become aware of a similar result obtained by J. Schwinger, to be published.

[7]W. A. Bardeen and B. W. Lee, Canadian Summer Institute in Nuclear and Particle Physics, Montreal, 1967 (to be published); L. S. Brown, Phys. Rev. 163, 1802 (1967); P. Chang and F. Gürsey, Phys. Rev. 164, 1752 (1967); S. Weinberg, to be published; J. Wess and B. Zumino, Phys. Rev. 163, 1727 (1967); J. A. Cronin, Phys. Rev. 161, 1483 (1967).

[8]Our metric and normalization conventions are those of J. D. Bjorken and S. D. Drell, Relativistic Quantum Fields (McGraw-Hill Book Company, Inc., New York, 1965).

[9]We define our Mandelstam variables by $s = (p+k)^2 = (q+l)^2$, $t = (k-q)^2 = (l-p)^2$, $u = (p-q)^2 = (l-k)^2$; and our nucleon and pion masses by M and μ, respectively.

[10]G. F. Chew and S. Mandelstam, Phys. Rev. 119, 467 (1960).

[11]M. G. Olsson, Phys. Rev. 162, 1338 (1967).

[12]H. J. Schnitzer, Phys. Rev. 158, 1471 (1967).

[13]We define our four-momenta by $p_i = (E_i, \vec{p}_i)$ for incoming nucleon, $p_f = (E_f, \vec{p}_f)$ for outgoing nucleon, $Q = (\omega_Q, \vec{Q})$ for incoming pion, $q_k = (\omega_k, \vec{q}_k)$ for outgoing pions.

[14]Yu. A. Batusov, S. A. Bunyatov, V. M. Sidorov, and V. A. Yarba, Yadern. Fiz. 1, 526 (1965) [translation: Soviet J. Nucl. Phys. 1, 374 (1965)]; M. G. Olsson and G. B. Yodh, Phys. Rev. 145, 1309 (1966).

[15]We base our calculations upon $G^2/4\pi = 14.6$, $|g_A/g_V| = 1.18$.

[16]We observe that our results for $\xi = 0$ agree with those obtained by L.-N. Chang, Phys. Rev. 162, 1497 (1967), who used the current-commutator theory of multiple-pion production of S. Weinberg, Phys. Rev. Letters 16, 879 (1966).