SHELL-MODEL CALCULATIONS OF $B(E2)$ VALUES IN s-d SHELL NUCLEI WITH $A = 18$-22†

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As part of an extensive theoretical investigation of s-d shell nuclei with masses from $A = 18$ to 22, we have calculated energy levels and transition rates by using a shell model with a "realistic" effective interaction. We give here some examples of the resulting energy spectra and the $B(E2)$ values for transitions between low-lying states, and we discuss the strong similarity between our shell-model results and those obtained within the framework of a Nilsson-rotational model.

In the calculation of energy levels and wave functions, we assume an inert ^{16}O core and let the active particles be distributed among the $d_{5/2}$, $s_{1/2}$, and $d_{3/2}$ single-particle orbits. For a given number of active particles, all possible configurations made from these three j shells, and all possible shell-model states within each configuration, are included in the basis space. For the shell-model Hamiltonian we use the effective interaction calculated by Kuo[1] for the oxygen region. This interaction is derived from the Hamada-Johnston potential via reaction-matrix techniques, and it includes some perturbative corrections associated with breakup of the 16-particle core. We use the same two-body interaction for all the nuclei 18-22. The single-particle energies are taken from the experimentally observed spectrum of ^{17}O, and they too are held constant. The energy matrices were constructed and diagonalized with the Oak Ridge–Rochester shell-model computer programs.[2] For the mass-22 nuclei, the dimensions of three of the matrices exceeded 500×500.

Agreement between the theoretical energy spectra and observed spectra is generally good. As examples, the results for ^{20}Ne and ^{22}Na are shown in Fig. 1. The ^{20}Ne results are typical of most of the results, while those for ^{22}Na are the least satisfactory.

A general feature of our energy-level results for all the nuclei $A = 18$-22 can be discussed in terms of a rotational scheme for classifying the nuclear levels. In the observed spectrum of each nucleus in this mass region, there is a series of levels with spin values obviously associated with a ground-state rotational band. In some cases the energy spacings within this series are markedly different from those predicted by a pure rotational model. But without exception, the observed excitations within this ground-state band are excellently reproduced by the shell-model calculations. In Fig. 1, levels within the ground-state bands are indicated by heavier lines. Even in ^{22}Na, where there are serious discrepancies for the $J = 1$ states, there is excellent agreement within the ground-state band.

We have used our shell-model eigenvectors to calculate theoretical $B(E2)$ values. For all the

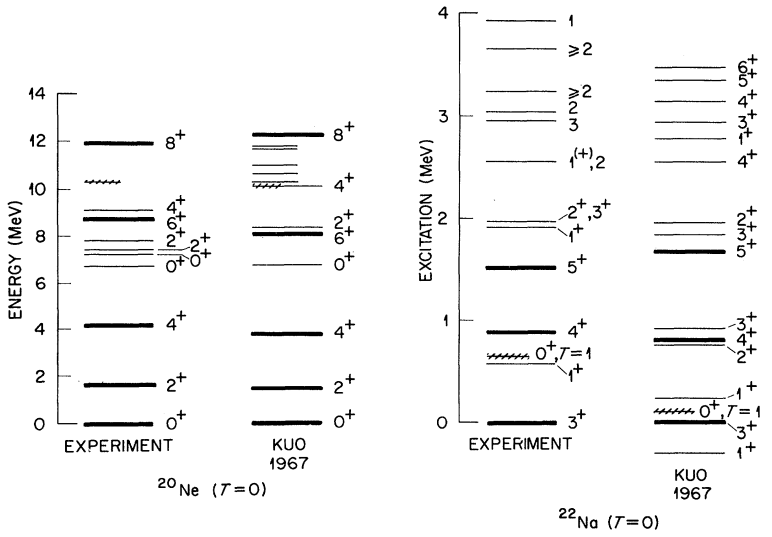

FIG. 1. The calculated and experimentally observed spectra of ^{20}Ne and ^{22}Na.

nuclei, we use an effective charge of $0.5e$ (i.e., the neutron has charge $0.5e$, and the proton has charge $1.5e$). This effective charge of $0.5e$ has been suggested by Wilkinson,[3] who discusses its use in light s-d shell nuclei to simulate the effects of exciting the ^{16}O core. The nucleon radial wave functions we take to be harmonic oscillator functions proportional to $e^{-\frac{1}{2}\nu r^2}$, where for each nucleus we evaluate ν from the relation $\nu = 0.971A^{-1/3}$ fm^{-2}.[4]

In the upper part of Table I we show calculated and observed $B(E2)$ values for transitions from the first excited state to the ground state in the nuclei with $A = 18$-22. The observed values are taken from the compilation of Skorka, Hertel, and Retz-Schmidt.[5] (Where one measured value is clearly better than the others, for a given transition, we list this favored value. Otherwise we list the averaged value shown in Skorka's table.) Over all, the agreement between calculated and measured values in Table I is very gratifying. The results do not suggest any need to vary the effective charge from nucleus to nucleus within this mass region. The calculated strengths for other $E2$ transitions are also in satisfactory agreement with observed strengths, especially for transitions between members of the ground-state rotational band. As examples, we list at the bottom of Table I calculated and observed $B(E2)$ values for the three transitions connecting known members of the $K = 3$ ground-state band in ^{22}Na.

There are two $E2$ transitions in Table I for which the theory and experiment are not in good agreement. One of these discrepancies is for ^{18}O. In our model of ^{18}O, all the active particles are neutrons; therefore, this calculated $B(E2)$ depends entirely on the effective-charge approximation. (The data for the other oxygen isotopes do not warrant any conclusions about systematic trends for these pure-neutron states.) The other discrepancy in Table I is for ^{21}Ne. Here there is some reason to suspect that the measured value is incorrect, and too high. The same experimental group that determined the observed $B(E2)$ listed for ^{21}Ne also measured a $B(E2)$ for the first-to-ground transition in ^{20}Ne. They obtained $92.7 \pm 34\%$ e^2 fm^4, but a subsequent measurement by a different experimental group gave $57.3 \pm 10\%$ e^2 fm^4.[6]

It is interesting to compare the shell-model results with $B(E2)$ values estimated by using a simple unified rotational model based on deformed single-particle orbits. The $B(E2)$ for a transition between two members of a pure rotational band is given by[4]

$$B(E2)_{i\text{-}f} = \frac{5}{16\pi} e^2 |Q_0|^2 \left| \begin{matrix} J_i & 2 & J_f \\ C_K & 0 & K \end{matrix} \right|^2, \qquad K \neq \tfrac{1}{2}, \quad K \neq 1. \tag{1}$$

Here Q_0 is the intrinsic electric quadrupole moment of the band head, and K is the Z projection of total angular momentum on the body-fixed axis. To evaluate Q_0,[7] we consider that the intrinsic states of the nuclei considered here are formed by putting the active particles in the lowest $K = \frac{1}{2}$ and $K = \frac{3}{2}$ s-d shell single-particle orbits. Deformed Hartree-Fock calculations[8] show that the single-particle mass quadrupole moments ϵ for these $K = \frac{1}{2}$ and $K = \frac{3}{2}$ orbits are close to +4 and +1, respectively (in units of $b^2 = \hbar/m\omega$). For a given nucleus, we form the intrinsic states by placing the appropriate number of neutrons and protons in the lowest allowed deformed orbits, and we assume (as above) an effective charge of $0.5e$ for both neutrons and protons. Then for each orbit, the single-particle intrinsic electric quadrupole moment is taken as the single-particle intrinsic mass quadrupole moment multiplied by the total charge (real plus effective) of the relevant particle. The total intrinsic electric quadrupole moment is obtained by summing the single-particle moments. With Q_0 deter-

Table I. Absolute value of $B(E2)$'s (in units of e^2 fm^4) for some $E2$ transitions between low-lying states. Except for ^{19}O, all transitions are from the first excited to the ground states. UL and LL stand for upper limit and lower limit, respectively.

Nucleus	Spins	Experimental	Shell model	Rotational
^{18}O	$2^+ \rightarrow 0^+$	$6.5 \pm 20\%$	3.0	3.2
^{18}F	$3^+ \rightarrow 1^+$	$16.4 \pm 10\%$	14.8	
^{19}F	$\frac{5}{2} \rightarrow \frac{1}{2}$	$20.8 \pm 20\%$	19.0	
^{19}O	$\frac{1}{2} \rightarrow \frac{5}{2}$	$(0.04 \rightarrow 1.1)_{\mathrm{LL}}$	2.5	
^{20}O	$2^+ \rightarrow 0^+$	$(38.7)_{\mathrm{UL}}$	4.7	3.9
^{20}F	$3^+ \rightarrow 2^+$		27.8	30.8
^{20}Ne	$2^+ \rightarrow 0^+$	$57.3 \pm 10\%$	48.3	40.0
^{21}Ne	$\frac{5}{2} \rightarrow \frac{3}{2}$	$161.8 \pm 20\%$	81.0	75.4
^{21}F	$\frac{1}{2} \rightarrow \frac{5}{2}$	$57.1 \pm 5\%$	68.8	
^{22}Ne	$2^+ \rightarrow 0^+$	$42.4 \pm 30\%$	55.4	48.1
^{22}Na	$1^+ \rightarrow 3^+$	$0.03 \pm <1\%$	0.7	
^{22}Na	$4^+ \rightarrow 3^+$	$108 \pm 17\%$	102	94.4
^{22}Na	$5^+ \rightarrow 3^+$	$25^{+28\%}_{-17\%}$	25.3	22.9
^{22}Na	$5^+ \rightarrow 4^+$	$76 \pm 33\%$	91.2	86.3

mined in this manner, we have used Eq. (1) to calculate the $B(E2)$ values for all the transitions listed in Table I for which Eq. (1) holds. The resulting numbers are listed under the heading "rotational." There is striking similarity between the two sets of calculated numbers. Note that for the ^{21}Ne transition discussed above, both methods of calculation lead to a $B(E2)$ that is much smaller than the measured value reported.

All these results indicate a close relationship between the shell-model and rotational pictures of the low-lying levels in light s-d shell nuclei.

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[1]T. T. S. Kuo, Nucl. Phys. A103, 71 (1967).

[2]J. B. French, E. C. Halbert, J. B. McGrory, and S. S. M. Wong, to be published.

[3]D. H. Wilkinson, Comments Nucl. Particle Phys. 1, 139 (1967).

[4]S. A. Moszkowski, Handbuch der Physik, edited by S. Flügge (Springer-Verlag, Berlin, Germany, 1967), Vol. 39, p. 411.

[5]S. J. Skorka, J. Hertel, and T. W. Retz-Schmidt, Nucl. Data 2, 347 (1967).

[6]K. H. Bhatt and J. C. Parikh, Phys. Letters 24B, 613 (1967).

[7]The method for evaluating Q_0 was suggested to us by K. H. Bhatt. It follows closely a method for determining the mass quadrupole moment in these nuclei as discussed previously by Parikh and Bhatt (Ref. 6).

[8]A. P. Stamp, Nucl. Phys. A105, 627 (1967).

POLARIZATION-VECTOR ANALYZING POWER OF THE REACTIONS ^{9}Be$(d,p)^{10}$Be AND ^{12}C$(d,p)^{13}$C

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Measurements have been made of the vector analyzing power for polarized deuterons of 12 MeV of the reactions ^{12}C$(d,p)^{13}$C, ^{12}C$(d,p)^{13}$C* (3.09-MeV state), and ^{9}Be$(d,p)^{10}$Be. For the first of these reactions a comparison has been made with proton-polarization measurements at a nearby energy.

Recently considerable interest has been shown in the usefulness of polarization measurements in (d,p) stripping reactions.[1-3] It has been suggested[1,2] that such studies should provide a more sensitive test for j dependence than cross-section measurements[4] and that they should give added insight into the reaction mechanism.[3,5] Most polarization experiments have examined $P_p(\theta)$, the proton polarization produced in the reaction using unpolarized deuterons,[6,7] but Yule and Haeberli[8] have recently reported proton asymmetries produced using a polarized deuteron beam. Their work on medium-weight nuclei ($54 > A > 24$) suggested that the proton-asymmetry angular distributions depended not only on l_n, the orbital angular momentum of the captured neutron, but also upon its total angular momentum j_n. The present work, which refers to light nuclei, strengthens the conclusions on j dependence and further allows a comparison to be made with a measurement of $P_p(\theta)$. This is of fundamental importance in understanding the reaction mechanism.[5]

Adopting the notation employed by Yule and Haeberli,[8] the proton asymmetry A_p produced by a deuteron beam of vector polarization p initiating a reaction of vector analyzing power $P_d(\theta)$ is

$$A_p = \tfrac{3}{2} p P_d(\theta),$$

where the polarized-beam scattering intensity is related to the unpolarized one by

$$I_{\text{pol}} = I_{\text{unpol}}(1 + A_p).$$

Yule and Haeberli[8] included a small term for tensor-polarization effects. The present work employed a 12-MeV pure vector-polarized beam from the Birmingham University radial ridge cyclotron[9]; the experimental technique has been fully described elsewhere.[10]

The vector analyzing power has been determined for the following reactions: ^{9}Be$(d,p)^{10}$Be (ground state, $l_n = 1$), ^{12}C$(d,p)^{13}$C (ground state, $l_n = 1$), and ^{12}C$(d,p)^{13}$C* (3.09-MeV state, $l_n = 0$).