structure is consistent with the predictions of the many-body calculations of Mahan and Conley.[19]

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†Present address: Physics Department, Osaka University, Osaka, Japan.

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SEMICLASSICAL TRANSPORT THEORY IN STRONG MAGNETIC FIELDS

H. F. Budd
Groupe de Physique des Solides de l'Ecole Normale Supérieure, Paris, France
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In this Letter we present a new formulation of the theory of galvanomagnetic phenomena in strong magnetic fields, where $\omega_c\bar{\tau} \gg 1$. (ω_c is the cyclotron frequency and $\bar{\tau}$ is some average relaxation time.) The present work differs from the usual treatments[1,2] in that it is not limited to Ohmic conductivity (linear response in the electric field), nor does it depend on the existence of a relaxation time. We consider instead the asymptotic state of the system when $\omega_c\bar{\tau} \to \infty$ and seek to determine the corresponding asymptotic distribution function (ADF), which serves as the zero-order function in a perturbation theory expansion in $1/\omega_c\bar{\tau}$. Only the semiclassical theory will be presented in this Letter.

The semiclassical description of transport phenomena is based on the Boltzmann equation, which is generally insoluble except in instances where the effect of collisions may be represented by a relaxation time. When such simplification is not possible, numerical techniques are required to solve even the Ohmic transport problem. An example is provided by polar optical-phonon interactions, where variational methods have been employed.[3]

We first sketch our theory for the case of classical statistics (no exclusion principle) and then present the modifications for Fermi statistics. The general characteristics of the ADF are established and the special case of Ohmic conductivity is treated in detail, where explicit expressions are derived for the transport coefficients. The case of polar optical-phonon interactions is treated as an example and exact formulas for the conductivity are given.

The steady-state Boltzmann equation to be solved is

$$e[\vec{E}+\vec{v}\times\vec{B}]\cdot\nabla_{\vec{p}}f = \hat{C}f = \int d\vec{p}'[f(\vec{p}')T_{\vec{p}'\vec{p}} - f(\vec{p})T_{\vec{p}\vec{p}'}], \qquad (1)$$

where $\hat{C}$ is the usual collision operator, and the electric ($\vec{E}$) and magnetic ($\vec{B}$) fields are taken in the x and z directions, respectively. In order to calculate the ADF we introduce the path-variable transformation,[4] for which we must first calculate the collision-free particle trajectories:

$$d\vec{p}/dt = e[\vec{E} + \vec{v} \times \vec{B}].$$

The transformed Boltzmann equation then becomes

$$f(\vec{p}) = \int_0^\infty ds \int d\vec{p}' f(\vec{p}') T_{\vec{p}'\vec{p}(s)} \exp\left[-\int_0^s \frac{ds'}{\tau(s')}\right] \quad (2)$$

with

$$1/\tau(s) \equiv \int d\vec{p}'\, T_{\vec{p}(s)\vec{p}'},$$

where $\vec{p}(s), \tau(s)$ indicate the respective quantities evaluated along the trajectory.[4] We consider only the case of closed orbits and denote by T the period of the orbit. Equation (2) may then be reduced to an integral over one period:

$$f(\vec{p}) = \frac{\int_0^T ds \int d\vec{p}' f(\vec{p}') T_{\vec{p}'\vec{p}(s)} \exp[-\int_0^s ds'/\tau(s')]}{1 - \exp[-\int_0^T ds'/\tau(s')]}. \quad (3)$$

We now consider the limit $\omega_c \tau \gg 1$ $(\tau/T \gg 1)$, so that Eq. (3) becomes:

$$f(\vec{p}) \int_0^T ds \int d\vec{p}' T_{\vec{p}(s)\vec{p}'} = \int_0^T ds \int d\vec{p}' f(\vec{p}') T_{\vec{p}'\vec{p}(s)}. \quad (4)$$

The analogy between this result and the thermal equilibrium condition $\hat{C}f = 0$ is striking, the only difference being that the transition rates appearing in Eq. (4), which determines the ADF, are averaged over one period of the collision-free trajectories. Thus the ADF is independent of the absolute coupling constants describing the interaction with the scattering system and depends only on the form of the interaction and the applied fields.

It is readily verified that the ADF, determined by Eq. (4), also satisfies

$$e(\vec{E} + \vec{v} \times \vec{B}) \cdot \nabla_p f = 0, \quad (5)$$

from which it follows immediately that the average velocity $\bar{v}_x = 0$ and $\bar{v}_y = E/B \equiv v_d$. Equation (5) follows directly from Eq. (4) and the average velocity is obtained by multiplying Eq. (5) by $\vec{p}$, and integrating over $\vec{p}$ with an integration by parts.

We may now carry out a perturbation theory about this asymptotic state. Only the lowest order $O(1/\omega_c\tau)$ terms will be considered here, and we therefore set $f = f_{\mathrm{ADF}} + f_1$, where $f_1 \sim O(1/\omega_c\tau)$ and is determined by

$$e[\vec{E} + \vec{v} \times \vec{B}] \cdot \nabla_p f_1 = \hat{C} f_{\mathrm{ADF}}. \quad (6)$$

We are particularly interested in the dissipative current in the x direction and may calculate $\bar{v}_x$ directly by the same moment procedure used above:

$$\bar{v}_x = \int d\vec{p}\, p_y \hat{C} f_{\mathrm{ADF}}/eB. \quad (7)$$

Since by Eq. (5) f_{ADF} is constant along the trajectory, we may restate the condition given by Eq. (4) as

$$\int_0^T ds [\hat{C}f]_{\vec{p}(s)} = 0 \text{ with } f(\vec{p}) = f(\vec{p}(s)). \quad (8)$$

A similar calculation for the case of Fermi-Dirac statistics yields the identical results given in Eqs. (7) and (8) except that $\hat{C}$ is replaced by

$$\hat{C}f = \int d\vec{p}' \{f(\vec{p}')[1 - f(\vec{p})] T_{\vec{p}'\vec{p}} - f(\vec{p})[1 - f(\vec{p}')] T_{\vec{p},\vec{p}'}\}. \quad (9)$$

We note that the condition (8) assures the solubility of Eq. (6) and thus guarantees the consistency of our perturbation theory. While no special application to the non-Ohmic problem will be made here, we illustrate the character of the ADF for the case of acoustic-phonon interactions, where an approximate solution has been obtained in the diffusion approximation[5]:

$$f_{\mathrm{ADF}} = e^{-\epsilon/kT^*}\left[1 + \frac{p_y v_d}{kT^*}\right]; \qquad \frac{T^*}{T} = 1 + \left(\frac{1}{3}\frac{v_d}{C}\right)^2, \quad (10)$$

where T is the lattice temperature and C is the sound velocity. The coupling constants are totally absent, and only the sound velocity enters explicitly in the ADF.

We now treat the Ohmic problem, where we keep only linear terms in E. Consider the function $f_0(\epsilon - p_y v_d) \approx f_0(\epsilon) - p_y v_d \partial f_0/\partial\epsilon$, where ϵ is the electron energy and f_0 is the thermal equilibrium distribution. Since $\epsilon - p_y v_d$ is a constant of motion, this function is constant along the trajectory and Eq. (8) becomes

$$\int_0^T ds \left[\hat{C}\left(f_0 p_y v_d \frac{\partial f_0}{\partial \epsilon}\right)\right]_{\vec{p}_0(s)} = 0, \quad (11)$$

where $\vec{p}_0(s)$ is now the trajectory in the absence of the electric field, since the integrand is already linear in E. A wide class of scattering mechanisms satisfies this condition; in particular any system which is isotropic (or has even cylindrical symmetry about B) will satisfy this condition. Even this condition can be considerably relaxed, but we shall not consider it in detail here. In any event Eq. (11) by no means suggests the existence of a relaxation time, as can readily be seen for the case of polar modes, and we take it to be satisfied.[6]

We then have $f_{\rm ADF} = f_0 - p_y v_d (\partial f_0/\partial\epsilon)$ and from Eq. (7),

$$\bar{v}_x = v_d \int d\vec{p}\, p_y \hat{C} p_y f_0 / eBkT,$$

$$= \frac{eE}{2kT} \iint d\vec{p}\, d\vec{p}\,' \left(\frac{p_y{}' - p_y}{eB}\right)^2 f_0(p) T_{\vec{p}\vec{p}\,'}, \qquad (12)$$

for classical statistics.[7] The similarity of this result with the quantum-orbit jump picture[8] is striking. An analogous result for Fermi statistics is also readily obtained, the only difference being the appearance of the exclusion factor $[1-f_0(p')]$ in Eq. (12).

We now calculate the conductivity for polar optical-phonon interactions, where no previous solutions of the Boltzmann equation have been obtained. Assuming a constant effective mass m and introducing the usual[9] interaction matrix elements into the transition rates $T_{pp'}$ in Eq. (12), we obtain for classical statistics

$$\bar{v}_x = \frac{4EE_0 N \delta e^{\frac{1}{2}\delta} K_1(\frac{1}{2}\delta)}{3B^2 [2\pi kT/m]^{\frac{1}{2}}};$$

$$N = \frac{1}{e^{\delta} - 1}, \quad \delta = \frac{\hbar\omega_0}{kT}, \qquad (13)$$

where $\hbar\omega_0$ is the optical-phonon energy, E_0 is related to the coupling constants,[9] and K_1 is the first-order modified Bessel function. This result is identical to the classical limit of the Kubo formula, which has been evaluated by Green's function techniques.[10] The corresponding expression for Fermi statistics will not be given here.

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