tion of M_1^+ across the resonance which was consistant with the behavior expected from the $P_{3/2,3/2}$ phase shift. However, at the two interintermediate points a more consistent behavior of M_1^+ was obtained when no correction was made for this term. We have therefore used one-half the correction at all four-momentum transfers and increased the errors appropriately.

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DOLEN-HORN-SCHMID DUALITY AND THE DECK EFFECT*

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An extension to multiperipheralism is made of the Dolen-Horn-Schmid duality argument relating Regge poles to low-energy resonances. The Deck model is thereby interpreted as predicting the existence of the A_1, rather than as undermining experimental evidence for this resonance. It is shown in general that Dolen-Horn-Schmid duality permits a vast simplification in the calculation of multiple-production processes.

A remark of profound import for strong-interaction theory has been made by Dolen, Horn, and Schmid[1] in connection with finite-energy sum rules. They have observed that high-energy Regge behavior is consistent with low-energy resonance behavior only if extrapolation of the smooth Regge representation down to low energy gives a certain semilocal average over the resonance peaks. In other words what is usually called the "peripheral" approximation to a reaction amplitude must, without containing energy poles, in a rough sense represent the resonances. (The converse presumably is also true.) We refer to this startling notion as "Dolen-Horn-Schmid duality." Its implication for bootstrap theory is being pursued vigorously by many authors[2]; our object here is to suggest relevance to what has been called the "Deck effect."[3] We argue that the Deck peripheral model for a reaction such as $\pi N \to \rho\pi N$, explaining a peak in the final $\pi\rho$ mass spectrum without explicit insertion therein of a resonance, fails to imply the absence of a resonance. On the contrary, Dolen-Horn-Schmid duality means that when peripheral models of this kind predict large cross sections at low subenergies (the term "subenergy" is used to mean energy of a subsystem), there probably are resonances present. Such reasoning leads to enormous simplification of multiperipheral calculations.

The step needed to relate Dolen-Horn-Schmid to Deck is the extension of single peripheralism to double peripheralism. Deck's model for the above reaction, for example, is depicted in Fig. 1, corresponding to a double Regge-pole representation,[4] a representation supposed to have validity when both the πN and $\pi\rho$ final subenergies are large.[5] The highest trajectory for the right-hand momentum transfer is the Pomeranchuk; the highest for the left-hand momentum transfer is not the π, but the small mass of the physical pion enhances the Regge residue so that this tra-

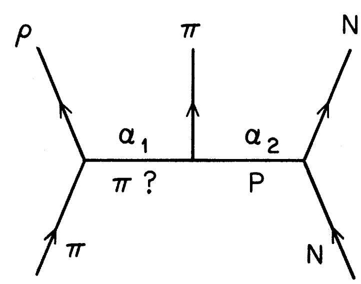

FIG. 1. Diagram representing the Deck doubly peripheral model for the reaction $\pi N \to \pi\rho N$.

jectory may well dominate at moderate energies. It will be seen that for our purposes here it does not matter if other trajectories play a significant role. The essential and almost trivial remark of this note is that it is possible to keep fixed all members of a complete set of variables except the $\pi\rho$ subenergy, thereby reducing to a singly-peripheral description, and to repeat the Dolen-Horn-Schmid reasoning. Thus if the Deck model is accurate for large values of the $\pi\rho$ subenergy, consistency considerations require the model to yield a semilocal average description of the cross section at low values of this subenergy even in the presence of resonances.

The above argument has been overlooked because in the experiment analyzed by Deck the $\pi\rho$ subenergy is varied, not by varying the incident (total) energy at fixed πN mass, but by varying the latter at fixed incident energy. However we shall show it possible to pass from the one type of variation to the other through the independence of appropriate variables[6] and the factorization attendant on simultaneous Regge expansions in both subenergies.

We begin by fixing both momentum transfers in Fig. 1 as well as the Toller angle of rotation about the internal vertex,[6] and noting that there is a functional relationship between the square of the total energy s, and the two subenergies, $s_{\pi\rho}$ and $s_{\pi N}$, a relation which can be inverted to express $s_{\pi N}$ in terms of s and $s_{\pi\rho}$. Now the dependence of the amplitude on $(s_{\pi\rho}, s_{\pi N})$ is assumed to be factorizable,

$$A(s_{\pi\rho}, s_{\pi N}) \sim g_{\pi\rho}(s_{\pi\rho}) g_{\pi N}(s_{\pi N}), \tag{1}$$

as either $s_{\pi\rho}$ or $s_{\pi N}$ becomes large, with each factor having Regge asymptotic behavior:

$$g_{\pi\rho}(s_{\pi\rho}) \sim C_{\pi\rho} s_{\pi\rho}^{\alpha_1}, \tag{2a}$$

$$g_{\pi N}(s_{\pi N}) \sim C_{\pi N} s_{\pi N}^{\alpha_2}. \tag{2b}$$

Let us identify particular finite values of $s_{\pi\rho}$ and and of $s_{\pi N}$, say $s_{\pi\rho} = N_{\pi\rho}$ and $s_{\pi N} = N_{\pi N}$, such that above these values Formulas (2a) and (2b) become an acceptably accurate approximation. Then, keeping $s_{\pi N}$ fixed at a value greater than $N_{\pi N}$, the Dolen-Horn-Schmid line of reasoning[1] leads to the conclusion that a certain average of $g_{\pi\rho}(s_{\pi\rho})$ over the range of $s_{\pi\rho}$ below $N_{\pi\rho}$ will be given correctly by Formula (2a).

For the required application it is necessary to keep s rather than $s_{\pi N}$ fixed, but from Formulas (1) and (2) we now explicitly calculate the resulting modification. Let us suppose s sufficiently large that $s_{\pi N}$ lies above $N_{\pi N}$ for all $s_{\pi\rho}$ below $N_{\pi\rho}$; then

$$A(s_{\pi\rho}, s) \underset{s\ \text{large}}{\sim} g_{\pi\rho}(s_{\pi\rho}) C_{\pi N} \times [s_{\pi N}(s, s_{\pi\rho})]^{\alpha_2}. \tag{3}$$

It follows that a modified amplitude defined by

$$\bar{A}(s_{\pi\rho}, s) \equiv [s_{\pi N}(s, s_{\pi\rho})]^{-\alpha_2} A(s_{\pi}, s) \tag{4}$$

exhibits the Dolen-Horn-Schmid phenomenon when averaged over low $s_{\pi\rho}$ at fixed (large) s. Since the extra factor in Formula (4) is positive definite and smoothly varying, we conclude that an average of $A(s_{\pi\rho}, s)$ itself over the low $s_{\pi\rho}$ region with s fixed at a large value is correctly given by the double-Regge representation. This is the desired result.

Were it required to replace the single-pole Formula (2b) by a sum over several poles, the single residue function $g_{\pi\rho}(s_{\pi\rho})$ would be replaced by a corresponding collection of residues, but for each of these separately the Dolen-Horn-Schmid average would apply. Evidently, by reversing the roles of $s_{\pi\rho}$ and $s_{\pi N}$ in the above argument we could show that low values of $s_{\pi N}$ also are correctly described in an average sense.

Since for singly peripheral models the prediction of large low-energy cross sections corresponds to the presence of resonances,[7] the same is likely for multiply-peripheral models. Thus, Deck's calculation[3] might be described as a prediction of the A_1! What is the source of large low-energy cross sections in a peripheral representation? It turns out to be the same in both singly peripheral and multiply peripheral situations. The Pomeranchuk trajectory has a small residue and produces no large low-energy cross section. It is lower lying trajectories with big residues that are responsible.[7] Residues turn out to be generally small except when magnified by nearby poles corresponding to low-mass particles on the trajectory. Thus a large low-energy peripheral cross section typically accompanies "exchanges" involving low-mass particles, exchanges which may be identified with the familiar "Yukawa forces." Dolen-Horn-Schmid duality at this point acquires a familiar dynamical interpretation because it is seen to predict resonances in precisely those situations where a preponderance of strong and attractive longrange forces occurs.[8] Note, however, that a nonrelativistic potential model does not correlate the input force and the output resonance in the direct fashion of Dolen, Horn, and Schmid. Their form of duality is an essentially relativistic phenomenon.

If the Deck model is to be regarded as giving an average description of the A_1 and other low-lying resonances decaying into $\pi\rho$, it might be expected that the predicted low $\pi\rho$ mass spectrum persists in its general form no matter how large the total reaction energy s. Such is in fact a feature of the doubly peripheral model. Using the results of Ref. 6 and integrating over all variables except the $\pi\rho$ total energy, one finds the fixed-s asymptotic spectrum

$$d\sigma \sim s_{\pi\rho}^{-2[\alpha_P(0)-\alpha_\pi(0)]} d(\ln s_{\pi\rho}), \tag{5}$$

or, setting $\alpha_P(0)=1$ and $\alpha_\pi(0)=0$,

$$d\sigma/d(\ln s_{\pi\rho}) \sim s_{\pi\rho}^{-2}. \tag{6}$$

Extending this spectrum right down to the $\pi\rho$ threshold, we may calculate the average $\pi\rho$ mass to be

$$\langle\sqrt{s_{\pi\rho}}\rangle \approx \tfrac{4}{3}(m_\pi + m_\rho) = 1200 \text{ MeV}.$$

Thus at fixed s the A_1 and A_2 are expected to dominate the $\pi\rho$ spectrum no matter how large s may be. Deck[3] found a sharpening of the "average $\pi\rho$ mass spectrum" with decreasing s due to dependence of the momentum-transfer lower limits on the $\pi\rho$ mass. Nevertheless, Formulas (5) and (6) show that when such transient phenomena have died away at very high s, there will remain a tendency for the $\pi\rho$ spectrum to concentrate near the A_1.[9]

Dolen-Horn-Schmid duality leads to an enormous simplification of multiperipheral calculations: To compute integrated cross sections, one need consider only final particles of low mass and can be guided in the choice of trajectories by experience with singly-peripheral phenomena at modest energies. Already in the Deck example we see how the A_1, A_2, etc., may be ignored in favor of π and ρ, but even the final ρ might be ignored if we replaced the doubly peripheral Fig. 1 with the triply peripheral Fig. 2. This would constitute a less accurate approximation than Fig. 1 when the $\pi_1\pi_2$ mass is near the ρ, but Fig. 2 roughly includes all the higher resonances that decay into $\pi_1\pi_2$.

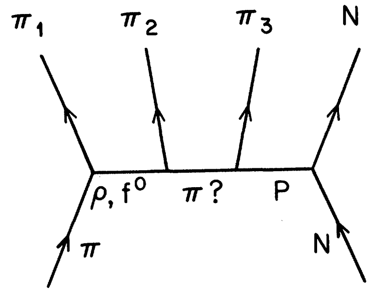

FIG. 2. Diagram representing a triply-peripheral representation for the reaction $\pi N \rightarrow 3\pi N$.

Dolen-Horn-Schmid duality thus opens the door to a simple description of high-energy multiple production if the detailed structure of final-particle spectra is not an issue. Questions such as total cross sections, multiplicity, and even the gross aspects of fireball structure become far more tractable than might have been imagined in the presence of an apparently unlimited spectrum of resonances.

The authors are indebted to C. Schmid for many illuminating conversations.

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[8]The forces must be "attractive" in elastic reactions if they are to augment rather than diminish the Pomeranchuk contribution which necessarily dominates at

very high energies. For inelastic reactions all interactions are well known to be effectively attractive.

[9]The model does not discriminate sharply between different resonances, giving only an average over them. The Deck width narrowing with decreasing total energy is to be interpreted as a decreasing role for resonancces lying above the A_1 because of phase-space limitation. Note that there is no reason for the width yielded by the model ever to be as narrow as the actual A_1 width.

$K_L{}^0 \to \mu^+\mu^-$ DECAY, $K_L{}^0$-$K_S{}^0$ MASS DIFFERENCE, AND WEAK-INTERACTION CUTOFF*

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A major problem in the theory of weak interactions is whether the universal (V,A) current-current interaction represents effectively a phenomenological description of lowest order weak processes or can be used to calculate higher order processes in a self-consistent way. The conservation of muon lepton number prevents this test of weak-interaction theory in the domain of purely leptonic processes[1] where the strong interaction is absent. One must therefore turn to semileptonic and nonleptonic weak processes for tests of the higher order theoretical predictions and attempt to isolate effects which are essentially independent of the strong interaction. Two second-order processes suitable for investigation in this respect are the decay $K_L{}^0 \to \mu^+\mu^-$ and the $K_L{}^0$-$K_S{}^0$ mass difference; in this Letter we report our main results.

The diagrams contributing to the decay $K_L{}^0 \to \mu^+\mu^-$ are given in Fig. 1. Figure 1(a) is first order in the weak and second order in the electromagnetic interaction and has been considered by Bég,[2] who finds this contribution to the branching ratio α $[\alpha \equiv (K_L{}^0 \to \mu^+\mu^-)/(K\to\mu\nu_\mu)]$ to be about 1% of the present experimental upper limit[3] ($\lesssim 1.6\times10^{-6}$). We have repeated Bég's calculation with the techniques of current algebra and reproduce his result to within a factor of 2.

Figure 1(b) is second order in the weak interaction on the basis of the intermediate-vector-boson (IVB) model and has been calculated by Ioffe[4] who finds that the most divergent term is independent of the strong interaction (and of the IVB mass); comparing this result with the experimental upper limit, Ioffe finds that the weak-interaction cutoff $\Lambda \lesssim 100$ BeV.

Figure 1(c) is the contribution in second order of the universal current-current interaction (UFI) theory to the decay $K_L{}^0(p) \to \mu^+(p_1)+\mu^-(p_2)$; we write for the matrix element the following:

$$\langle \mu^+(p_1)\mu^-(p_2)|K_L{}^0\rangle = (2\pi)^4\delta^4(p-p_1-p_2)\mathfrak{M},$$

$$\mathfrak{M} = \tfrac{1}{2}G^2\sin\theta\cos\theta\left(\frac{m_\mu{}^2}{2p_0p_{10}p_{20}V^3}\right)^{1/2}\int\frac{d^4q}{(2\pi)^4}\bar{u}(p_1)\gamma_\lambda(\not{q}+\not{p}_1)^{-1}\gamma_\nu(1+\gamma_5)v(p_2)M_{\lambda\nu}(q,p), \tag{1}$$

where

$$M_{\lambda\nu}(q,p) = i(2p_0V)^{\frac{1}{2}}\int d^4x\, e^{iq\cdot x}\langle 0|T\{J_{\lambda2}{}^1(x)J_{\nu1}{}^3(0)\}|\bar{K}_0(p)\rangle, \tag{2}$$

and $J_{\lambda2}{}^1$ and $J_{\lambda1}{}^3$ are the hypercharge-conserving and hypercharge-changing currents, respectively; θ is the Cabibbo angle.

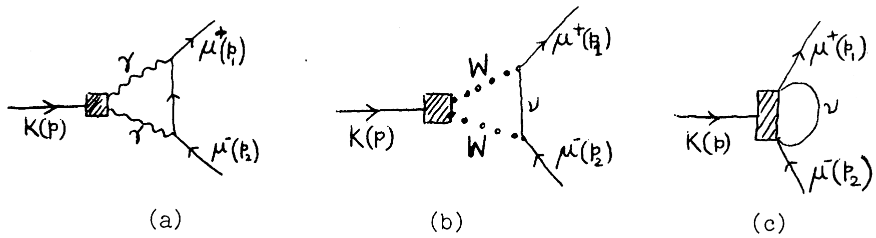

FIG. 1. Diagrams contributing to the decay.